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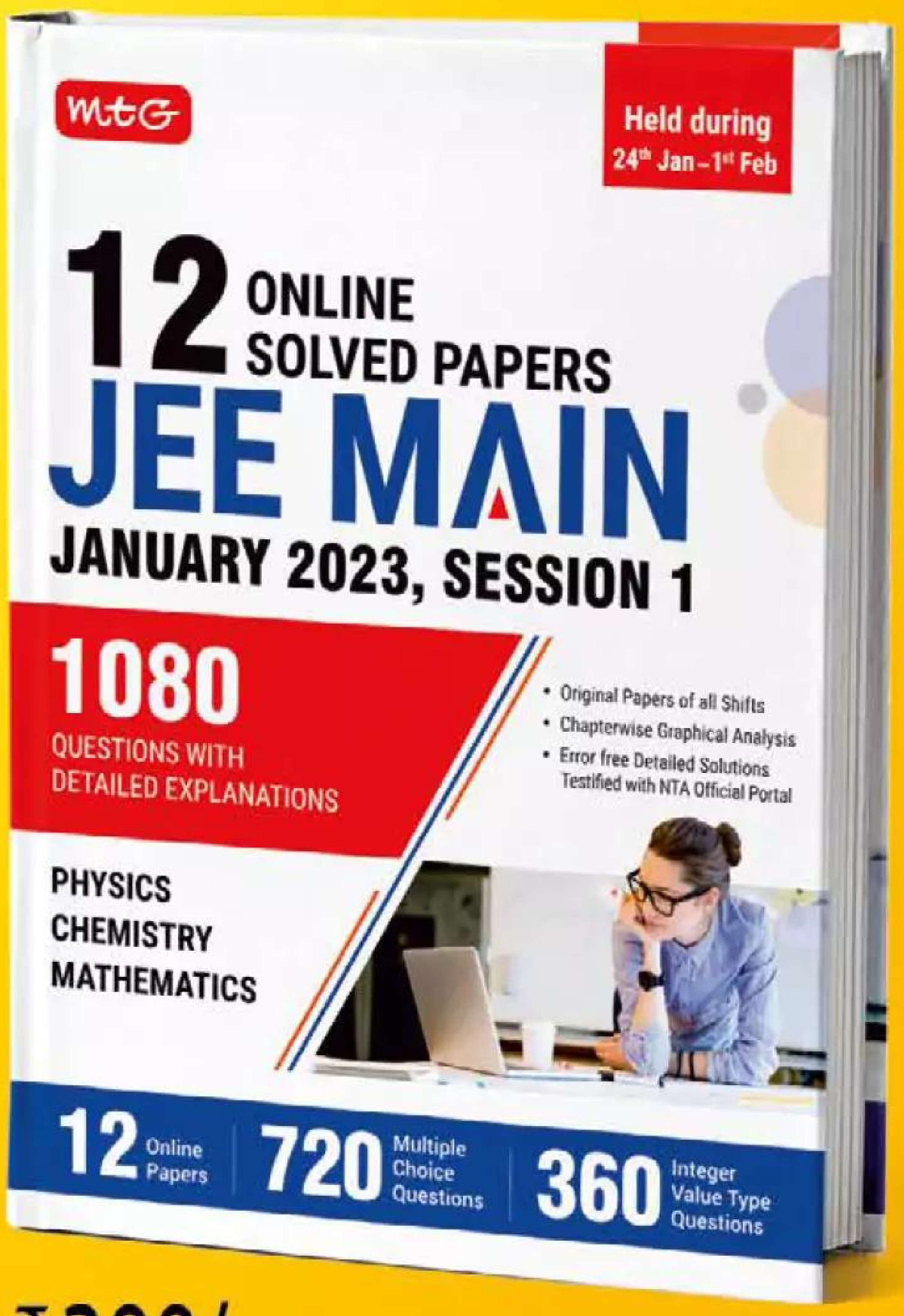
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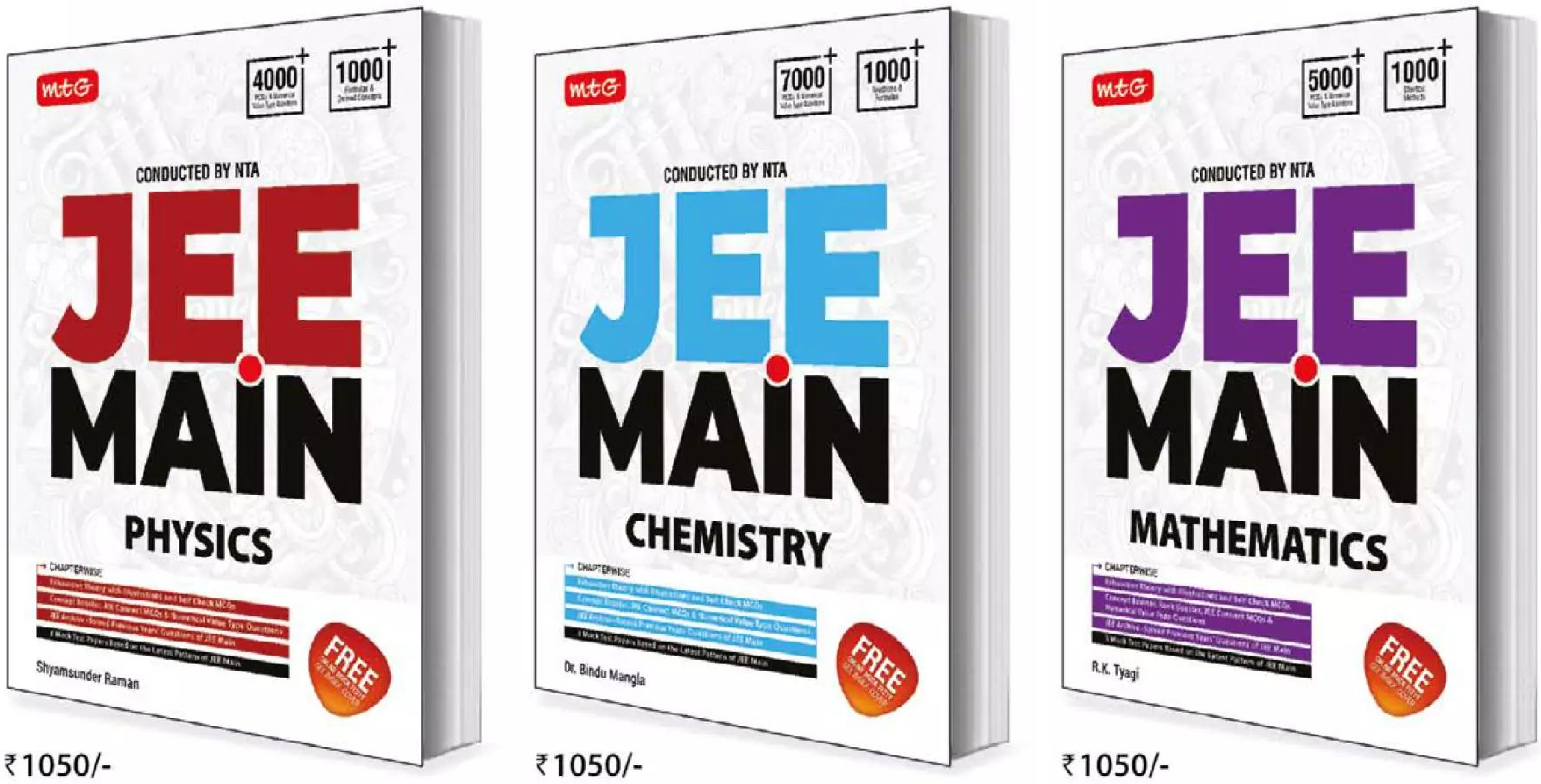
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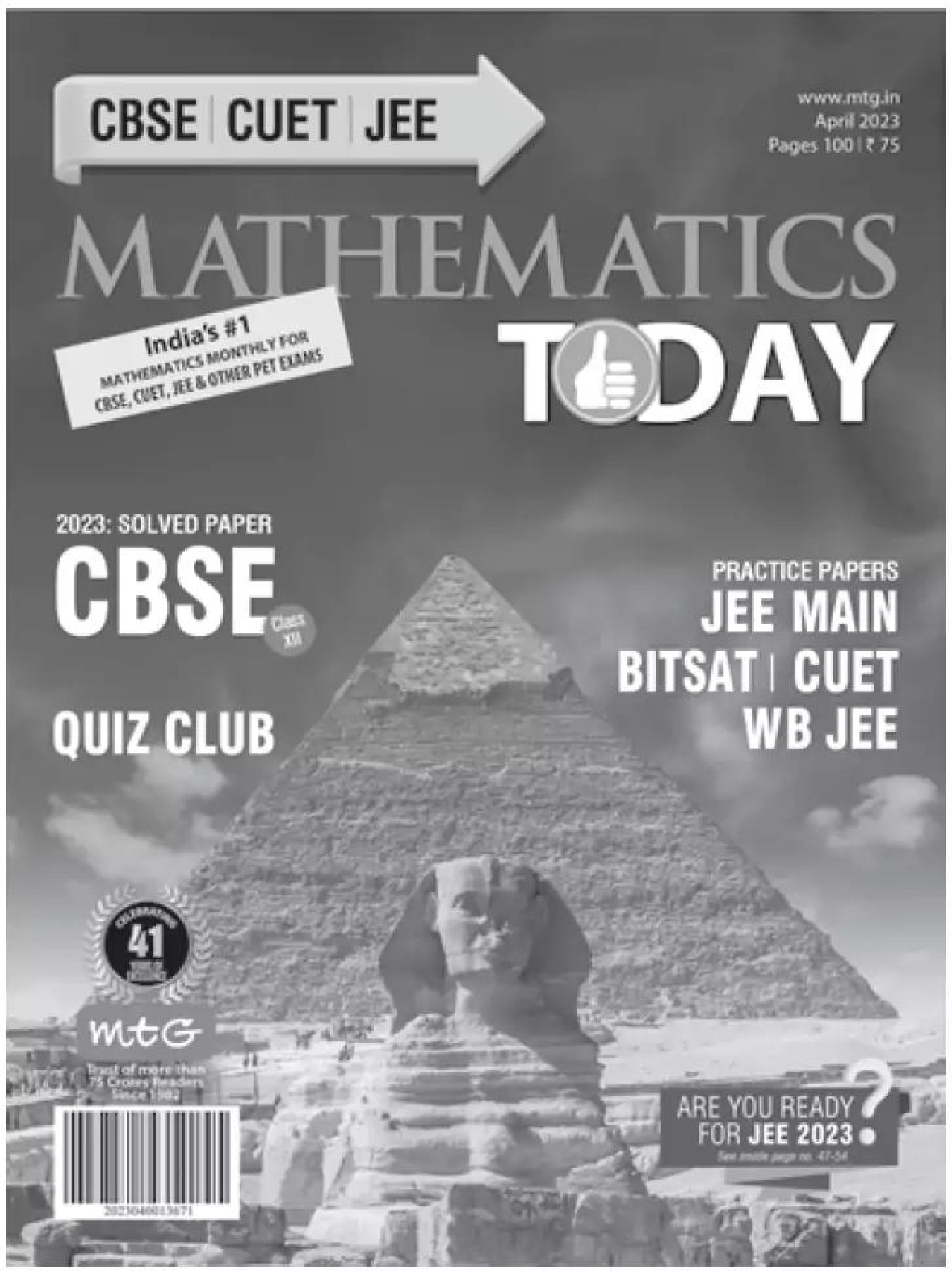
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MATHEMATICS today

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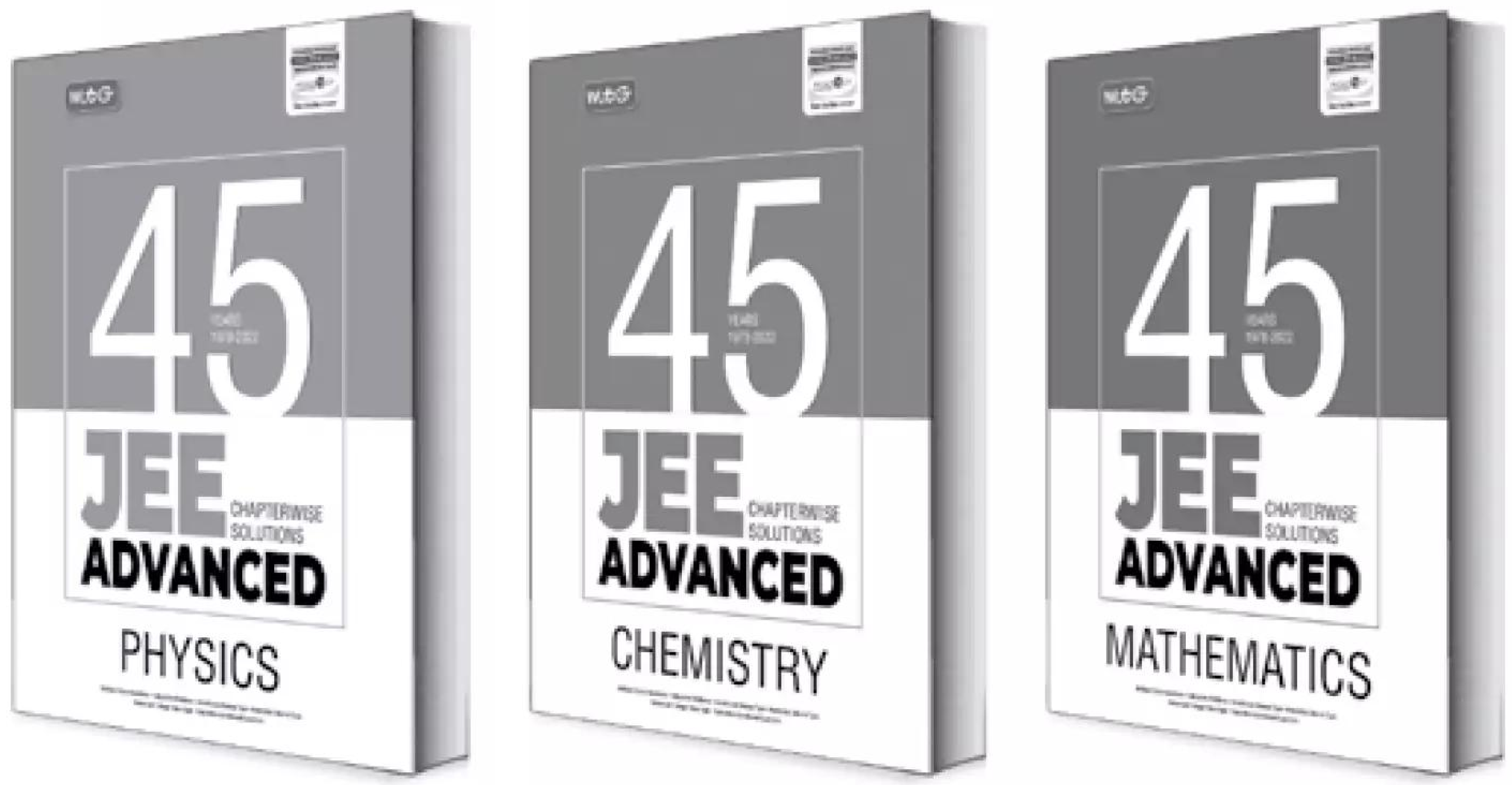
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SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. The value of $\sum_{r=0}^{n}(r)\,{}^{2n}C_r$ is equal to

(a) $2n(2^{2n-2}+{}^{2n-1}C_{n-1})$
(b) $n(2^{2n-2}+{}^{2n-1}C_{n})$
(c) $2n(2^{2n-2}+{}^{2n-1}C_{n+1})$
(d) none of these

2. Total number of ways, in which 22 different books can be given to 5 students, so that two students get 5 books each and all the remaining students get 4 books each, is equal to

(a) $\frac{22!}{3!2!5!(4!)^3}$ (b) $\frac{22!}{(3!)^2\cdot 2!\cdot 5!}$
(c) $\frac{22!}{3!\,2!\,5!4!}$ (d) none of these

3. Let $\vec{a},\vec{b},\vec{c}$ be three unit vectors such that $\vec{a}+5\vec{b}+3\vec{c}=\vec{0}$, then $\vec{a}\cdot(\vec{b}\times\vec{c})$ is equal to

(a) $\vec{a}\cdot\vec{b}$ (b) $\vec{a}\cdot(\vec{b}+2\vec{c})$
(c) $\vec{b}\cdot(\vec{a}+\vec{c})$ (d) none of these

4. Consider the set of integers {10, 11, 12, ..., 98, 99}. By seeing the number a person *A* will laugh if the product of the digits of the number is 12. He chooses three numbers one at a time from this set of integers, randomly and with replacement. The probability that he will laugh atleast once, is equal to

(a) $1-\left(\frac{43}{45}\right)^3$ (b) $1-\left(\frac{3}{5}\right)^3$
(c) $1-\left(\frac{4}{25}\right)^3$ (d) none of these

5. If A and B are square matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2$ is always equal to

(a) $2AB$ (b) $2BA$ (c) $A + B$ (d) AB

6. The number of complex numbers z such that $|z - 1| = |z + 1| = |z - i|$ equals

(a) 0 (b) 1 (c) 2 (d) ∞

7. $\lim_{x\to\pi/2}\frac{[1-\tan(x/2)][1-\sin x]}{[1+\tan(x/2)][\pi-2x]^3}=$

(a) 0 (b) 1/32 (c) ∞ (d) 1/8

8. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in

(a) $\left(0,\frac{\pi}{2}\right)$ (b) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$
(c) $\left(\frac{\pi}{4},\frac{\pi}{2}\right)$ (d) $\left(-\frac{\pi}{2},\frac{\pi}{4}\right)$

9. The range of the function $F(x) = {}^{7-x}P_{x-3}$ is

(a) {1, 2, 3, 4} (b) {1, 2, 3, 4, 5, 6}
(c) {1, 2, 3} (d) {1, 2, 3, 4, 5}

10. The integral $\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx$ equals

(a) $\frac{2\pi}{3}-4-4\sqrt{3}$ (b) $4\sqrt{3}-4$
(c) $4\sqrt{3}-4-\frac{\pi}{3}$ (d) $\pi - 4$

11. The differential equation of all circles passing through the origin and having their centres on the x-axis is

(a) $y^2=x^2+2xy\frac{dy}{dx}$ (b) $y^2=x^2-2xy\frac{dy}{dx}$
(c) $x^2=y^2+xy\frac{dy}{dx}$ (d) $x^2=y^2+3xy\frac{dy}{dx}$

12. Minimum area of the triangle by any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with the coordinate axes is

(a) $\frac{a^2+b^2}{2}$ (b) $\frac{(a+b)^2}{2}$
(c) ab (d) $\frac{(a-b)^2}{2}$

13. A line is drawn through a fixed point $P(\alpha, \beta)$ to cut the circle $x^2 + y^2 = r^2$ at A and B. Then $PA \cdot PB$ is equal to

(a) $(\alpha + \beta)^2 - r^2$ (b) $\alpha^2 + \beta^2 - r^2$
(c) $(\alpha - \beta)^2 + r^2$ (d) None of these

14. If $f(x) : [1, 10] \rightarrow Q$ be a continuous function. If $f(x)$ takes rational value for all x and $f(2) = 5$, then the equation whose roots are $f(3)$ and $f(\sqrt{10})$ is

(a) $x^2 - 10x + 25 = 0$ (b) $x^2 - 3x + 2 = 0$
(c) $x^2 - 6x + 5 = 0$ (d) None of these

15. If a chord joining $P(a \sec \theta, a \tan \theta)$, $Q(a \sec \alpha, a \tan \alpha)$ on the hyperbola $x^2 - y^2 = a^2$ is normal at P, then $\tan \alpha =$

(a) $\tan \theta\,(4 \sec^2 \theta + 1)$ (b) $\tan \theta\,(4 \sec^2 \theta - 1)$
(c) $\tan \theta\,(2 \sec^2 \theta - 1)$ (d) $\tan \theta\,(1 - 2 \sec^2 \theta)$

16. The lengths of two opposite edges of a tetrahedron are a and b. Their shortest distance is d and the angle between them is θ. Then its volume is

(a) $\frac{1}{2}abd\sin\theta$ (b) $\frac{1}{3}abd\cos\theta$
(c) $\frac{1}{6}abd\sin\theta$ (d) $\frac{1}{6}abd\cos\theta$

17. The mean of 6 distinct observations is 6.5 and their variance is 10.25. If 4 out of 6 observations are 2, 4, 5 and 7, then the remaining two observations are

(a) 8, 13 (b) 1, 20 (c) 10, 11 (d) 3, 18

18. Let a be a positive real number such that $\int_0^a e^{x-[x]}dx = 10e - 9$, where $[x]$ is the greatest integer less than or equal to x. Then a is equal to

(a) $10 + \log_e 2$ (b) $10 + \log_e 3$
(c) $10 + \log_e(1 + e)$ (d) $10 - \log_e(1 + e)$

19. Words with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the letter M appears at the fourth position in any such word is

(a) $\frac{2}{11}$ (b) $\frac{1}{11}$ (c) $\frac{1}{9}$ (d) $\frac{1}{66}$

20. The Boolean expression $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$ is equivalent to

(a) $\sim q \Rightarrow p$ (b) $p \Rightarrow \sim q$ (c) $p \Rightarrow q$ (d) $q \Rightarrow p$

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

21. If the shortest distance between the lines $\vec{r}_1 = \alpha\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k}), \lambda \in R, \alpha > 0$ and $\vec{r}_2 = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k}), \mu \in R$ is 9, then, α is equal to ______.

22. Let a, b, c, d be in arithmetic progression with common difference λ. If $\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix} = 2$, then value of λ^2 is equal to ______.

23. Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ and $B = 7A^{20} - 20A^7 + 2I$, where I is an identity matrix of order 3×3. If $B = [b_{ij}]$, then b_{13} is equal to _____.

24. The number of three-digit even numbers, formed by the digits 0, 1, 3, 4, 6, 7 if the repetition of digits is not allowed, is ________.

25. If $y = y(x)$ is an implicit function of x such that $\log_e(x + y) = 4xy$, then $\frac{d^2y}{dx^2}$ at $x = 0$ is equal to _______.

26. If ${}^1P_1 + 2 \cdot {}^2P_2 + 3\ {}^3P_3 + \cdots + 15 \cdot {}^{15}P_{15} = {}^qP_r - s$' $0 \le s \le 1$, then ${}^{q+s}C_{r-s}$ is equal to ______.

27. The area of the region $S = \{(x, y) : 3x^2 \le 4y \le 6x + 24\}$ is _______.

28. The area (in sq. units) of the region bounded by the parabola, $y = x^2 + 2$ and the lines, $y = x + 1$, $x = 0$ and $x = 3$, is $\lambda/2$ sq. units, then value of λ is ________.

29. Let $S_k = \frac{1+2+3+\ldots+k}{k}$. If $S_1^2 + S_2^2 + \ldots + S_{10}^2 = \frac{5}{12}A$, then A is equal to ________.

30. Let $P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ and $Q = [q_{ij}]$ be two 3×3 matrices such that $Q - P^5 = I_3$. Then $\frac{q_{21}+q_{31}}{q_{32}}$ is equal to ________.

SOLUTIONS

1. (d): We have, $\sum_{r=0}^{n} r\cdot{}^{2n}C_r = \sum_{r=1}^{n} 2n\cdot{}^{2n-1}C_{r-1}$

$= 2n({}^{2n-1}C_0 + {}^{2n-1}C_1 + ... + {}^{2n-1}C_{n-1})$

$= 2n(\frac{1}{2}(2^{2n-1})) = 2n(2^{2n-2}) = n \cdot 2^{2n-1}$

2. (a): It is same as dividing 22 books in 5 packets such that two packets have 5 books each and remaining three packets have four books each and then distributing these packets.

Thus, total ways of distributing these books

$$=\frac{22!}{(5!)^2\cdot(4!)^3\cdot 2!3!}.(5!)=\frac{22!}{2!3!5!(4!)^3}$$

3. (d): $\vec{a}+5\vec{b}+3\vec{c}=\vec{0}$

Thus $\vec{a}, \vec{b}, \vec{c}$ are coplanar. Hence $\vec{a}\cdot(\vec{b}\times\vec{c})=0$

4. (a): Person will laugh if the selected number is 43 or 34 or 62 or 26.

Thus, probability that person will not laugh on seeing the number $=\frac{86}{90}=\frac{43}{45}$

Thus, required probability $= 1-\left(\frac{43}{45}\right)^3$

5. (c): Given, $AB = B$, $BA = A$

Now, $A^2 + B^2 = AA + BB$

$= A(BA) + B(AB) = (AB)A + (BA)B$

$= BA + AB = A + B$

6. (b): We have, $|z - 1| = |z + 1|$

$\Rightarrow |z-1|^2 = |z+1|^2 \Rightarrow (z-1)(\bar{z}-1)=(z+1)(\bar{z}+1)$

$\Rightarrow z\bar{z}-z-\bar{z}+1=z\bar{z}+z+\bar{z}+1$

$\Rightarrow z+\bar{z}=0$ (z being purely imaginary)

Thus, $x = 0$

Again, $|z-1|^2 = |z-i|^2 \Rightarrow (x-1)^2 + y^2 = x^2 + (y-1)^2$

$\Rightarrow 1 + y^2 = (y-1)^2$ $(\because x = 0)$

$\Rightarrow 1 + y^2 = y^2 - 2y + 1$

$\therefore y = 0$

Thus, (0, 0) is the desired point.

7. (b): $$\lim_{x\to\pi/2}\frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)(1-\sin x)}{4\cdot\left(\frac{\pi-2x}{4}\right)(\pi-2x)^2}$$

$$=\lim_{x\to\pi/2}\frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{4\cdot\left(\frac{\pi}{4}-\frac{x}{2}\right)}\cdot\frac{1-\cos\left(\frac{\pi}{2}-x\right)}{(\pi-2x)^2}$$

$$=\lim_{x\to\pi/2}\frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{4\cdot\left(\frac{\pi}{4}-\frac{x}{2}\right)}\frac{2\sin^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}{4^2\left(\frac{\pi-2x}{4}\right)^2}=\frac{1}{4}\times\frac{2}{16}=\frac{1}{32}$$

8. (d): $f'(x)=\frac{1}{1+(\sin x+\cos x)^2}.(\cos x-\sin x)$

$\Rightarrow f'(x)=\frac{\cos x-\sin x}{2+\sin 2x}$

If $f'(x) > 0$ then $f(x)$ is an increasing function.

For $-\frac{\pi}{2}<x<\frac{\pi}{4}, \cos x>\sin x$

Hence, $y=f(x)$ is increasing in $\left(-\frac{\pi}{2},\frac{\pi}{4}\right)$.

9. (c): $F(x)$ to be defined for $x \in N$.

(i) $\therefore 7 - x > 0 \Rightarrow x < 7$

(ii) $x - 3 \ge 0 \Rightarrow x \ge 3$

(iii) $x - 3 \le 7 - x \Rightarrow x \le 5$

$\therefore$ From (i), (ii) and (iii), $x = 3, 4, 5$

$\therefore F(3) = {}^4P_0, F(4) = {}^3P_1, F(5) = {}^2P_2$

$\therefore$ {1, 2, 3} is required range.

10. (c): Let $I=\int_0^{\pi}\sqrt{1+4\sin^2\frac{x}{2}-4\sin\frac{x}{2}}\,dx$

$$=\int_0^{\pi}\sqrt{\left(1-2\sin\frac{x}{2}\right)^2}\,dx=\int_0^{\pi}\left|1-2\sin\frac{x}{2}\right|dx$$

$$=\int_0^{\pi/3}\left(1-2\sin\frac{x}{2}\right)dx+\int_{\pi/3}^{\pi}\left(2\sin\frac{x}{2}-1\right)dx$$

$$=\left[\left(x+4\cos\frac{x}{2}\right)\right]_0^{\pi/3}+\left[\left(-4\cos\frac{x}{2}-x\right)\right]_{\pi/3}^{\pi}$$

$$=-\frac{\pi}{3}+8\cdot\frac{\sqrt{3}}{2}-4=4\sqrt{3}-4-\frac{\pi}{3}$$

11. (a): General equation of all such circles is

$(x-h)^2 + (y-0)^2 = h^2$ ....(i) where h is parameter

$\Rightarrow (x-h)^2 + y^2 = h^2$

Differentiating w.r.t. x, we get

$2(x-h)+2y\frac{dy}{dx}=0 \Rightarrow h=x+y\frac{dy}{dx}$

Putting the value of h in (i), we get

$y^2=x^2+2xy\frac{dy}{dx}$

12. (c) : Equation of tangent at $(a\cos\theta, b\sin\theta)$ is

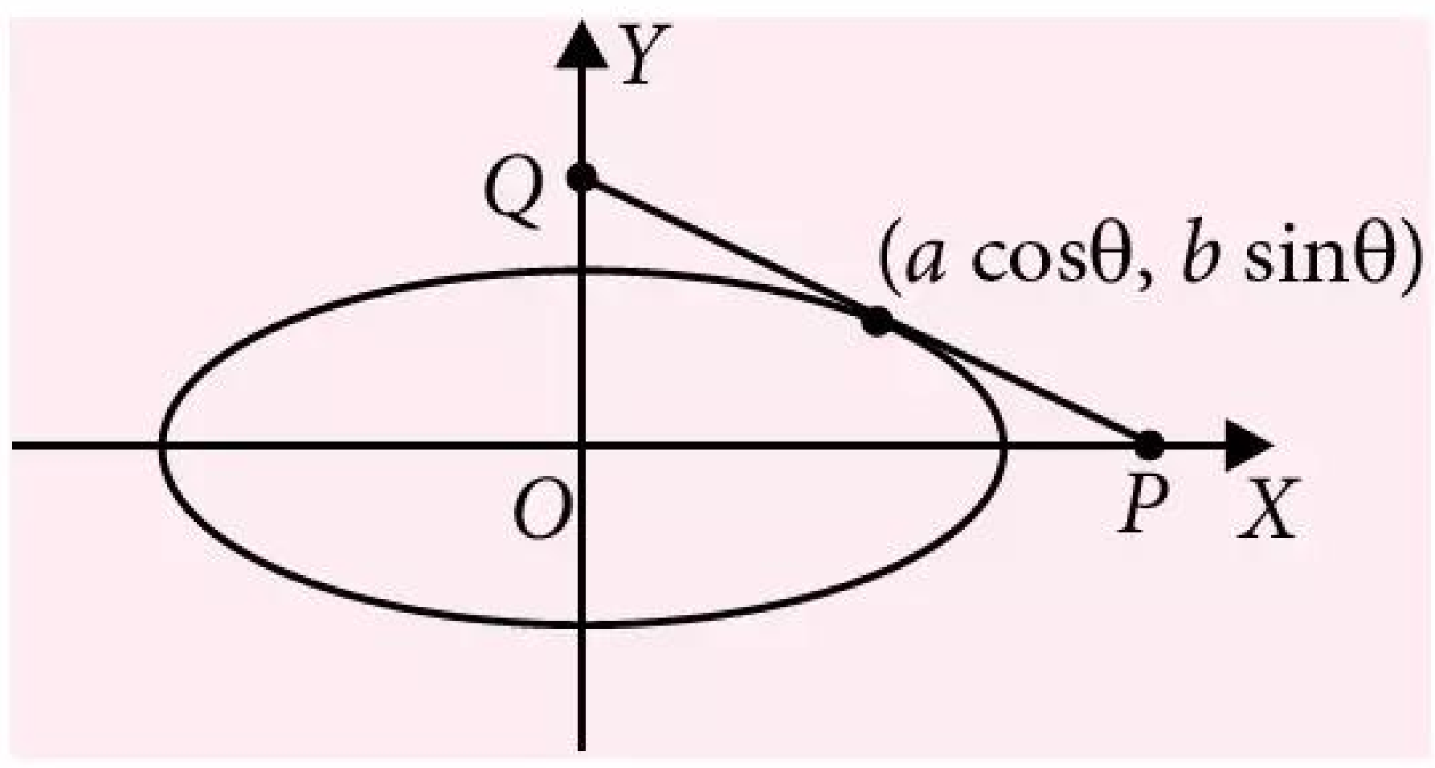
$$\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$$

$$P=\left(\frac{a}{\cos\theta},0\right)$$

$$Q=\left(0,\frac{b}{\sin\theta}\right)$$

Area of $\Delta OPQ=\frac{1}{2}\left|\left(\frac{a}{\cos\theta}\right)\left(\frac{b}{\sin\theta}\right)\right|=\frac{ab}{|\sin 2\theta|}$

$\therefore$ $(\text{Area})_{\min}=ab.$



13. (b): Let the equation of line through the point (α, β) be $\frac{x-\alpha}{\cos\theta}=\frac{y-\beta}{\sin\theta}=k$ (say) ...(i)

where k is the distance of any point (x, y) on the line from the point $P(\alpha, \beta)$. Let this line meets the circle $x^2+y^2=r^2$ at $(\alpha+k\cos\theta, \beta+k\sin\theta)$.

$\therefore$ $(\alpha+k\cos\theta)^2+(\beta+k\sin\theta)^2=r^2$

or $k^2+2(\alpha\cos\theta+\beta\sin\theta)k+(\alpha^2+\beta^2-r^2)=0,$

which is a quadratic in k. If k_1 and k_2 are its roots and the line (i) meets circle at A and B, then $PA=k_1$ and $PB=k_2$.

$\therefore$ $PA.PB=k_1k_2=$ Products of roots $=\alpha^2+\beta^2-r^2$.

14. (a): Since $f(x)$ is continuous in [1, 10].

$f(x)$ will attain all values between $f(1)$ and $f(10)$. If $f(1)\neq f(10)$, then $f(x)$ will attain innumerable irrational values between $f(1)$ and $f(10)$.

But given that $f(x)$ attains rational values only, then we must have $f(1)=f(10)$, infact $f(x)$ = constant for $x\in[1, 10]$.

Since $f(2)=5$, $f(x)=f(2)=5$, $\forall$ x.

Hence the equation whose roots are $f(3)$ and $f(\sqrt{10})$ is $x^2-(5+5)x+25=0$.

15. (b): Slope of chord joining P and Q = slope of normal at P

$$\frac{\tan\alpha-\tan\theta}{\sec\alpha-\sec\theta}=-\frac{\tan\theta}{\sec\theta}\Rightarrow\tan\alpha-\tan\theta=-k\tan\theta$$

and $\sec\alpha-\sec\theta=k\sec\theta$

$\therefore$ $(1-k)\tan\theta=\tan\alpha$ and $(1+k)\sec\theta=\sec\alpha$

$\Rightarrow$ $[(1+k)\sec\theta]^2-[(1-k)\tan\theta]^2=\sec^2\alpha-\tan^2\alpha$

$\Rightarrow$ $k=-2(\sec^2\theta+\tan^2\theta)=-4\sec^2\theta+2$

$\tan\alpha=\tan\theta(1+4\sec^2\theta-2)=\tan\theta(4\sec^2\theta-1)$

16. (c) : Consider $OABC$, $\overrightarrow{OA}=\vec{\alpha}, \overrightarrow{OB}=\vec{\beta}, \overrightarrow{OC}=\vec{\gamma}$

And OA, BC as a pair of opposite edges.

$$|\overrightarrow{OA}|=a, |\overrightarrow{BC}|=b$$

Equation of OA is $\vec{r}=\vec{O}+t\vec{\alpha}$

Equation of BC is $\vec{r}=\vec{\beta}+s(\vec{\beta}-\vec{\gamma})$

$$d=\frac{-[\vec{\alpha}\,\vec{\beta}\,\vec{\gamma}]}{|\vec{\alpha}||\vec{\beta}-\vec{\gamma}|\sin\theta}\Rightarrow V=\frac{abd\sin\theta}{6}$$

17. (c) : Let the remaining two observations be a and b.

Then, mean $=\frac{2+4+5+7+a+b}{6}=6.5$ (Given)

$\Rightarrow$ $a+b=21$ ...(i)

Also, variance = 10.25 (Given)

$\Rightarrow$ $\frac{1}{6}(2^2+4^2+5^2+7^2+a^2+b^2)-(6.5)^2=10.25$

$\Rightarrow$ $a^2+b^2=315-94=221$ ...(ii)

Now, $(a+b)^2+(a-b)^2=2(a^2+b^2)$

$\Rightarrow$ $(a-b)^2=2\times221-(21)^2$ [From (i) and (ii)]

$\Rightarrow$ $(a-b)^2=1$ $\Rightarrow$ $a-b=\pm1$

If $a-b=1$, then $a=11$, $b=10$

If $a-b=-1$, then $a=10$, $b=11$

18. (a) : a is positive integer *i.e.* $a>0$ $\therefore$ $a=[a]+\{a\}$, where $[\cdot]$ and $\{.\}$ denote the G.I.F. and fractional part

Let $n\le a<n+1$, $n\in W$

Here, $[a]=n$

Now $\int_0^a e^{x-[x]}dx=10e-9$

$\Rightarrow$ $\int_0^n e^{\{x\}}dx+\int_n^a e^{x-[x]}dx=10e-9$

$\therefore$ $n\int_0^1 e^x dx+\int_n^a e^{x-n}dx=10e-9$

$\Rightarrow$ $n(e-1)+(e^{a-n}-1)=10e-9$

$\therefore$ $n=10$ and $\{a\}=\log_e 2$

So, $a=[a]+\{a\}=10+\log_e 2$

19. (b): In the word EXAMINATION, "A, N and I" appears two times.

$\therefore$ Total number of words with M at fourth place $=\frac{10!}{2!\cdot2!\cdot2!}$

Also, total number of words formed using all letters of word EXAMINATION $=\frac{11!}{2!\cdot2!\cdot2!}$

$\therefore$ Required probability $=\frac{10!}{11!}=\frac{1}{11}$

20. (c) :

p	q	$\sim p$	$\sim q$	$p\wedge\sim q$	$q\vee\sim p$	$(p\wedge\sim q)\Rightarrow(q\vee\sim p)$	$p\Rightarrow q$
T	F	F	T	T	F	F	F
T	T	F	F	F	T	T	T
F	F	T	T	F	T	T	T
F	T	T	F	F	T	T	T

21. (6) : We know that, if $\vec{r}_1 = \vec{a} + \lambda\vec{b}$ and $\vec{r}_2 = \vec{c} + \lambda\vec{d}$, then shortest distance between two lines is given by

$$d = \frac{|(\vec{b}\times\vec{d})\cdot(\vec{a}-\vec{c})|}{|\vec{b}\times\vec{d}|}$$

Now, $\vec{a}-\vec{c} = (\alpha+4)\hat{i} + 2\hat{j} + 3\hat{k}$

Also, $\vec{b}\times\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 2 \\ 3 & -2 & -2 \end{vmatrix} = 8\hat{i} + 8\hat{j} + 4\hat{k}$

$\therefore \quad |\vec{b}\times\vec{d}| = 12$

Now, $\dfrac{\vec{b}\times\vec{d}}{|\vec{b}\times\vec{d}|} = \dfrac{2\hat{i}+2\hat{j}+\hat{k}}{3}$

$\therefore$ Required distance

$$= \frac{((\alpha+4)\hat{i}+2\hat{j}+3\hat{k})\cdot(2\hat{i}+2\hat{j}+\hat{k})}{3} = 9$$

$\Rightarrow \quad 2\alpha + 8 + 4 + 3 = 27 \;\Rightarrow\; 2\alpha = 27 - 15$

$\Rightarrow \quad 2\alpha = 12 \Rightarrow \alpha = 6$

22. (1) : We have, $\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix}$

Applying $c_2 \to c_2 - c_3$, we get

$$\begin{vmatrix} x-2\lambda & \lambda & x+a \\ x-1 & \lambda & x+b \\ x+2\lambda & \lambda & x+c \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1, R_3 \to R_3 - R_1,$ we get

$$\begin{vmatrix} x-2\lambda & \lambda & x+a \\ 2\lambda-1 & 0 & \lambda \\ 4\lambda & 0 & 2\lambda \end{vmatrix}$$

$= -\lambda(4\lambda^2 - 2\lambda - 4\lambda^2) = 2\lambda^2 = 2$ (Given)

$\therefore \quad \lambda^2 = 1$

23. (910) : Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix} = I + P,$

where $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, P = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}$

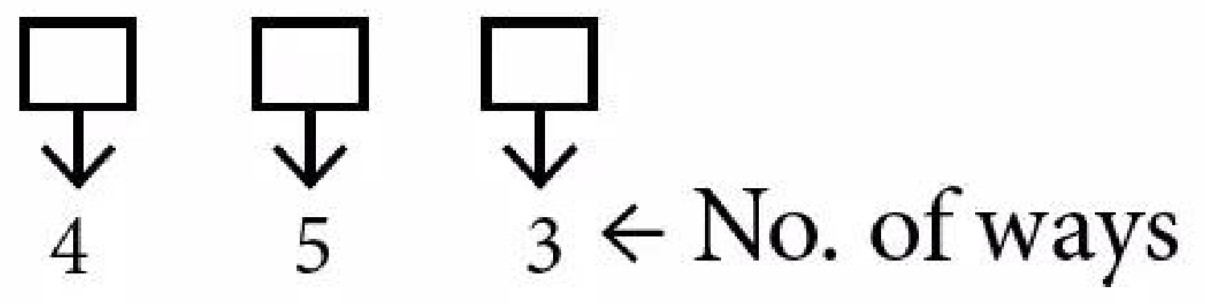
$$P^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, P^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Now, $B = 7A^{20} - 20A^7 + 2I = 7(I+P)^{20} - 20(I+P)^7 + 2I$

$= 7(I + 20P + {}^{20}C_2P^2) - 20(I + 7P + {}^7C_2P^2) + 2I$

$\therefore \quad b_{13} = 7 \times {}^{20}C_2 - 20 \times {}^7C_2 = 910$

24. (52) : The units place must be either 0, 4 or 6 for the number to be even.



Number of 3 digit numbers which are even

$= 4 \times 5 \times 3 = 60$

But this includes even numbers with 0 at hundreds place.

Number of such numbers $= 2 \times 4 = 8$

$\therefore$ Required number of 3 digit even numbers

$= 60 - 8 = 52$

25. (40) : We have, $\log_e(x+y) = 4xy$

Substituting $x = 0$, we get

$\log_e y = 0 \Rightarrow y = 1$

Thus, $x = 0 \Rightarrow y = 1$

Clearly, $\dfrac{1}{(x+y)}\left(1 + \dfrac{dy}{dx}\right) = 4 \times \left(y + \dfrac{x\,dy}{dx}\right)$ ...(i)

At $x = 0, y = 1$, we have

$1 + \dfrac{dy}{dx} = 4 \times \left(1 + 0\cdot\dfrac{dy}{dx}\right) \Rightarrow \dfrac{dy}{dx} = 3$

Also, from (i), we have $1 + \dfrac{dy}{dx} = 4(x+y)\left(y + \dfrac{x\,dy}{dx}\right)$

$$\Rightarrow \frac{d^2y}{dx^2} = 4\left(1+\frac{dy}{dx}\right)\left(y + \frac{x\,dy}{dx}\right) + 4(x+y)\left(\frac{dy}{dx} + \frac{dy}{dx} + \frac{xd^2y}{dx^2}\right)$$

SOLUTIONS TO MARCH 2023 QUIZ CLUB

1. $\dfrac{8}{7}$
2. 6
3. 6
4. B
5. 0 or $\dfrac{2\pi}{3}$
6. 4920
7. 28
8. 9
9. 0
10. ABA
11. $y = -4x + 5$
12. 1
13. 1
14. $\tan^{-1}\dfrac{3}{10}$
15. x
16. -1
17. $\dfrac{3}{4}$
18. $\dfrac{15}{4}$
19. $\sqrt{3}$
20. $\dfrac{1}{10}$

Substituting $x = 0$, $y = 1$ and $\frac{dy}{dx} = 3$, we get

$$\frac{d^2y}{dx^2} = 4(1+3)\left(1+0\cdot\frac{dy}{dx}\right)+4(0+1)\left(2\times3+0\cdot\frac{d^2y}{dx^2}\right)$$

$= 4 \times 4 + 4 \times 6 = 16 + 24 = 40$

26. (136 or 680) : Let $t_n = n\cdot{}^nP_n$

$= n\times\frac{n!}{(n-n)!} = n\cdot n! = (n+1-1)n!$

$= (n + 1)! - n!$

$t_1 = 2! - 1!$

$t_2 = 3! - 2!$

$t_3 = 4! - 3!$ ...., $t_{15} = 16! - 15!$

$\therefore \sum_{n=1}^{15} t_n = 16! - 1 = {}^qP_r - s$

$\Rightarrow {}^qP_r = 16! - 1 + s$

Since, ${}^qP_r$ is an integer, s can take values 1 or 0.

$s = 0$ is not possible as there is no such q and r for which ${}^qP_r = 16! - 1$

$s = 1 \Rightarrow {}^qP_r = 16! \Rightarrow q = r = 16$ or $q = 16$ and $r = 15$

$\therefore {}^{q+s}C_{r-s} = {}^{17}C_{15}$ or ${}^{17}C_{14}$, *i.e.*, 136 or 680

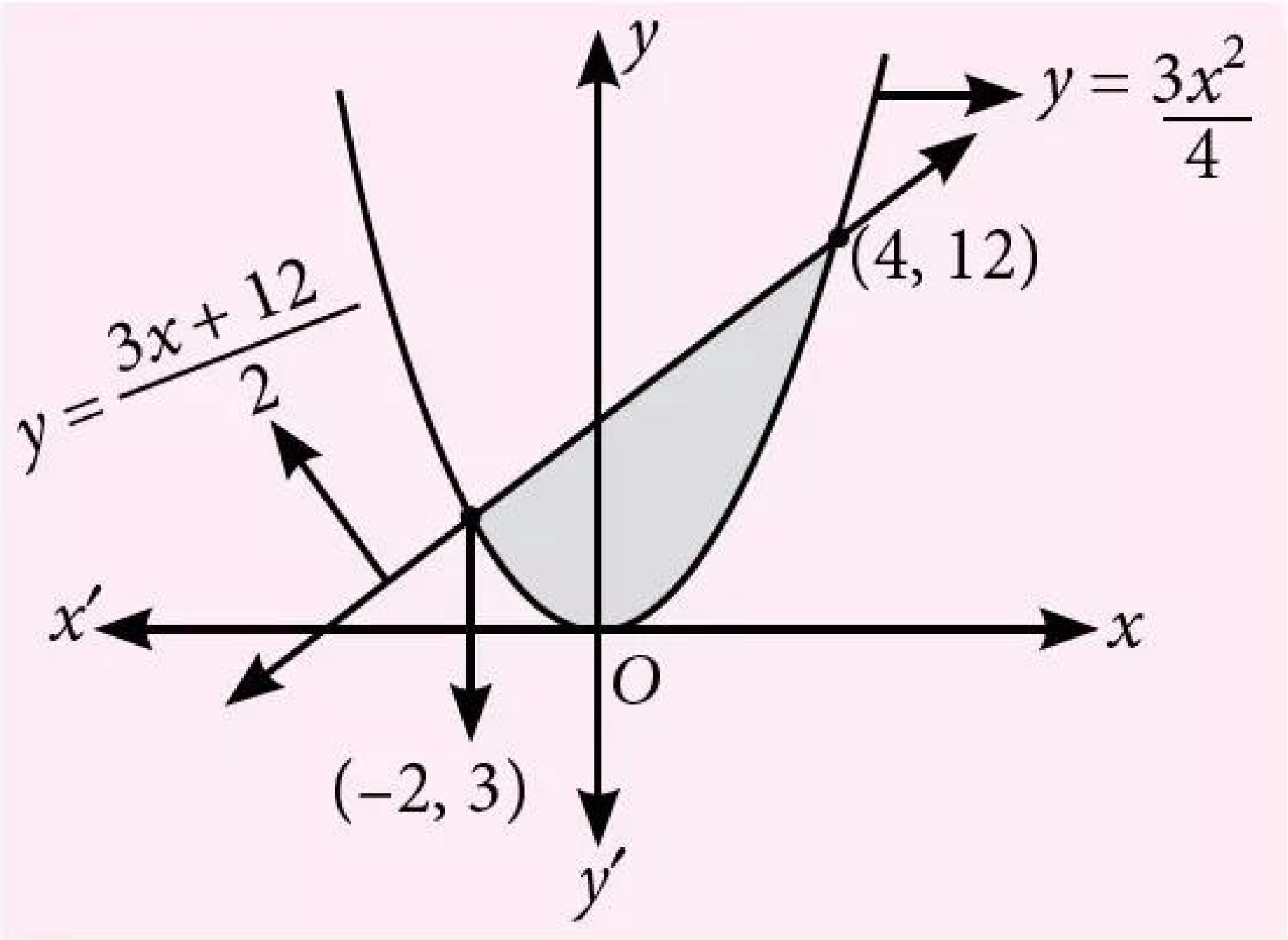
27. (27) : We have, $\frac{3x^2}{4} \le y \le \frac{3x+12}{2}$

For point of intersection, we have $3x^2 = 6x + 24$

$\Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow (x - 4)(x + 2) = 0$

$\Rightarrow x = 4$ and $x = -2$

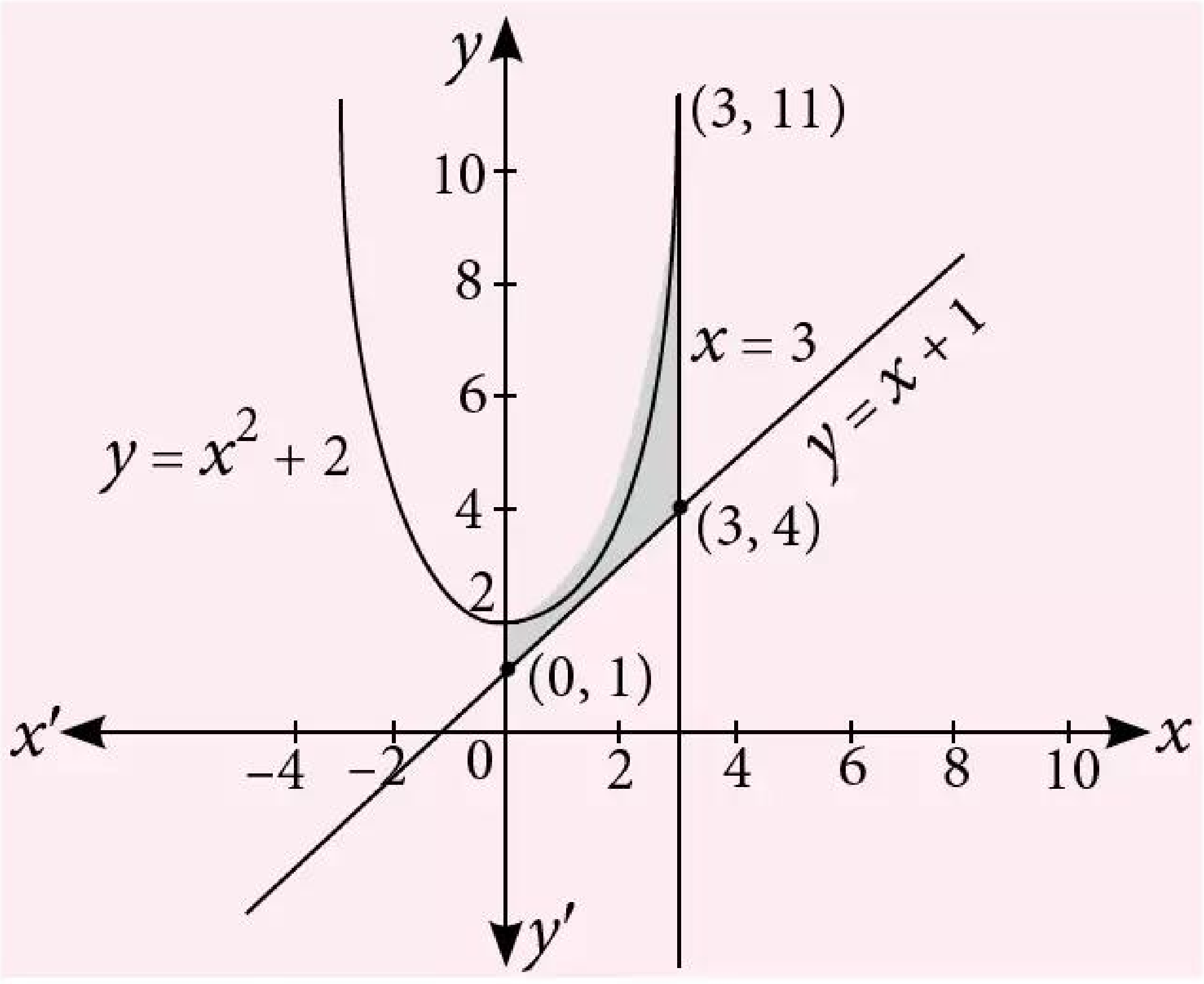


$\therefore$ Required area $= \int_{-2}^{4}\left(\frac{3x+12}{2} - \frac{3x^2}{4}\right)dx$

$= \left[\frac{3x^2}{4} + 6x - \frac{x^3}{4}\right]_{-2}^{4}$

$= (12 + 24 - 16) - (3 - 12 + 2) = 20 - (-7) = 27$

28. (15) : The shaded region represents the required area in the figure.



$$\text{Required area} = \int_0^3 [(x^2+2)-(x+1)]dx = \int_0^3 (x^2-x+1)dx$$

$$= \left[\frac{x^3}{3} - \frac{x^2}{2} + x\right]_0^3 = 9 - \frac{9}{2} + 3 = \frac{15}{2}\text{ sq. units}$$

$\therefore \lambda = 15$

29. (303) : $S_k = \frac{k(k+1)}{2k} = \frac{k+1}{2}$

Now, $\sum_{k=1}^{10}(S_k)^2 = \frac{5}{12}A \Rightarrow \sum_{k=1}^{10}\left(\frac{k+1}{2}\right)^2 = \frac{5}{12}A$

$\Rightarrow \frac{1}{4}(2^2 + 3^2 + ... + 11^2) = \frac{5}{12}A$

$\Rightarrow \frac{1}{4}\left(\frac{11\times12\times23}{6} - 1\right) = \frac{5}{12}A$

$\Rightarrow \frac{505}{4} = \frac{5}{12}A \Rightarrow A = 303$

30. (10) : Here, $P = \begin{bmatrix}1 & 0 & 0\\3 & 1 & 0\\9 & 3 & 1\end{bmatrix}$ and $Q = P^5 + I_3$

$$\therefore P^2 = \begin{bmatrix}1 & 0 & 0\\3 & 1 & 0\\9 & 3 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\3 & 1 & 0\\9 & 3 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\6 & 1 & 0\\27 & 6 & 1\end{bmatrix}$$

$$P^3 = \begin{bmatrix}1 & 0 & 0\\6 & 1 & 0\\27 & 6 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\3 & 1 & 0\\9 & 3 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\9 & 1 & 0\\54 & 9 & 1\end{bmatrix}$$

Similarly, $P^5 = \begin{bmatrix}1 & 0 & 0\\15 & 1 & 0\\135 & 15 & 1\end{bmatrix}$

$$\therefore Q = \begin{bmatrix}2 & 0 & 0\\15 & 2 & 0\\135 & 15 & 2\end{bmatrix} \Rightarrow \frac{q_{21}+q_{31}}{q_{32}} = \frac{15+135}{15} = 10$$

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No matter how advanced the world becomes but the unmatched feeling of holding a book, flipping its pages, highlighting the points we want to come back to; can never be compared to kindle or other digital devices. We must realise the magical power of books because they create a connection between past, present and future. Books have the power to bridge the gap between generations.

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Y?U ASK
WE ANSWER

Do you have a question that you just can't get answered?

Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough.

The best questions and their solutions will be printed in this column each month.

1. Evaluate the following indefinite integral:

$$\int \frac{dx}{(x+1)^5\sqrt{x^2+2x}}$$

(Priya Malhotra, Gujarat)

Ans. Let $I = \int \frac{dx}{(x+1)^5\sqrt{x^2+2x}} = \int \frac{dx}{(x+1)^5\sqrt{(x+1)^2-1}}$

$$= \int \frac{dx}{(x+1)^6\sqrt{1-\frac{1}{(x+1)^2}}} = \int \frac{-t^4\,dt}{\sqrt{1-t^2}} \quad \left[\text{Putting } \frac{1}{x+1} = t\right]$$

Here, t^4 is a polynomial of degree 4.

$$\therefore \quad I = \int \frac{-t^4\,dt}{\sqrt{1-t^2}} = (At^3 + Bt^2 + Ct + D)\sqrt{1-t^2} + \lambda \int \frac{dt}{\sqrt{1-t^2}} \quad \ldots(\text{i})$$

Differentiating both sides and multiplying by $\sqrt{1-t^2}$, we get

$-t^4 = (3At^2 + 2Bt + C)(1 - t^2) - t(At^3 + Bt^2 + Ct + D) + \lambda$

Comparing the coefficients, we have

$-4A = -1, 3B = 0, 3A - 2C = 0, 2B - D = 0$ and $C + \lambda = 0$

$\Rightarrow A = 1/4, B = 0, C = 3/8, D = 0$ and $\lambda = -3/8$

From (i) $I = \left(\frac{1}{4}t^3 + \frac{3}{8}t\right)\sqrt{1-t^2} - \frac{3}{8}\int \frac{dt}{\sqrt{1-t^2}}$

$= \frac{1}{8}(2t^3 + 3t)\sqrt{1-t^2} - \frac{3}{8}\sin^{-1} t + C$, where $t = \frac{1}{x+1}$.

2. If $f(x)$ is a polynomial of degree <3, then prove that

$$\begin{vmatrix} 1 & a & \frac{f(a)}{x-a} \\ 1 & b & \frac{f(b)}{x-b} \\ 1 & c & \frac{f(c)}{x-c} \end{vmatrix} \div \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \frac{f(x)}{(x-a)(x-b)(x-c)}.$$

(Ankit, U.P.)

Ans. $\frac{f(x)}{(x-a)(x-b)(x-c)} = \frac{A}{(x-a)} + \frac{B}{(x-b)} + \frac{C}{(x-c)}$

[By partial fractions]

i.e., $f(x) = A(x-b)(x-c) + B(x-a)(x-c) + C(x-a)(x-b)$ ...(i)

Putting $x = a$ in equation (i), we have

$$f(a) = A(a-b)(a-c) \Rightarrow A = \frac{f(a)}{(a-b)(a-c)}$$

Similarly, putting $x = b, c$ in equation (i) respectively, we have $B = \frac{f(b)}{(b-a)(b-c)}$ and $C = \frac{f(c)}{(c-a)(c-b)}$

Hence, we have $\frac{f(x)}{(x-a)(x-b)(x-c)}$

$$= \frac{(c-b)\frac{f(a)}{x-a} - (c-a)\frac{f(b)}{(x-b)} + (b-a)\frac{f(c)}{(x-c)}}{(a-b)(b-c)(c-a)}$$

$$= \begin{vmatrix} 1 & a & \frac{f(a)}{x-a} \\ 1 & b & \frac{f(b)}{x-b} \\ 1 & c & \frac{f(c)}{x-c} \end{vmatrix} \div \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$, which is the desired result.

3. If $\sin^{-1}a + \sin^{-1}b + \sin^{-1}c = \pi$, then find the value of

$$a\sqrt{(1-a^2)} + b\sqrt{(1-b^2)} + c\sqrt{(1-c^2)}.$$

(Prateush Sharma)

Ans. Let $\sin^{-1}a = A \Rightarrow a = \sin A$ ...(i)

$\sin^{-1}b = B \Rightarrow b = \sin B$ ...(ii)

and $\sin^{-1}c = C \Rightarrow c = \sin C$ ...(iii)

$\therefore \quad \sin^{-1}a + \sin^{-1}b + \sin^{-1}c = \pi$

$\Rightarrow \quad A + B + C = \pi$ ...(iv)

Now, $a\sqrt{1-a^2} + b\sqrt{1-b^2} + c\sqrt{1-c^2}$

$= \sin A\sqrt{1-\sin^2 A} + \sin B\sqrt{1-\sin^2 B} + \sin C\sqrt{1-\sin^2 C}$

$= \sin A \cos A + \sin B \cos B + \sin C \cos C$

$= \frac{\sin 2A + \sin 2B + \sin 2C}{2}$

$= \frac{2\sin(A+B)\cos(A-B) + \sin 2C}{2}$

$= \frac{2\sin(\pi - C)\cos(A-B) + 2\sin C\cos C}{2}$ [Using (iv)]

$= \frac{2\sin C\cos(A-B) + 2\sin C\cos C}{2}$

$= \frac{2\sin C\left[\cos(A-B) + \cos(\pi - (A+B))\right]}{2}$

$= \sin C(\cos(A - B) - \cos(A + B))$

$= 2\sin C \sin A \sin B = 2abc$

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- Project Manager
- Consultant
- Analyst
- Teaching Assistant
- Software Engineer
- Security Review Operations Analyst

Eligibility Criteria

- **Certificate :** Pass in class 10th and 12th
- **UG :** Pass in 10+2 with Physics, Chemistry and Maths stream.

Recipe for Success

> “ All birds find shelter during a rain. But eagle avoids rain by flying above the clouds. Problems are common but attitude makes the difference!! ”
>
> *– A.P.J. Abdul Kalam*

PRACTICE PAPER

BITSAT

Exam Dates
Session-I
22nd May to 26th May
Session-II
18th June to 22nd June

1. The modulus of $[1 - \cos\theta + i\sin\theta]^{-1}$ is

(a) $\frac{1}{2}\operatorname{cosec}\frac{\theta}{2}$ (b) $\operatorname{cosec}\frac{\theta}{2}$

(c) $\frac{1}{2}\left|\operatorname{cosec}\frac{\theta}{2}\right|$ (d) $\left|\operatorname{cosec}\frac{\theta}{2}\right|$

2. If $a_1, a_2, \ldots, a_n$ are in A.P. where $a_i > 0$ for all i, then

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}=$$

(a) $\frac{1}{\sqrt{a_1}+\sqrt{a_n}}$ (b) $\frac{n}{\sqrt{a_1}+\sqrt{a_n}}$

(c) $\frac{n+1}{\sqrt{a_1}+\sqrt{a_n}}$ (d) $\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$

3. There are 3 copies each of 4 different books. The number of ways they can be arranged in a shelf is

(a) 369600 (b) 400400 (c) 420600 (d) 440720

4. If $x = \left(\sqrt{2}+1\right)^6$, then the integer part $[x]$ is

(a) 98 (b) 197 (c) 196 (d) 198

5. A and B alternately cut a pack of cards which is shuffled after each cut. The game is started by A and continuous till one of the player cuts a club. The probability, that B wins the game, is

(a) $\frac{1}{7}$ (b) $\frac{2}{7}$ (c) $\frac{3}{7}$ (d) $\frac{2}{5}$

6. The domain of $\frac{\log_2(x+3)}{x^2+3x+2}$ is

(a) $R - \{-1, -2\}$ (b) $[-2, \infty)$

(c) $R - \{-1, -2, -3\}$ (d) $(-3, \infty) - \{-1, -2\}$

7. Let $P(n)$: $n^2 + n$ is an odd integer and $P(k) \Rightarrow P(k+1)$ is true, $P(n)$ is true for all

(a) $n > 2$ (b) $n > 1$

(c) n (d) None of these

8. $2\sin^2\theta + 4\cos(\theta+\alpha)\sin\alpha\sin\theta + \cos 2(\theta+\alpha) =$

(a) $\sin 2\alpha$ (b) $\cos 2\alpha$ (c) $\sin^2\alpha$ (d) $\cos^2\alpha$

9. If $\begin{bmatrix} 1 & -3 & 2 \\ 3 & 4 & 1 \\ 4 & -2 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \\ -2 \end{bmatrix}$, then $x =$

(a) 3 (b) 4 (c) 5 (d) 7

10. If $\sin^{-1}\frac{2\alpha}{1+\alpha^2} - \sin^{-1}\frac{2\beta}{1+\beta^2} = 2\tan^{-1}x$, then $x =$

(a) $\alpha+\beta$ (b) $\frac{\alpha+\beta}{\alpha-\beta}$ (c) $\frac{\alpha+\beta}{1-\alpha\beta}$ (d) $\frac{\alpha-\beta}{1+\alpha\beta}$

11. Let $A = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ such that $|A| = 0$. If a, b, c are distinct, then the sum of the coordinates of the fixed point through which the line $ax + by + c = 0$ passes is

(a) 2 (b) 3 (c) 4 (d) 5

12. If $x^m y^n = (x+y)^{m+n}$, then $\frac{dy}{dx} =$

(a) xy (b) $\frac{x}{y}$ (c) $\frac{y}{x}$ (d) $\frac{x+y}{xy}$

13. The distance of the plane $x + 2y - z = 2$ from the point $(2, -1, 3)$ measured in the direction with direction ratios 2, 2, 1 is

(a) 1 (b) 2 (c) 3 (d) $\frac{5}{\sqrt{6}}$

14. If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y - 2 = 0$ meets the line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then the length PQ is

(a) 4 (b) $2\sqrt{5}$ (c) 5 (d) $3\sqrt{5}$

15. In an experiments with 15 observation on x, the following results were available.

$\Sigma x^2 = 2830$, $\Sigma x = 170$

One observation that was 20 was found to be wrong and was replaced by the correct value 30. Then the corrected variance is

(a) 78 (b) 188.66 (c) 177.33 (d) 8.33

16. A bag contains 50 tickets numbered 1, 2, 3, ..., 50 of which five are drawn at random and arranged in ascending order of magnitude ($x_1 < x_2 < x_3 < x_4 < x_5$) then the probability that $x_3 = 30$ is

(a) $\dfrac{{}^{29}C_2 \times {}^{20}C_2}{{}^{50}C_5}$ (b) $\dfrac{{}^{30}C_1 \times {}^{20}C_1}{{}^{50}C_5}$

(c) $\dfrac{{}^{5}C_1 \times {}^{50}C_2}{{}^{50}C_5}$ (d) $\dfrac{{}^{50}C_2 \times {}^{20}C_1}{{}^{50}C_5}$

17. If $\log_{30} 5 = a$, $\log_{30} 3 = b$, then $\log_{30} 8 =$

(a) $a + b$ (b) ab

(c) $3(1 - a - b)$ (d) $3(1 + a + b)$

18. The maximum value of $z = 2x + 3y$ subject to $x \geq 3y$, $3x + 4y \leq 12$, $x, y \geq 0$ is

(a) 8 (b) 9 (c) $\dfrac{108}{13}$ (d) $\dfrac{113}{13}$

19. Three balls are drawn simultaneously from a bag containing 5 white and 4 red balls. Let X be the number of red balls drawn. Find the variance of X.

(a) $\dfrac{4}{3}$ (b) $\dfrac{5}{3}$ (c) $\dfrac{4}{9}$ (d) $\dfrac{5}{9}$

20. $y \sec x = \tan x + c$ is a solution of the differential equation

(a) $\dfrac{dy}{dx} + y\tan x + \sec x = 0$

(b) $\dfrac{dy}{dx} + y\tan x - \sec x = 0$

(c) $\dfrac{dy}{dx} - y\tan x + \sec x = 0$

(d) $\dfrac{dy}{dx} - y\tan x - \sec x = 0$

21. If $y = \sin^{-1}(2^x)$, then find $\dfrac{dy}{dx}$.

(a) $\dfrac{2^x \log 2}{\sqrt{1-4^x}}$ (b) $\dfrac{2^x}{\sqrt{1-4^x}}$

(c) $\dfrac{\log 2}{\sqrt{1+4^x}}$ (d) $\dfrac{2^x \log x}{\sqrt{1-2^x}}$

22. Find the equations of tangent and normal to the curve $x = \sin\theta$, $y = \cos 2\theta$ at $\theta = \dfrac{\pi}{6}$.

(a) $4x + 2y - 3 = 0$; $2x - 4y + 1 = 0$

(b) $4x + 2y + 3 = 0$; $2x - 4y + 1 = 0$

(c) $4x + 2y - 3 = 0$; $2x + 4y + 1 = 0$

(d) $4x - 2y + 3 = 0$; $2x - 4y + 1 = 0$

23. The value of c in LMVT for the function $f(x) = 6x^2 - x^3$, $x \in [0, 6]$ is

(a) 1 (b) 4 (c) 5 (d) 3

24. The function $f(x) = \dfrac{7}{x} - 3$, $x \in R$, $x \neq 0$ is

(a) decreasing (b) increasing

(c) neither increasing nor decreasing

(d) none of these

25. Dividing the number 20 into two parts such that their product is maximum. The two parts are

(a) 11, 9 (b) 12, 8 (c) 13, 7 (d) 10, 10

26. $\displaystyle\int \frac{dx}{\sin^2 x \cos^2 x} =$

(a) $2\tan 2x + c$ (b) $-2\tan 2x + c$

(c) $-2\cot 2x + c$ (d) $2\cot 2x + c$

27. $\displaystyle\int \frac{dx}{x^{1/5}[1+x^{4/5}]^{1/2}} =$

(a) $\sqrt{1+x^{4/5}} + c$ (b) $\dfrac{5}{2}\sqrt{1+x^{4/5}} + c$

(c) $x^{4/5}(1 + x^{4/5}) + c$ (d) $\dfrac{2}{5}x^{4/5}(1+x^{4/5}) + c$

28. $\displaystyle\int \frac{3\sin x + 2\cos x}{3\cos x + 2\sin x}\,dx =$

(a) $\dfrac{12}{13}x - \dfrac{5}{13}x\log|3\cos x - 2\sin x| + c$

(b) $\dfrac{12}{13}x - \dfrac{5}{13}\log|3\cos x + 2\sin x| + c$

(c) $\dfrac{12}{13}x + \dfrac{5}{13}\log|3\cos x - 2\sin x| + c$

(d) $\dfrac{12}{13}x + \dfrac{5}{13}\log|2\cos x - 3\sin x| + c$

29. If $\displaystyle\int_0^{b-c} f(x+c)\,dx = a\int_b^c f(x)\,dx$, then $a =$

(a) 0 (b) -1 (c) 2 (d) $b - c$

30. Find the area bounded by the curve $y = x^2$, $x = 1$, $x = 2$ and X-axis.

(a) $\dfrac{4}{3}$ sq. units (b) $\dfrac{3}{7}$ sq. units

(c) $\dfrac{8}{3}$ sq. units (d) $\dfrac{7}{3}$ sq. units

31. The combined equation of the lines passing through the origin and having inclinations $\dfrac{\pi}{3}$ and $\dfrac{5\pi}{3}$ is

(a) $x^2 + 3y^2 = 0$ (b) $x^2 + y^2 = 0$

(c) $3x^2 - y^2 = 0$ (d) $x^2 - 3y^2 = 0$

32. If the vectors $3\hat{i}+2\hat{j}-\hat{k}$ and $6\hat{i}-4x\hat{j}+y\hat{k}$ are parallel, then the values of x and y respectively are
(a) $-1, -2$ (b) $1, -2$ (c) $-1, 2$ (d) $1, 2$

33. If $[\vec{a}\ \vec{b}\ \vec{c}] \neq 0$, then $\dfrac{[\vec{a}+\vec{b}\ \ \vec{b}+\vec{c}\ \ \vec{c}+\vec{a}]}{[\vec{a}\ \vec{b}\ \vec{c}]}$ is
(a) 1 (b) 2 (c) 5 (d) 7

34. If $ABCD$ is a square, then $\overrightarrow{AB}+2\overrightarrow{BC}+3\overrightarrow{CD}+4\overrightarrow{DA}=$
(a) $4\overrightarrow{AC}$ (b) $3\overrightarrow{AC}$ (c) $2\overrightarrow{CA}$ (d) $\overrightarrow{AC}$

35. If a line makes angles 3α, 3β and 3γ with the coordinate axes, find the value of $\cos 6\alpha + \cos 6\beta + \cos 6\gamma$.
(a) -1 (b) 0 (c) 1 (d) 8

36. The distance of point (4, 1, 1) from the line $\dfrac{x-7}{2}=\dfrac{y-7}{2}=\dfrac{z-6}{3}$ is
(a) $\sqrt{\dfrac{110}{17}}$ units (b) $\sqrt{\dfrac{101}{19}}$ units
(c) $\sqrt{\dfrac{101}{17}}$ units (d) $\sqrt{\dfrac{110}{19}}$ units

37. The circle passing through the distinct points $(t, 1)$, (t, t), $(1, t)$ for all values of t passes through the point
(a) $(1, -1)$ (b) $(-1, 1)$ (c) $(-1, -1)$ (d) $(1, 1)$

38. If the eccentricity of $\dfrac{x^2}{a^2}-\dfrac{y^2}{b^2}=1$ is $\dfrac{5}{4}$, then the eccentricity of $\dfrac{x^2}{b^2}-\dfrac{y^2}{a^2}=1$ is
(a) $\dfrac{5}{4}$ (b) $\dfrac{5}{3}$ (c) $\dfrac{4}{3}$ (d) $\dfrac{9}{4}$

39. If $\{x, y\}$ is a subset of the first 30 natural numbers, then the probability, that $x^3 + y^3$ is divisible by 3, is
(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{3}$ (c) $\dfrac{1}{5}$ (d) $\dfrac{1}{6}$

40. The number of values of x such that $4\sin^{-1}x + \cos^{-1}x = \pi$ is
(a) 3 (b) 2 (c) 1 (d) 0

SOLUTIONS

1. (c) : $\left|\dfrac{1}{1-\cos\theta+i\sin\theta}\right| = \dfrac{1}{|1-\cos\theta+i\sin\theta|}$

$$= \frac{1}{\sqrt{2-2\cos\theta}} = \frac{1}{2|\sin(\theta/2)|} = \frac{1}{2}\left|\operatorname{cosec}\frac{\theta}{2}\right|.$$

2. (d): Let d be the common difference

$$\Rightarrow \frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}$$
$$= \frac{\sqrt{a_2}-\sqrt{a_1}}{d}+\frac{\sqrt{a_3}-\sqrt{a_2}}{d}+\ldots+\frac{\sqrt{a_n}-\sqrt{a_{n-1}}}{d}$$
$= \dfrac{\sqrt{a_n}-\sqrt{a_1}}{d}$, cancelling the terms
$$= \frac{a_n-a_1}{(\sqrt{a_n}+\sqrt{a_1})d} = \frac{(n-1)}{\sqrt{a_n}+\sqrt{a_1}}.$$

3. (a): There are 4 sets of 3 identical books. The desired number is $\dfrac{12!}{(3!)^4} = 369600$

4. (b): If f is the fractional part of x, then
$[x]+f=\left(\sqrt{2}+1\right)^6, 0<f<1$
Let $f'=\left(\sqrt{2}-1\right)^6, 0<f'<1$
Adding, $[x]+f+f'=\left(\sqrt{2}+1\right)^6+\left(\sqrt{2}-1\right)^6$
$$=2\left[\binom{6}{0}\sqrt{2}^6+\binom{6}{2}\sqrt{2}^4+\binom{6}{4}\sqrt{2}^2+\binom{6}{6}\right]$$
$= 2[8 + 15 \times 4 + 15 \times 2 + 1] = 198.$
But $f + f' = 1 \therefore [x] = 197.$

5. (c) : The required sequence is FS, $FFFS$, …, where $P(S) = \dfrac{1}{4}$

The probability $= \dfrac{3}{4}\cdot\dfrac{1}{4}+\left(\dfrac{3}{4}\right)^3\dfrac{1}{4}+\ldots$
$$= \frac{3}{16} \div \left(1-\frac{9}{16}\right)=\frac{3}{7}.$$

6. (d): The domain excludes the roots of $x^2 + 3x + 2 = 0$ *i.e.*, $-1, -2$. Further $x + 3 > 0 \Rightarrow x > -3$.
The domain is $(-3, \infty) - \{-1, -2\}$.

7. (d): Since $P(1) = 1 + 1 = 2$, not an odd number, $P(1)$ is not true. Therefore $P(n)$ cannot be true for $n \in N$.

8. (b): $2\sin^2\theta + 2\cos(\theta+\alpha)\cdot 2\sin\alpha\sin\theta + 2\cos^2(\theta+\alpha) - 1$
$= 2\sin^2\theta + 2\cos(\theta+\alpha)[\cos(\theta-\alpha) - \cos(\theta+\alpha)] + 2\cos^2(\theta+\alpha) - 1$
$= 2\sin^2\theta + 2(\cos^2\theta - \sin^2\alpha) - 1$
$= 2 - 1 - 2\sin^2\alpha = 1 - 2\sin^2\alpha = \cos 2\alpha.$

9. (d): $\begin{bmatrix}1 & -3 & 2\\ 3 & 4 & 1\\ 4 & -2 & 9\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}8\\ 5\\ -2\end{bmatrix}$

$x - 3y + 2z = 8;\ 3x + 4y + z = 5$
$4x - 2y + 9z = -2$
Solving, $x = 7, y = -3, z = -4$

10. (d): $\sin^{-1}\dfrac{2\alpha}{1+\alpha^2} = \tan^{-1}\dfrac{2\alpha}{1-\alpha^2} = 2\tan^{-1}\alpha$

$\sin^{-1}\dfrac{2\beta}{1+\beta^2} = 2\tan^{-1}\beta$

So, $2\tan^{-1}\alpha - 2\tan^{-1}\beta = 2\tan^{-1}x$

$\therefore \quad \tan^{-1}x = \tan^{-1}\alpha - \tan^{-1}\beta = \tan^{-1}\dfrac{\alpha-\beta}{1+\alpha\beta}$

$\Rightarrow \quad x = \dfrac{\alpha-\beta}{1+\alpha\beta}.$

11. (a) : $A = \dfrac{1}{2}(a+b+c)\,[(a-b)^2+(b-c)^2+(c-a)^2] = 0$

The only way this can equal 0 is, if $a + b + c = 0$

$\Rightarrow \quad a(1) + b(1) + c = 0$

The fixed point is therefore, (1, 1). The required sum is 1 + 1 = 2.

12. (c) : By logarithmic differentiation, we get $m\ln x + n\ln y = (m+n)\ln(x+y)$,

$$\Rightarrow \quad \frac{m}{x}+\frac{n}{y}y' = \frac{m+n}{x+y}(1+y')$$

$$\Rightarrow \quad \left(\frac{m+n}{x+y}-\frac{n}{y}\right)y' = \frac{m}{x}-\frac{m+n}{x+y}$$

$$\Rightarrow \frac{my-nx}{y}\cdot y' = \frac{my-nx}{x} \quad \Rightarrow \quad y' = \frac{y}{x}$$

13. (c) : *d.r.'s* are 2, 2, 1. *d.c.'s* are $\dfrac{2}{3}, \dfrac{2}{3}, \dfrac{1}{3}$

The line is $\dfrac{x-2}{2/3} = \dfrac{y+1}{2/3} = \dfrac{z-3}{1/3} = r$, where r is the distance.

$\therefore \quad x = 2 + \dfrac{2r}{3},\ y = -1 + \dfrac{2r}{3},\ z = 3 + \dfrac{r}{3}$

$x + 2y - z = 2$

$\Rightarrow \quad 2 + \dfrac{2r}{3} - 2 + \dfrac{4r}{3} - 3 - \dfrac{r}{3} = 2$

$\Rightarrow \quad \dfrac{5r}{3} = 5 \Rightarrow r = 3.$

14. (c) : The line $5x - 2y + 6 = 0$ meets the y-axis at $Q(0, 3)$.

PQ = length of the tangent from (0, 3)

$= \sqrt{S_{11}} = \sqrt{3^2 + 18 - 2} = 5.$

15. (a) : $n = 15$, $\Sigma x^2 = 2830$, $\Sigma x = 170$

(New)$\Sigma x^2 = 2830 - 400 + 900 = 3330$

(New)$\Sigma x = 170 - 20 + 30 = 180$

$$\sigma^2 = \frac{\sum x^2}{n} - \left(\frac{\sum x}{n}\right)^2 = \frac{3330}{15} - \left(\frac{180}{15}\right)^2 = 78.0$$

16. (a) : Number of ways of drawing two tickets numbered 1 to 29 = ${}^{29}C_2$

Number of ways of drawing two tickets numbered 31 to 50 = ${}^{20}C_2$

Number of ways of drawing a ticket numbered 30 = 1

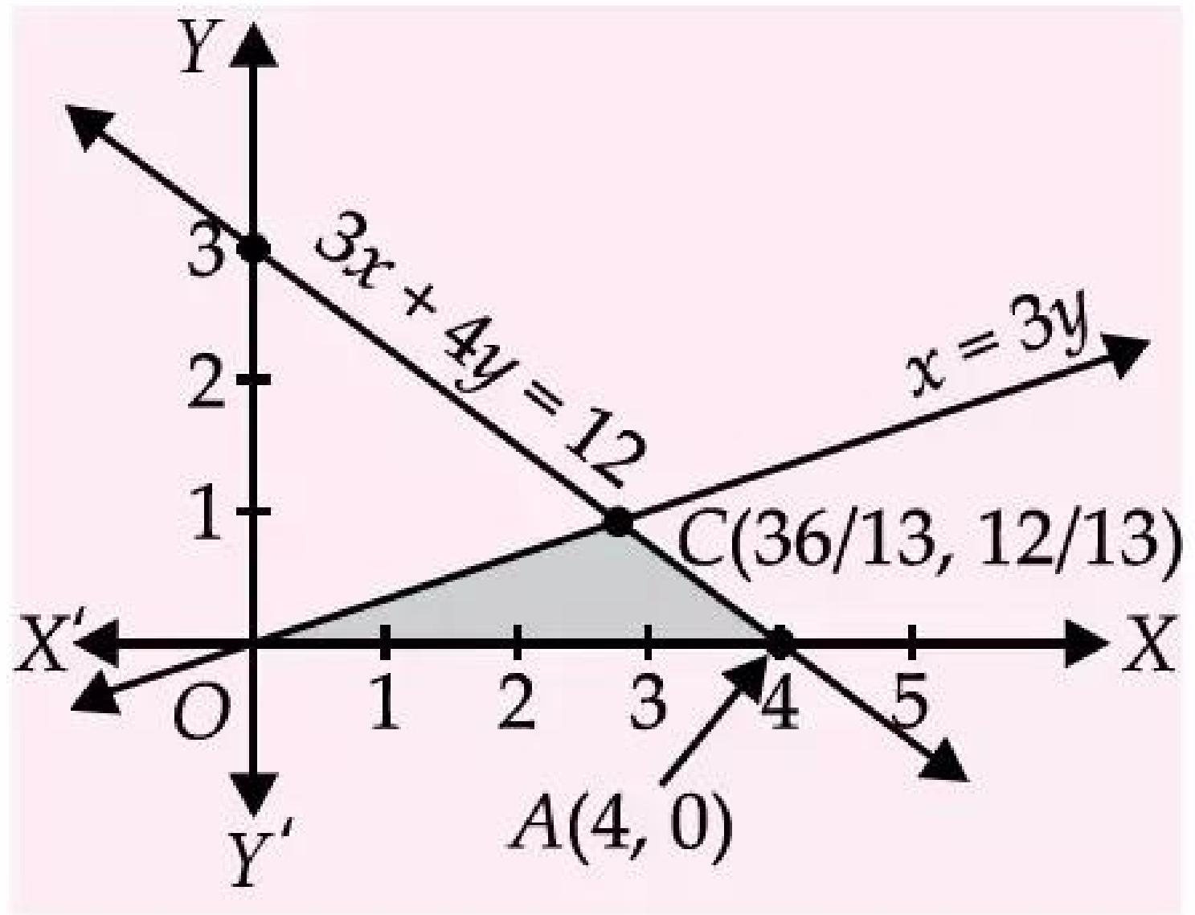
Hence required probability = $\dfrac{{}^{29}C_2 \times 1 \times {}^{20}C_2}{{}^{50}C_5}$

$= \dfrac{{}^{29}C_2 \times {}^{20}C_2}{{}^{50}C_5}$

17. (c) : $\log_{30} 8 = 3\log_{30} 2 = 3\log_{30}(30/15)$

$= 3(1 - \log_{30} 15) = 3(1 - \log_{30} 5 - \log_{30} 3) = 3(1 - a - b).$

18. (c) : The corners of the feasible bounded region are $O(0, 0)$, $A(4, 0)$, $C\left(\dfrac{36}{13}, \dfrac{12}{13}\right)$.



$z = 2x + 3y$

$\Rightarrow \quad z(O) = 0,\ z(A) = 8,$

$z(C) = \dfrac{108}{13}$

$\therefore \quad$ Maximum $z = \dfrac{108}{13}$ at C.

19. (d): X can take values 0, 1, 2 or 3

$$P(X=0) = \frac{{}^5C_3}{{}^9C_3} = \frac{5}{42},\ P(X=1) = \frac{({}^4C_1 \times {}^5C_2)}{{}^9C_3} = \frac{10}{21},$$

$$P(X=2) = \frac{({}^4C_2 \times {}^5C_1)}{{}^9C_3} = \frac{5}{14}, P(X=3) = \frac{{}^4C_3}{{}^9C_3} = \frac{1}{21}$$

Mean, $\mu = \Sigma p_i x_i$

$$= \left(\frac{5}{42}\times 0\right) + \left(\frac{10}{21}\times 1\right) + \left(\frac{5}{14}\times 2\right) + \left(\frac{1}{21}\times 3\right)$$

$$= \frac{20+30+6}{42} = \frac{56}{42} = \frac{4}{3}$$

Variance, $\sigma^2 = \Sigma p_i x_i^2 - \mu^2$

$$= \frac{5}{42}(0)^2 + \frac{10}{21}(1)^2 + \frac{5}{14}(2)^2 + \frac{1}{21}(3)^2 - \left(\frac{4}{3}\right)^2$$

$$= \frac{10}{21} + \frac{20}{14} + \frac{9}{21} - \frac{16}{9} = \frac{60+180+54-224}{126}$$

$$= \frac{70}{126} = \frac{5}{9}$$

20. (b): We have $y\sec x = \tan x + c$ ...(i)

Differentiating (i) w.r.t. x, we get

$$\Rightarrow \quad \frac{dy}{dx}\sec x + y\sec x\tan x = \sec^2 x$$

$$\Rightarrow \quad \frac{dy}{dx}+y\tan x-\sec x=0$$

21. (a) : Let $y = \sin^{-1}(2^x)$

$$\therefore \quad \frac{dy}{dx}=\frac{d}{dx}[\sin^{-1}(2^x)] = \frac{1}{\sqrt{1-(2^x)^2}}\cdot\frac{d}{dx}(2^x)$$

$$=\frac{1}{\sqrt{1-4^x}}\times 2^x\log 2=\frac{2^x\log 2}{\sqrt{1-4^x}}$$

22. (a)

23. (b) : (i) $f(x)$ is continuous on $[0, 6]$

(ii) $f(x)$ is differentiable on $(0, 6)$

Thus, all the conditions of Lagrange's Mean value theorem are satisfied.

We want to find $c \in (0, 6)$ such that

$$\frac{f(6)-f(0)}{6-0} = f'(c) \Rightarrow \frac{0-0}{6-0} = f'(c) \Rightarrow f'(c) = 0$$

Now $f(x) = 6x^2 - x^3 \Rightarrow f'(x) = 12x - 3x^2$

$\Rightarrow \quad f'(c) = 12c - 3c^2 \Rightarrow 0 = 12c - 3c^2$

$\Rightarrow \quad 3c^2 = 12c \Rightarrow c = 4$ which lies in the interval $(0, 6)$.

24. (a) : $f(x) = \frac{7}{x} - 3 \quad \therefore \ f'(x) = 7\left(\frac{-1}{x^2}\right) < 0$

for all $x \in R, x \neq 0$

$\therefore \quad f(x)$ is decreasing for all $x \in R$, where $x \neq 0$.

25. (d) : Let the first part of 20 be x.

Then the second part is $20 - x$.

$\therefore \quad$ Their product $= x(20 - x) = 20x - x^2 = f(x)$ (say)

$\therefore \quad f'(x) = 20 - 2x$ and $f''(x) = -2$

Now, $f'(x) = 0, \Rightarrow 20 - 2x = 0 \Rightarrow x = 10$

and $f''(10) = -2 < 0$

$\therefore \quad$ By the second derivative test, $f(x)$ is maximum at $x = 10$.

$\therefore \quad$ The parts of 20 are 10 and 10.

26. (c) : Let $I = \int\frac{dx}{\sin^2 x\cos^2 x} = \,= \int\frac{4}{4\sin^2 x\cos^2 x}dx$

$$= 4\int \operatorname{cosec}^2 2x\, dx = -2\cot 2x + c$$

27. (b) : Let $I = \int\frac{dx}{x^{\frac{1}{5}}\sqrt{1+x^{\frac{4}{5}}}}$

Putting $1 + x^{4/5} = t \Rightarrow \frac{4}{5}x^{-1/5}dx = dt$

$$\therefore \quad I = \frac{5}{4}\int\frac{dt}{\sqrt{t}} = \frac{5}{4}\int t^{-1/2}dt = \frac{5}{4}\cdot\frac{t^{1/2}}{1/2}+c$$

$$=\frac{5}{2}\sqrt{1+x^{4/5}}+c$$

28. (b) : $I = \int\frac{3\sin x+2\cos x}{3\cos x+2\sin x}dx$

Let $3\sin x + 2\cos x = A(3\cos x + 2\sin x) + B\frac{d}{dx}(3\cos x + 2\sin x)$

$\Rightarrow \quad 3\sin x + 2\cos x = A(3\cos x + 2\sin x) + B(-3\sin x + 2\cos x)$

$$-3B+2A=3 \text{ and } 2B+3A=2 \Rightarrow A=\frac{12}{13} \text{ and } B=-\frac{5}{13}$$

$$\therefore I = \int\frac{A(3\cos x+2\sin x)+B(-3\sin x+2\cos x)}{3\cos x+2\sin x}dx$$

$$= A\int 1\,dx + B\int\frac{-3\sin x+2\cos x}{3\cos x+2\sin x}dx$$

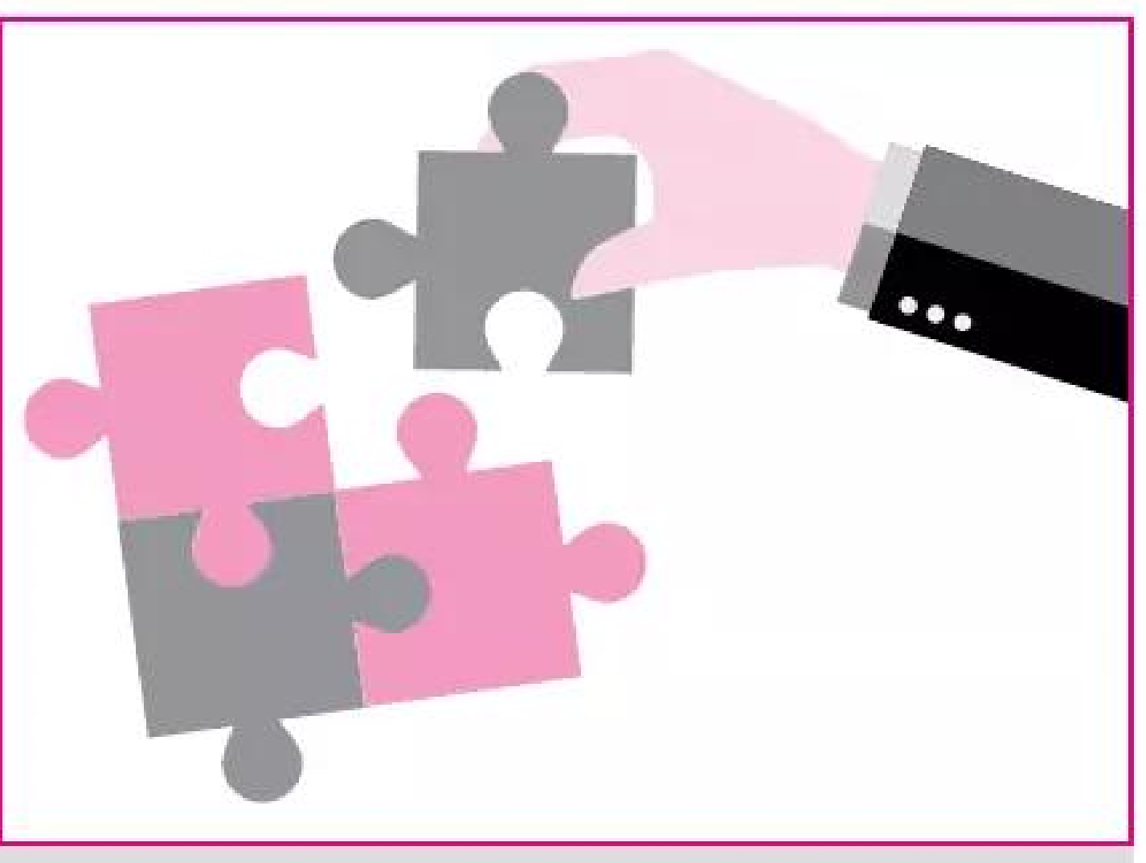
$$= \frac{12}{13}x - \frac{5}{13}\log|3\cos x + 2\sin x| + c$$

29. (b) : Let $t = x + c \ \therefore \ dt = dx$

Also, $x = b - c \Rightarrow t = (b - c) + c = b$

$x = 0 \Rightarrow t = 0 + c = c$

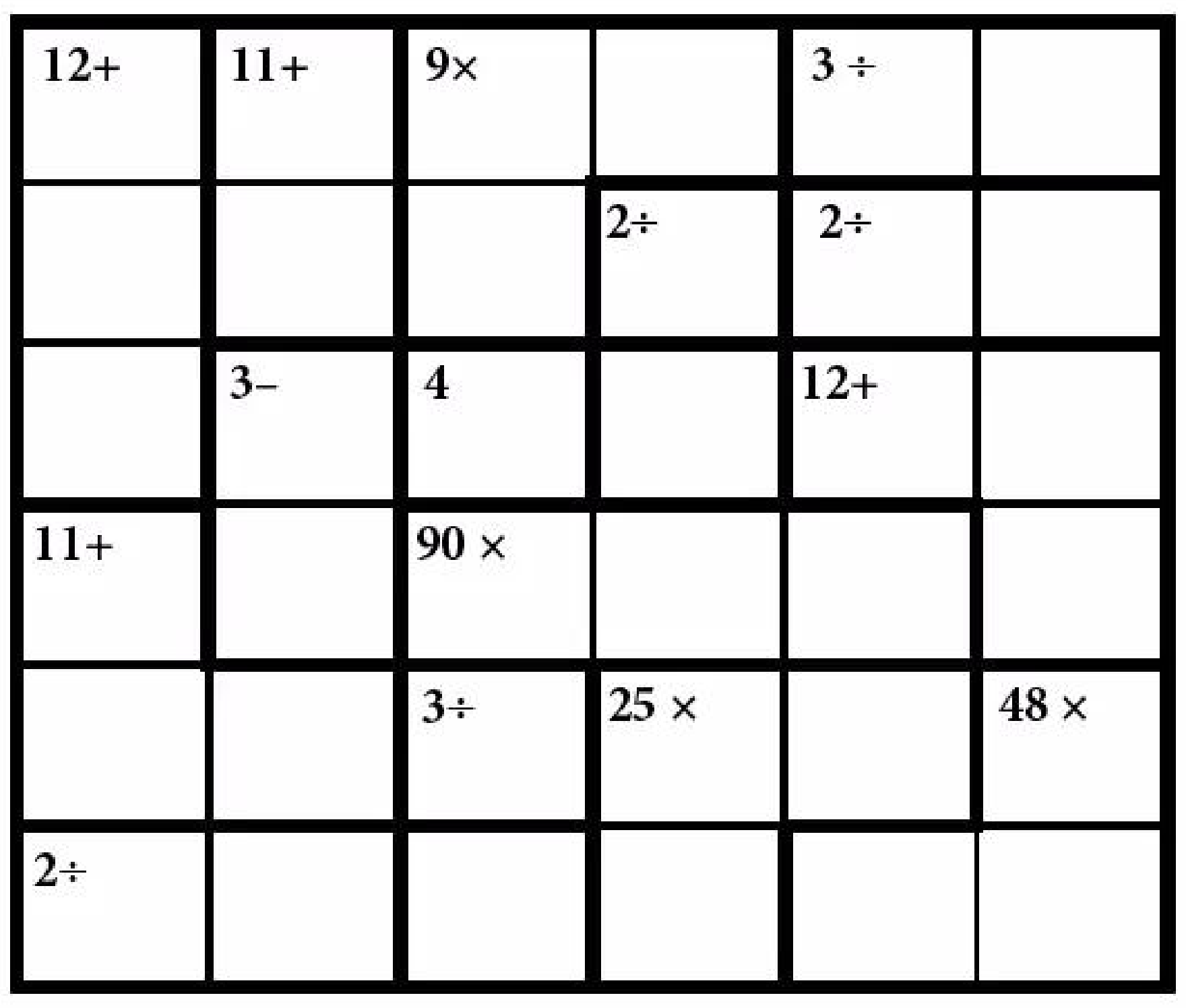
PUZZLE CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.



12+	11+	9×		3 ÷	
			2÷	2÷	
	3–	4		12+	
11+		90 ×			
		3÷	25 ×		48 ×
2÷					

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

$$\therefore \int_0^{b-c} f(x+c)dx = \int_c^b f(t)\,dt = -\int_b^c f(x)dx$$

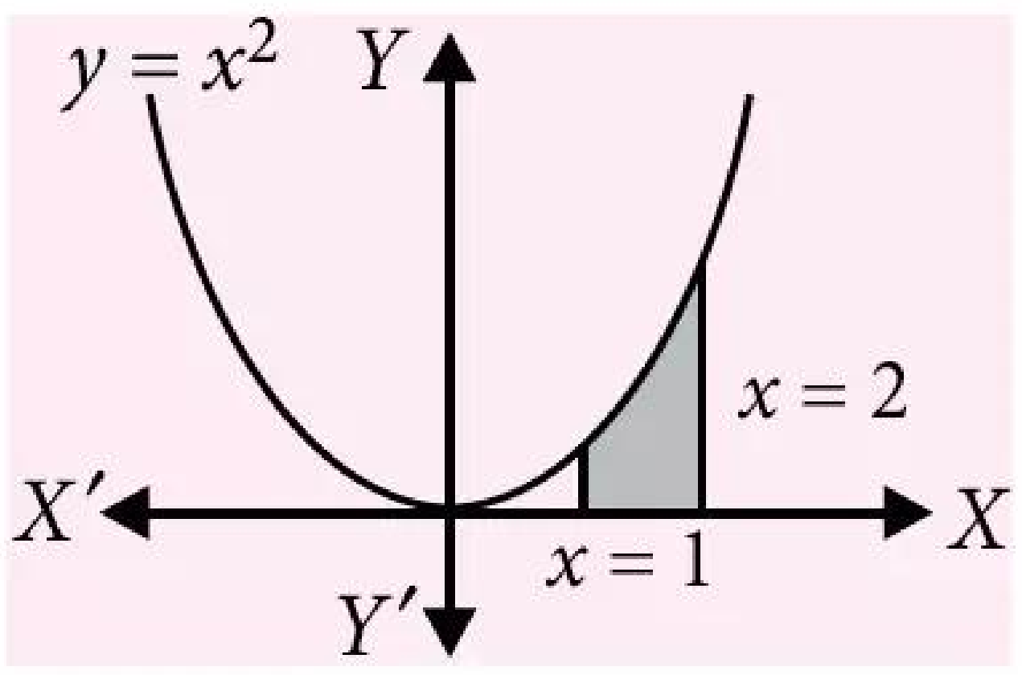
$$\therefore a\int_b^c f(x)\,dx = -\int_b^c f(x)dx \quad \therefore a = -1$$

30. (d): Given curve is $y = x^2$, and $x = 1, x = 2$

$$\therefore \quad \text{Required area} = \int_1^2 x^2\,dx$$

$$= \left[\frac{x^3}{3}\right]_1^2 = \left(\frac{8}{3} - \frac{1}{3}\right)$$

$$= \frac{7}{3}\text{ sq. units}$$



31. (c) : Slope of lines are

$$m_1 = \tan\left(\frac{\pi}{3}\right) = \sqrt{3} \text{ and } m_2 = \tan\left(\frac{5\pi}{3}\right) = -\sqrt{3}$$

Equation of the lines through origin is

$y = m_1x = \sqrt{3}x$ and $y = m_2x = -\sqrt{3}x$

Joint equation is

$$(\sqrt{3}x - y)(\sqrt{3}x + y) = 0 \Rightarrow 3x^2 - y^2 = 0$$

32. (a) : Let $\vec{a} = 3\hat{i} + 2\hat{j} - \hat{k}, \vec{b} = 6\hat{i} - 4x\hat{j} + y\hat{k}$

Since, $\vec{a}$ and $\vec{b}$ are parallel

$$\therefore \quad \vec{a} = m\vec{b} \Rightarrow 3\hat{i} + 2\hat{j} - \hat{k} = m(6\hat{i} - 4x\hat{j} + y\hat{k})$$

$$\Rightarrow \quad 3 = 6m \Rightarrow m = \frac{1}{2}$$

$$\text{Also, } -4xm = 2 \Rightarrow \frac{-4x}{2} = 2 \Rightarrow x = -1$$

$$\text{and } ym = -1 \Rightarrow \frac{y}{2} = -1 \Rightarrow y = -2$$

33. (b): We have,

$$[\vec{a}+\vec{b} \;\; \vec{b}+\vec{c} \;\; \vec{c}+\vec{a}] = (\vec{a}+\vec{b}) \cdot [(\vec{b}+\vec{c}) \times (\vec{c}+\vec{a})]$$

$$= (\vec{a}+\vec{b}) \cdot [\vec{b} \times \vec{c} + \vec{b} \times \vec{a} + \vec{c} \times \vec{a}]$$

$$= \vec{a} \cdot (\vec{b} \times \vec{c}) + \vec{b} \cdot (\vec{c} \times \vec{a}) = 2[\vec{a} \;\; \vec{b} \;\; \vec{c}]$$

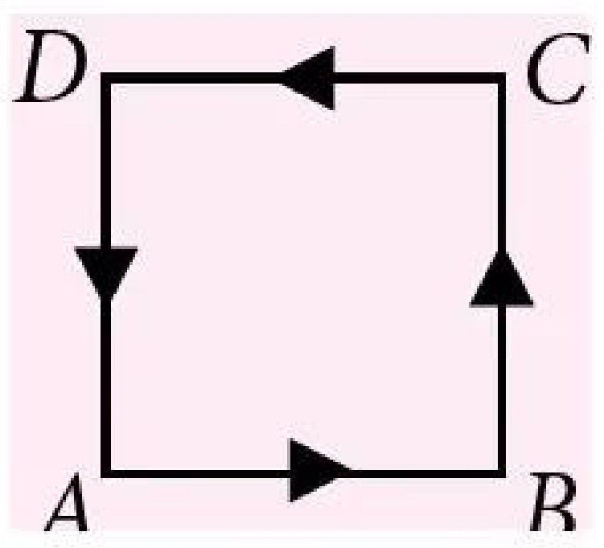
$$\therefore \quad \frac{[\vec{a}+\vec{b} \;\; \vec{b}+\vec{c} \;\; \vec{c}+\vec{a}]}{[\vec{a} \;\; \vec{b} \;\; \vec{c}]} = 2$$

34. (c) : $\overrightarrow{AB} + 2\overrightarrow{BC} + 3\overrightarrow{CD} + 4\overrightarrow{DA}$

$$= (\overrightarrow{AB} + \overrightarrow{BC}) + (\overrightarrow{BC} + \overrightarrow{DA}) + 3(\overrightarrow{CD} + \overrightarrow{DA})$$

$$= \overrightarrow{AC} + (\overrightarrow{BC} - \overrightarrow{BC}) + 3\overrightarrow{CA}$$

$$= -\overrightarrow{CA} + 3\overrightarrow{CA} = 2\overrightarrow{CA}.$$



35. (a) : We have, $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 1$

Now, $\cos 6\alpha + \cos 6\beta + \cos 6\gamma$

$= \cos 2(3\alpha) + \cos 2(3\beta) + \cos 2(3\gamma)$

$= 2\cos^2 3\alpha - 1 + 2\cos^2 3\beta - 1 + 2\cos^2 3\gamma - 1$

$= 2 \times 1 - 3 = -1$

36. (c) : Let $\vec{\alpha}$ be the position vector of point $P(4, 1, 1)$

$$\therefore \quad \vec{\alpha} = 4\hat{i} + \hat{j} + \hat{k}$$

Given equation of line in vector form is

$$(7\hat{i} + 7\hat{j} + 6\hat{k}) + \lambda(2\hat{i} + 2\hat{j} + 3\hat{k})$$

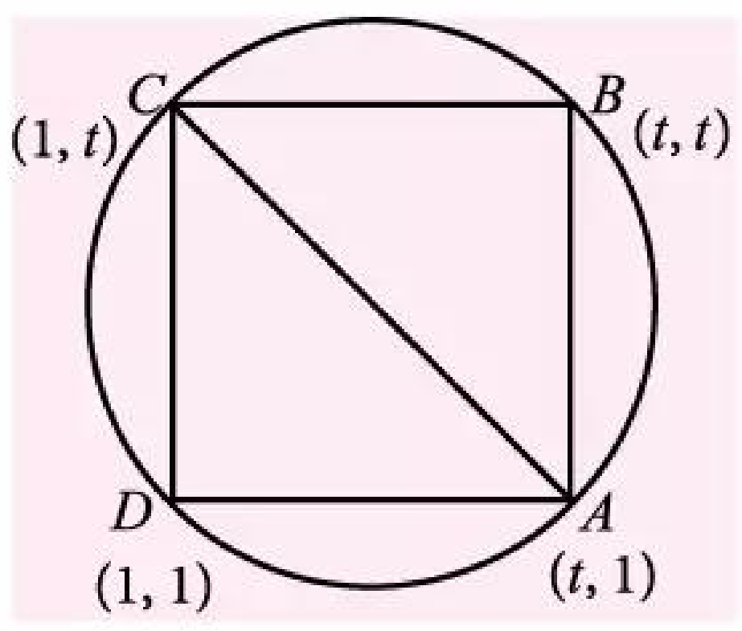
$$\therefore \quad \vec{a} = 7\hat{i} + 7\hat{j} + 6\hat{k}, \vec{b} = 2\hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{\alpha} - \vec{a} = -3\hat{i} - 6\hat{j} - 5\hat{k},$$

$$|(\vec{\alpha} - \vec{a}) \times \vec{b}| = \sqrt{101}, \; |\vec{b}| = \sqrt{17}$$

$$\therefore \quad \text{Required distance} = \left|\frac{(\vec{\alpha} - \vec{a}) \times \vec{b}}{\sqrt{\vec{b} \cdot \vec{b}}}\right| = \sqrt{\frac{101}{17}} \text{ units}$$

37. (d): The triangle ABC is isosceles right angled. Its circumcircle passes through the point $D(1, 1)$.



38. (b): $b^2 = a^2(e^2 - 1) = a^2\left(\frac{25}{16} - 1\right) = \frac{9a^2}{16}$

If e_1 is the eccentricity of the second curve,

$$a^2 = b^2(e_1^2 - 1) \text{ or } \frac{16}{9} = e_1^2 - 1$$

$$\Rightarrow \quad e_1^2 = \frac{16}{9} + 1 = \frac{25}{9} \Rightarrow e_1 = \frac{5}{3}.$$

39. (b): $x^3 + y^3$ is divisible by 3

$\Rightarrow x + y$ is divisible by 3 $\Rightarrow$ x, y are multiples of 3 or one leaves remainder 1 and the other 2 when divided by 3.

3, 6, 9, …, 30 are multiples of 3; 1, 4, 7, …, 28 leave remainder 1. 2, 5, 8, …, 29 leave remainder 2.

$$\therefore \text{ Required probability} = \frac{\binom{10}{1}\binom{10}{1} + \binom{10}{2}}{\binom{30}{2}}$$

$$= \frac{145}{15 \times 29} = \frac{1}{3}$$

40. (c) : $3\sin^{-1}x + (\sin^{-1}x + \cos^{-1}x) = \pi$

$$\therefore \quad 3\sin^{-1}x = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

$$\Rightarrow \quad \sin^{-1}x = \frac{\pi}{6} \Rightarrow x = \frac{1}{2}.$$

◆◆

MONTHLY TEST DRIVE CLASS XI ANSWER KEY

1.	(d)	**2.**	(d)	**3.**	(d)	**4.**	(a)	**5.**	(a)
6.	(c)	**7.**	(c)	**8.**	(c,d)	**9.**	(a,b)	**10.**	(a,c)
11.	(a,c)	**12.**	(a,b)	**13.**	(a,d)	**14.**	(c)	**15.**	(c)
16.	(a)	**17.**	(3)	**18.**	(4)	**19.**	(9)	**20.**	(2)

PRACTICE PAPER 2023

CUET (UG)

Exam on
21st to 31st
May 2023

General Instructions

This practice paper contains two sections i.e. Section A and Section B [B1 and B2].

Section A has 15 questions covering both i.e. Mathematics/Applied Mathematics which is compulsory for all candidates.

Section B1 has 30 questions from Mathematics out of which 20 questions need to be attempted.

Section B2 has 30 questions purely from Applied Mathematics out of which 20 question need to be attempted.

SECTION A

1. $\int x^2(ax+b)^{-2}dx$ is equal to
   (a) $\frac{2}{a^2}\left(x-\frac{b}{a}\log(ax+b)\right)+C$
   (b) $\frac{1}{a^3}\left(ax+b-\frac{b^2}{ax+b}-2b\log(ax+b)\right)+C$
   (c) $\frac{1}{a^2}\left(x+\frac{b}{a}\log(ax+b)\right)+\frac{x^2}{a(ax+b)}+C$
   (d) $\frac{1}{a^3}\left(x+\frac{b}{a}\log(ax+b)\right)-\frac{x^2}{a(ax+b)}+C$

2. Consider the following statements and choose the correct option.
   I. If A is any square matrix, then $\frac{A+A'}{2}$ is always skew-symmetric.
   II. If A and B are invertible matrices of the same order, then $A + B$ is also invertible.
   III. If A, B, C are three matrices such that both AB and AC are defined and are equal, then B and C are equal matrices.
   IV. For any matrix A, AA' is always defined and is a square matrix.
   V. For any square matrix A with real entries, $A-A'$ is a skew symmetric matrix.
   (a) Only IV and V are correct
   (b) Only II, III and IV are correct
   (c) Only I, III and IV are correct
   (d) All are correct

3. **Statement-I :** The area of ellipse $2x^2 + 3y^2 = 6$ will be more than the area bounded by $2|x| + 3|y| \le 6$.
   Statement-II : The length of major axis of the ellipse $2x^2 + 3y^2 = 6$ is less than the distance between the points of $2|x| + 3|y| \le 6$ on X-axis.
   In the light of above statements, choose the correct answer from the options given below.
   (a) Both statement I and statement II are true
   (b) Both statement I and statement II are false
   (c) Statement I is true but statement II is false
   (d) Statement I is false but statement II is true

4. The maximum value of $\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1+\cos\theta & 1 & 1 \end{vmatrix}$ is (θ is real number)
   (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) $\frac{2\sqrt{3}}{4}$

5. Three coins are tossed. Consider the events E : 'three heads or three tails', F : 'atleast two heads', G : 'atmost two heads'. Which of the following events are independent?
   (a) E and F (b) E and G
   (c) F and G (d) All of these

6. **Statement-I :** Given E_i (i = 1, 2, ..., n) are n independent events such that , $P(E_i) = \frac{i}{i+1}$, $1 \le i \le n$, the probability that none of the n events occur is $\frac{1}{n}$.
   Statement-II : Probability of simultaneous occurrence of independent events is equal to the product of the probabilities of these events.
   In the light of above statements, choose the correct answer from the options given below.
   (a) Both statement I and statement II are true
   (b) Both statement I and statement II are false
   (c) Statement I is true but statement II is false
   (d) Statement I is false but statement II is true

7. If $A^2 - A + I = O$, then the inverse of A is
   (a) $I - A$ (b) $A - I$ (c) A (d) $A + I$

8. Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Then, the probability that there is atleast one defective egg is

(a) $1-\frac{7^{10}}{10^{10}}$　(b) $1+\frac{7^{10}}{10^{10}}$

(c) $1+\frac{9^{10}}{10^{10}}$　(d) $1-\frac{9^{10}}{10^{10}}$

9. The mean and the variance of a binomial distribution are 4 and 2 respectively. Find the probability of atleast 6 successes.

(a) $\frac{37}{256}$　(b) $\frac{32}{255}$　(c) $\frac{34}{259}$　(d) $\frac{31}{256}$

10. Find the combined resistance R of two resistors R_1 and $R_2(R_1, R_2 > 0)$ is given by $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$.

If $R_1 + R_2 = C$ (a constant), then maximum resistance R is obtained if

(a) $R_1 > R_2$　(b) $R_1 < R_2$

(c) $R_1 = R_2$　(d) None of these

11. Match Column I with Column II and select the correct answer by choosing an appropriate option.

Column I		Column II	
(P)	The area bounded by the curve $x = 4 - y^2$, the Y-axis is	(1)	$\frac{14}{3}$ sq. units
(Q)	The area between parabolas $y^2 = 7x$ and $x^2 = 7y$ is	(2)	$\frac{8}{15}$ sq. unit
(R)	The area bounded by the parabola $y = 4x^2$ and the lines $y = 1$, $y = 4$ is	(3)	$\frac{49}{3}$ sq. units
(S)	The area of the loop of the curve $y^2 = x(1 - x)^2$ is	(4)	$\frac{32}{3}$ sq. units

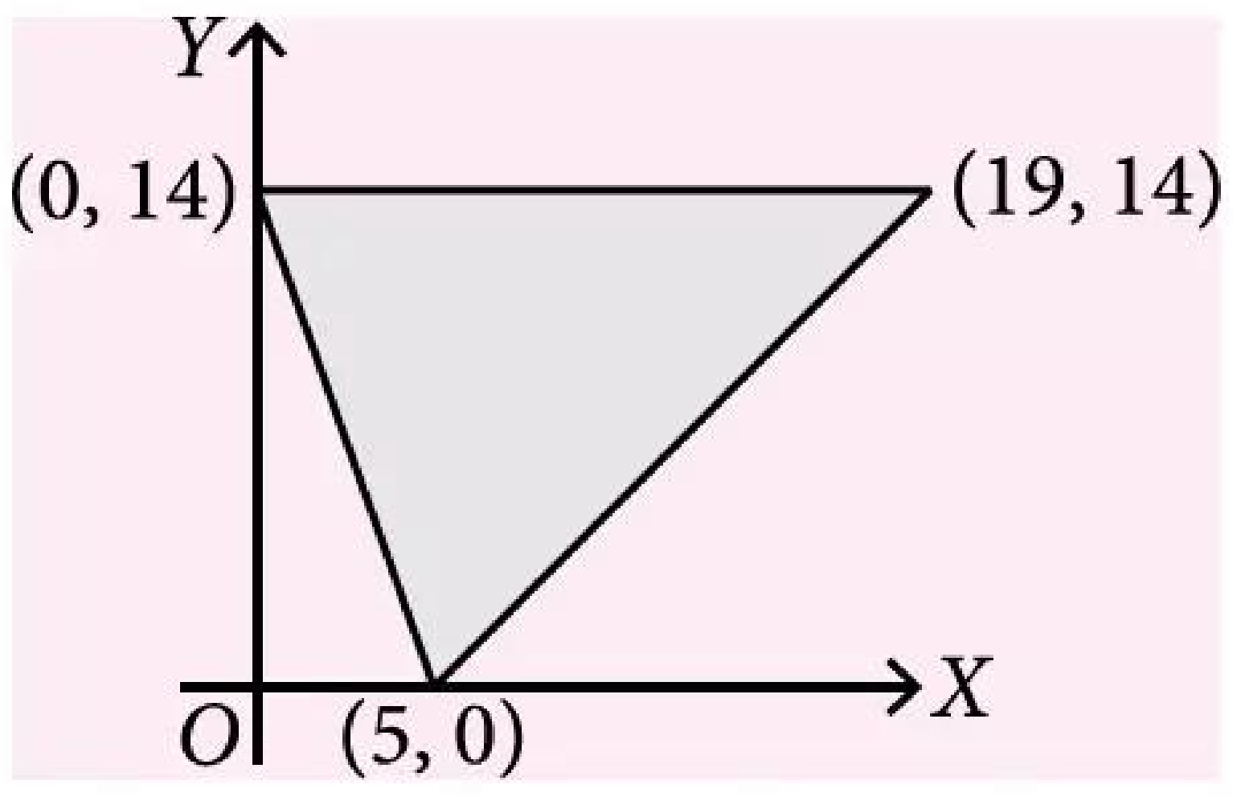
(a) (P) - (4), (Q) - (3), (R) - (1), (S) - (2)
(b) (P) - (2), (Q) - (1), (R) - (4), (S) - (3)
(c) (P) - (1), (Q) - (4), (R) - (2), (S) - (3)
(d) (P) - (3), (Q) - (4), (R) - (2), (S) - (1)

12. Match the differential equation given in Column II with their respective solution given in Column I.

Column I		Column II	
(P)	$y = ae^{3x} + be^{5x}$	(1)	$a + b[(y')^2 + yy''] = 0$
(Q)	$ax^2 + by^2 = 1$	(2)	$(y^2 - 2y + 2)y'' + (2y - 2)(y')^2 + e^y y' = 0$
(R)	$xe^y - y^2 = 2$	(3)	$y'' - 8y' + 15y = 0$

(a) (P) - (2), (Q) - (3), (R) - (1)
(b) (P) - (3), (Q) - (2), (R) - (1)
(c) (P) - (3), (Q) - (1), (R) - (2)
(d) (P) -(1), (Q) - (2), (R) - (3)

13. The shaded region shown in fig. is given by the inequation



(a) $14x + 5y \geq 70$, $y \leq 14$ and $x - y \geq 5$
(b) $14x + 5y \geq 70$, $y \geq 14$ and $x - y \geq 5$
(c) $14x + 5y \geq 70$, $y \leq 14$ and $x - y \leq 5$
(d) $14x + 5y \leq 70$, $y \leq 14$ and $x - y \geq 5$

14. Consider the following statements and choose the correct option.

I. The solution of $\frac{dy}{dx}=\left(\frac{y}{x}\right)^{1/3}$ is $y^{2/3} + x^{2/3} = c$

II. Number of arbitrary constants in the particular solution of a differential equation of order two is two.

III. The differential equation representing the family of circles $x^2 + (y - a)^2 = a^2$ will be of order two.

IV. The solution of the differential equation $\frac{dy}{dx}=\frac{x+2y}{x}$ is $x + y = kx^2$.

V. There is no arbitrary constants in the particular solution of a differential equation.

(a) I, II and III are correct
(b) III, IV and V are correct
(c) IV and V are correct
(d) All are correct

15. It is given that at $x = 1$, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval [0, 2]. Find the value of a.

(a) 100　(b) 120　(c) 140　(d) 160

SECTION B1 (MATHEMATICS)

16. The domain of the function

$$f(x)=\frac{1}{\sqrt{\{\sin x\}+\{\sin(\pi+x)\}}}$$

where {·} denotes fractional part, is

(a) $[0, \pi]$　(b) $(2n + 1)\pi/2, n \in Z$

(c) $(0, \pi)$　(d) None of these

17. $\cos^{-1}\left\{\frac{1}{2}x^2+\sqrt{1-x^2}\sqrt{1-\frac{x^2}{4}}\right\}=\cos^{-1}\frac{x}{2}-\cos^{-1}x$

holds for

(a) $|x| \le 1$ (b) $x \in R$
(c) $0 \le x \le 1$ (d) $-1 \le x \le 0$

18. If ω is a complex cube root of unity, then the matrix

$$A = \begin{vmatrix} 1 & \omega^2 & \omega \\ \omega^2 & \omega & 1 \\ \omega & 1 & \omega^2 \end{vmatrix} \text{ is}$$

(a) symmetric matrix
(b) diagonal matrix
(c) skew-symmetric matrix
(d) none of these

19. The cofactors of each element of the first column of the matrix $A = \begin{bmatrix} g & f & c \\ h & b & f \\ a & h & g \end{bmatrix}$

(a) $(bg - hf), (ch - fg), 1$
(b) $(bg - hf), (ch - fg), (f^2 - bc)$
(c) $0, 0, 0$
(d) $(bg - hf), (ch - fg), 0$

20. Let $f(x) \cdot f(y) = f(x + y)$ for all x, y. If $f(5) = 2$ and $f'(0) = 3$, then $f'(5)$ is equal to

(a) 5 (b) 6
(c) 0 (d) None of these

21. If $y = \cos^2\left(\frac{3x}{2}\right) - \sin^2\left(\frac{3x}{2}\right)$, then $\frac{d^2y}{dx^2}$ is equal to

(a) $-3\sqrt{1-y^2}$ (b) $9y$
(c) $-9y$ (d) $3\sqrt{1-y^2}$

22. Consider the following statements and choose the correct option.

I. $\int \frac{\sin x + \cos x}{\sqrt{1+\sin 2x}} dx = x + C$

II. $\int \frac{dx}{1+\cos x} = \tan\frac{x}{2} + C$

III. $\int \left(\frac{x^2+2}{x+1}\right) dx = \frac{x^2}{2} - x + \log|x+1| + C$

IV. $\int \frac{dx}{\sqrt{16-9x^2}} = \frac{1}{4}\sin^{-1}\left(\frac{3x}{4}\right) + C$

V. $\int \frac{x+3}{(x+4)^2} e^x dx = e^x\left(\frac{1}{x+4}\right) + C$

(a) II, III and IV are correct
(b) II, IV and V are correct
(c) I, II and V are correct
(d) All are correct.

23. Let a solution $y = y(x)$ of the differential equation $x\sqrt{x^2-1}\,dy - y\sqrt{y^2-1}\,dx = 0$ satisfy $y(2) = \frac{2}{\sqrt{3}}$.

Statement-I : $y(x) = \sec\left(\sec^{-1} x - \frac{\pi}{6}\right)$.

Statement-II : $y(x)$ is given by $\frac{1}{y} = \frac{2\sqrt{3}}{x} - \sqrt{1 - \frac{1}{x^2}}$

In the light of above statements, choose the correct answer from the options given below.

(a) Both statement I and statement II are true
(b) Both statement I and statement II are false
(c) Statement I is true but statement II is false
(d) Statement I is false but statement II is true

24. Match the following :

	Column I		Column II
(P)	The function $f(x) = -\tan^{-1} x$ is	(1)	$k^2 = 8$
(Q)	The function $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is, where $x \in (1, \infty)$	(2)	decreasing
(R)	The curves $2x = y^2$ and $2xy = k$ cut orthogonally if	(3)	increasing

(a) (P) → (2), (Q) → (3), (R) → (1)
(b) (P) → (1), (Q) → (2), (R) → (2)
(c) (P) → (3), (Q) → (2), (R) → (1)
(d) (P) → (2), (Q) → (1), (R) → (3)

25. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linearly dependent vectors and $|\vec{c}| = \sqrt{3}$, then

(a) $\alpha = 1, \beta = -1$ (b) $\alpha = 1, \beta = \pm 1$
(c) $\alpha = -1, \beta = \pm 1$ (d) $\alpha = \pm 1, \beta = 1$

26. **Statement-I :** The equation of the plane through the intersection of the planes $x + y + z = 6$ and $2x + 3y + 4z + 5 = 0$ and the point $(4, 4, 4)$ is $29x + 23y + 17z = 276$.

Statement-II : Equation of the plane through the line of intersection of the planes $P_1 = 0$ and $P_2 = 0$ is $P_1 + \lambda P_2 = 0, \lambda \ne 0$.

In the light of above statements, choose the correct answer from the options given below.

(a) Both statement I and statement II are true
(b) Both statement I and statement II are false
(c) Statement I is true but statement II is false
(d) Statement I is false but statement II is true

27. In solving the LPP : "minimize $f = 6x + 10y$ subject to constraints $x \ge 6, y \ge 2, 2x + y \ge 10, x \ge 0, y \ge 0$" redundant constraints are

(a) $x \ge 6, y \ge 2$ (b) $2x + y \ge 10, x \ge 0, y \ge 0$
(c) $x \ge 6$ (d) none of these

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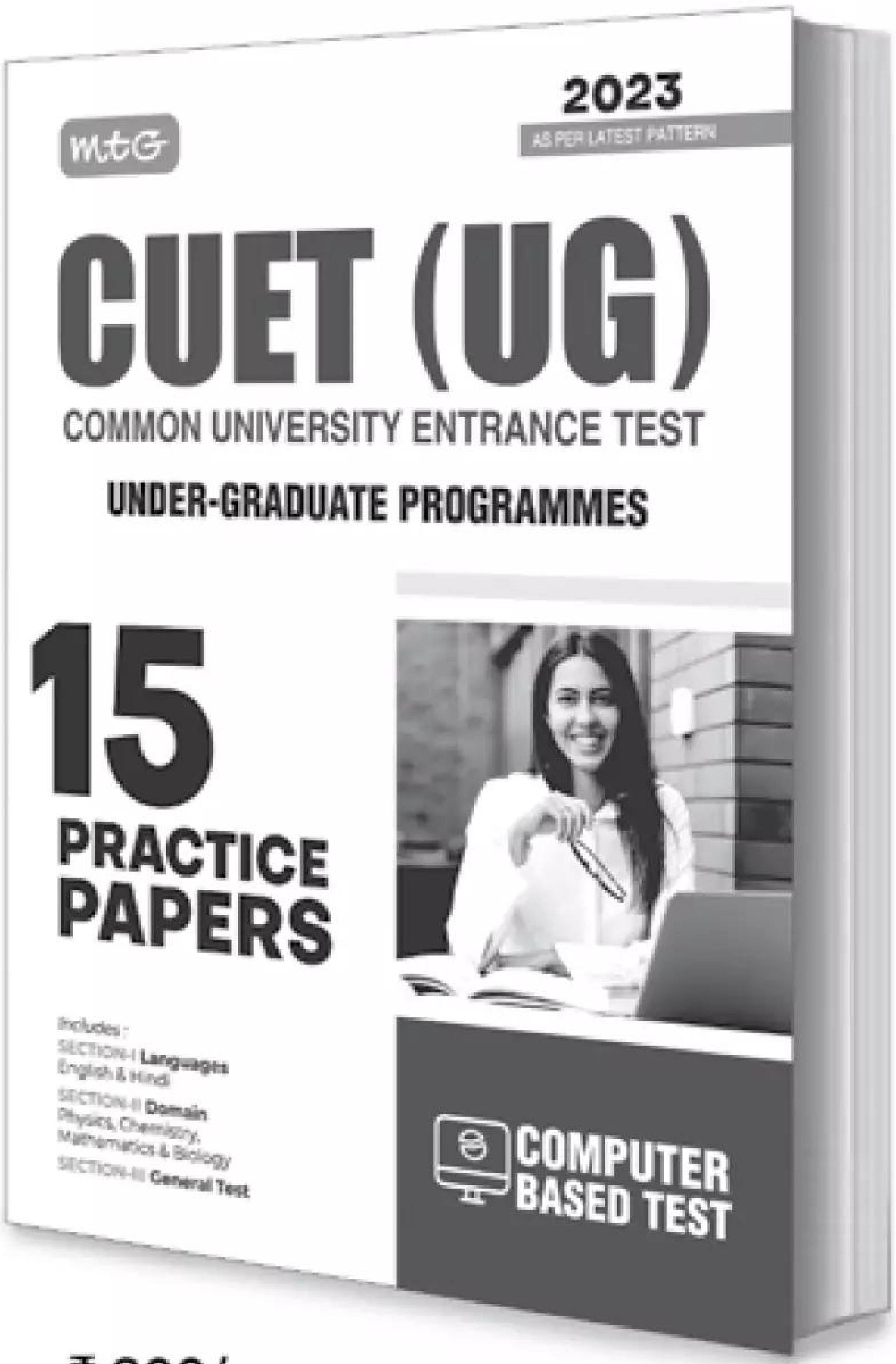

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28. A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
$P(X)$	a	$4a$	$3a$	$7a$	$8a$	$10a$	$6a$	$9a$

Find $P(X < 3)$, $P(X \geq 4)$, $P(0 < X < 5)$ respectively.

(a) $\frac{1}{6}, \frac{11}{24}, \frac{33}{48}$ (b) $\frac{1}{6}, \frac{33}{48}, \frac{11}{24}$

(c) $\frac{1}{4}, \frac{11}{26}, \frac{21}{44}$ (d) $\frac{11}{26}, \frac{1}{4}, \frac{21}{44}$

29. If $f(x) = (ax^2 + b)^3$, then the function g such that $f(g(x)) = g(f(x))$ is given by

(a) $g(x) = \left(\frac{b - x^{1/3}}{a}\right)$ (b) $g(x) = \frac{1}{(ax^2 + b)^3}$

(c) $g(x) = (ax^2 + b)^{1/3}$ (d) $g(x) = \left(\frac{x^{1/3} - b}{a}\right)^{1/2}$

30. If the equations $x + ay - z = 0$, $2x - y + az = 0$, $ax + y + 2z = 0$ have non-trivial solutions, then $a =$

(a) 2 (b) -2 (c) $\sqrt{3}$ (d) $-\sqrt{3}$

31. **Statement-I :** If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non-coplanar vectors, such that $\vec{r} \cdot \vec{a} = \vec{r} \cdot \vec{b} = \vec{r} \cdot \vec{c} = 0$, then $\vec{r}$ is the null vector.

Statement-II : If $\vec{r}$ be a non-zero vector satisfying $\vec{r} \cdot \vec{a} = \vec{r} \cdot \vec{b} = \vec{r} \cdot \vec{c} = 0$ for given non-zero vectors $\vec{a}, \vec{b}$ and $\vec{c}$, then $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar vectors.

In the light of above statements, choose the correct answer from the options given below.

(a) Both statement I and statement II are true
(b) Both statement I and statement II are false
(c) Statement I is true but statement II is false
(d) Statement I is false but statement II is true

32. Match the following :

	Column I		**Column II**
(i)	The equation of the plane through the points $(2,1,0), (3,-2,-2)$ and $(3, 1, 7)$ is	(p)	$x + y + 2z - 19 = 0$
(ii)	The equation of a plane which bisects perpendicularly the line joining the points $A(2, 3, 4)$ and $B(4, 5, 8)$ is	(q)	$3x - 2y + 6z - 27 = 0$
(iii)	The equation of the plane through the points $(2,1,-1), (-1,3,4)$ and perpendicular to the plane $x - 2y + 4z = 10$ is	(r)	$7x + 3y - z - 17 = 0$
(iv)	If the line drawn from the point $(-2, -1, -3)$ meets a plane at right angle at the point $(1, -3, 3)$, then the equation of the plane is	(s)	$18x + 17y + 4z - 49 = 0$

(a) (i)$\rightarrow$(q), (ii)$\rightarrow$(r), (iii)$\rightarrow$(s), (iv)$\rightarrow$(p)
(b) (i)$\rightarrow$(r), (ii)$\rightarrow$(p), (iii)$\rightarrow$(s), (iv)$\rightarrow$(q)
(c) (i)$\rightarrow$(p), (ii)$\rightarrow$(q), (iii)$\rightarrow$(r), (iv)$\rightarrow$(s)
(d) (i)$\rightarrow$(r), (ii)$\rightarrow$(p), (iii)$\rightarrow$(q), (iv)$\rightarrow$(s)

33. Consider the following statements and choose the correct option.

I. At (3, 2) and (–1, 2) on the curve $x^2 + y^2 - 2x - 4y + 1 = 0$, the tangents are parallel to y-axis.
II. $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function on $(0, \pi/4)$.
III. $f(x) = \sin x + \sqrt{3}\cos x$ has maximum value at $\frac{\pi}{3}$.
IV. $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function on $(0, \pi)$
V. $f(x) = \sin x + \sqrt{3}\cos x$ has maximum value at $\frac{\pi}{6}$

(a) II, III and IV are correct
(b) I, II and V are correct
(c) III, IV and V are correct
(d) All are correct.

34. If $x \cos \frac{y}{x}(x dy + y dx) = y \sin \frac{y}{x}(x\, dy - y\, dx)$, then

(a) $\cos \frac{y}{x} = cxy$ (b) $\sec \frac{y}{x} = cxy$

(c) $x \cos(xy) = cy$ (d) $x \sec(xy) = cy$

35. The area of the region enclosed by the lines $y = x$, $x = e$ and curve $y = \frac{1}{x}$ and the positive x-axis is

(a) 1 sq. unit (b) $\frac{3}{2}$ sq. units

(c) $\frac{5}{2}$ sq. units (d) $\frac{1}{2}$ sq. unit

Case Based MCQs

Case I : The integral $\int f(x)dx$ can be transformed into another form by changing the independent variable x to t by substituting $x = g(t)$.

Consider $I = \int f(x)dx$

Put $x = g(t)$ so that $\frac{dx}{dt} = g'(t)$

We write $dx = g'(t)dt$

Thus, $I = \int f(x)dx = \int f(g(t))g'(t)dt$

This change of variable formula is one of the important tools available to us in the name of integration by substitution.

Base on the above information, answer the questions, from 36 to 40.

36. Evaluate : $\int 2x\sin(x^2+1)dx$.

(a) $-\sin(x^2 + 1) + C$ (b) $-\cos(x^2 + 1) + C$
(c) $\sin(x^2 + 1) + C$ (d) None of these

37. Evaluate : $\int \frac{x}{\sqrt{32-x^2}}dx$

(a) $-\sqrt{32-x^2}+C$ (b) $\sqrt{32+x^2}+C$
(c) $\sqrt{64-x^2}+C$ (d) $\sqrt{32-x^2}+C$

38. Evaluate : $\int \frac{\sin(2\tan^{-1}x)}{(1+x^2)}dx$

(a) $-\frac{\sin 2(\tan^{-1}x)}{2}+C$ (b) $-\frac{\cos 2(\tan^{-1}x)}{2}+C$
(c) $\frac{\cos 2(\tan^{-1}x)}{2}+C$ (d) $\frac{\sin 2(\tan^{-1}x)}{2}+C$

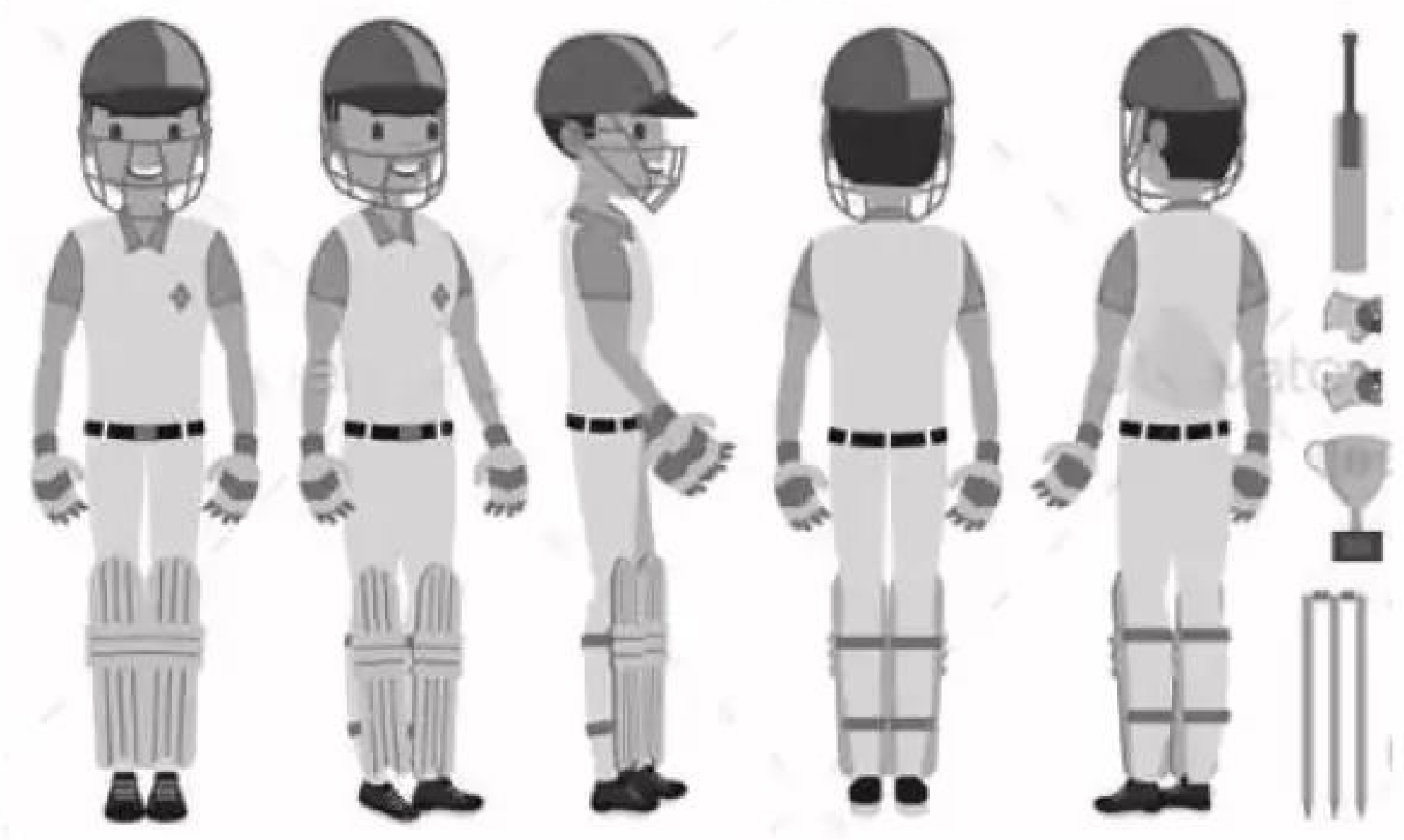
39. Solve : $\int \frac{\sqrt{1+x^2}}{x^4}dx$

(a) $-\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2}+C$ (b) $\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2}+C$
(c) $\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{1/2}+C$ (d) None of these

40. Evaluate: $\int \frac{\cos^{-1}x}{\sqrt{1-x^2}}dx$

(a) $\frac{(\sin^{-1}x)^2}{2}+C$ (b) $\frac{(-\sin^{-1}x)^2}{2}+C$
(c) $\frac{-(\cos^{-1}x)^2}{2}+C$ (d) $\frac{(\tan^{-1}x)^2}{2}+C$

Case II : A cricket match is organised between two Clubs *A* and *B* for which a team from each club is chosen. Remaining players of Club *A* and Club *B* are respectively sitting on the plane represented by the equation. $\vec{r}\cdot(2\hat{i}-\hat{j}+\hat{k})=3$ and $\vec{r}\cdot(\hat{i}+3\hat{j}+2\hat{k})=8$ to cheer the team of their own clubs.



Base on the above information, answer the questions from 41 to 45.

41. The Cartesian equation of the plane on which players of Club *A* are seated, is
(a) $2x - y + z = 3$ (b) $2x - y + 2z = 3$
(c) $2x - y + z = -3$ (d) $x - y + z = 3$

42. The magnitude of the normal to the plane on which players of Club *B* are seated, is
(a) $\sqrt{15}$ (b) $\sqrt{14}$ (c) $\sqrt{17}$ (d) $\sqrt{20}$

43. The intercept form of the equation of the plane on which players of Club *B* are seated, is

(a) $\frac{x}{8}+\frac{y}{8/3}+\frac{z}{2}=1$ (b) $\frac{x}{5}+\frac{y}{8/3}+\frac{z}{3}=1$
(c) $\frac{x}{8}+\frac{y}{\frac{8}{3}}+\frac{z}{4}=1$ (d) $\frac{x}{8}+\frac{y}{7}+\frac{z}{2}=1$

44. Which of the following is a player of Club *B*?
(a) Player sitting at (1, 2, 1)
(b) Player sitting at (0, 1, 2)
(c) Player sitting at (1, 4, 1)
(d) Player sitting at (1, 1, 2)

45. The distance of the plane, on which players of Club *B* are seated, from the origin is

(a) $\frac{8}{\sqrt{14}}$ units (b) $\frac{6}{\sqrt{14}}$ units
(c) $\frac{7}{\sqrt{14}}$ units (d) $\frac{9}{\sqrt{14}}$ units

SECTION B2 (APPLIED MATHEMATICS)

16. Three varieties *A*, *B* and *C* of tea are mixed together in the ratio 1 : 2 : 1 respectively. The price of tea *A* is ₹120 per kg and that of tea *C* is ₹ 130 per kg. If the price of the mixture is ₹ 150 per kg, then find the price per kg of tea *B*.
(a) ₹ 150 per kg (b) ₹ 175 per kg
(c) ₹ 190 per kg (d) None of these

17. Find the absolute maximum of $f(x) = 2x^3 - 9x^2 + 12x - 5$ in [0, 3].
(a) 4 (b) –5 (c) 3 (d) 0

18. Find the values of *a* and *b* for which the following hold:

$$\begin{bmatrix} a & b \\ -a & 2b \end{bmatrix}\begin{bmatrix} 2 \\ -1 \end{bmatrix}=\begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

(a) $a = 1, b = 2$ (b) $a = -2, b = 3$
(c) $a = 1, b = -3$ (d) $a = -2, b = -3$

19. Mr. Sharma borrowed ₹1000000 from a bank to purchase a house and decided to repay the loan

by equal monthly installments in 10 years. If bank charges interest at 9% p.a. compounded monthly, calculate the EMI. (Given $(1.0075)^{120} = 2.4514$)
(a) ₹12667 (b) ₹12825 (c) ₹13502 (d) ₹13648

20. For the probability distribution:

X	-4	-3	-2	-1	0
$P(X)$	0.1	0.2	0.3	0.2	0.2

$E(X)$ is equal to
(a) 0 (b) -1 (c) -2 (d) -1.8

21. Match the following column

Column I		Column II	
(P)	21 mod (2)	(1)	8
(Q)	(8×7) mod 6	(2)	1
(R)	12 mod 4	(3)	2
(S)	$(9 + 23)$ mod 12	(4)	0

(a) (P) → (1), (Q) → (3), (R) → (4), (S) → (2)
(b) (P) → (2), (Q) → (3), (R) → (4), (S) → (1)
(c) (P) → (1), (Q) → (4), (R) → (3), (S) → (2)
(d) (P) → (2), (Q) → (4), (R) → (3), (S) → (1)

22. Find the solution set of system of linear inequalities $4x - 5 < 11, -3x - 4 \geq 8$.
(a) $[4, \infty)$ (b) $[-4, 4]$
(c) $(-\infty, -4]$ (d) None of these

23. If the total cost function is given by $C = \frac{x^3}{4} + 9x^2 - 7$, then find the average cost.
(a) $\frac{x^4}{4} + 9x^3 - 7x$ (b) $\frac{3x^2}{4} + 18x$
(c) $\frac{x^2}{4} + 9x - 7$ (d) $\frac{x^2}{4} + 9x - \frac{7}{x}$

24. Consider the following statements:
Statement-I: The term population means only group of people or animate creatures.
Statement-II: A sample is a smaller group of members of a population selected to represent the population
In the light of above statements, choose the correct answer from the options given below.
(a) Both statement I and II are correct.
(b) Both statement I and II are incorrect.
(c) Statement I is correct but statement II is incorrect.
(d) Statement II is correct but statement I is incorrect.

25. A bond that matures in 5 years has coupon rate of 10% per annum and has a face value of ₹10000. Find the fair value of bond if the yield to maturity is 8%.
(a) ₹10798 (b) ₹11798
(c) ₹12987 (d) None of these

26. A man can swim a certain distance downstream in 4 hours and returns back the same distance upstream in 8 hours. If the speed of stream is 2km/h, then the speed of man in still water is
(a) 4 km/h (b) 6 km/h (c) 8 km/h (d) 12 km/h

27. Consider the following LPP
Maximize $Z = 5x + 2y$, subject to the constraints $3x + 5y \leq 15, 5x + 2y \leq 10, x, y \geq 0$.
Z is maximum at
(a) $(2, 0)$ (b) $\left(\frac{20}{8}, \frac{45}{8}\right)$
(c) $(6, 0)$ (d) $(0, 3)$

28. The function $f(x) = x^3 - 3x^2 + 3x$ is increasing on
(a) $[-1, 1]$ (b) $[0, \infty)$ (c) $(-\infty, 1)$ (d) R

29. Ramesh and Mohan invest ₹3500000 and ₹4200000 respectively in a business. If the profit during the year was ₹220000, then find Ramesh's share of profit.
(a) 50000 (b) 90000 (c) 100000 (d) 120000

30. If $y = xe^x - e^{-x}$, then the value of $y'' - y$ at $x = 0$ is
(a) e (b) 2 (c) 1 (d) 0

31. The annual depreciation of a scooter is ₹ 12000. If the scrap value of the scooter after 10 years is ₹ 55000, find the original cost of scooter.
(a) ₹150000 (b) ₹165000
(c) ₹175000 (d) ₹180000

32. The random variable X can take only the values 0, 1, 2, 3. Given that $P(X = 0) = P(X = 1) = p$ and $P(X = 2) = P(X = 3)$ such that $\sum p_i x_i^2 = 2 \sum p_i x_i$, find the value of p.
(a) $\frac{1}{7}$ (b) $\frac{2}{5}$ (c) $\frac{1}{9}$ (d) $\frac{3}{8}$

33. $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ satisfies $A^T A = I_3$ then
I. Value of x is $\pm\frac{1}{2}$
II. Value of y is $\pm\frac{1}{\sqrt{6}}$
III. Value of z is $\pm\frac{1}{\sqrt{3}}$
IV. $2x^2 + 6y^2 + 3z^2 = 1$
V. $x^2 + y^2 + z^2 = 1$
Which of the given statements are true?
(a) II, III, V (b) I, II, III, V
(c) II, III, IV (d) All of these

34. Generation of electricity from solar PV system is which of the following time series component.
(a) Cyclical compound (b) Seasonal component
(c) Irregular compound (d) Trend component

35. Consider the following statements
I. The general LPP calls for optimizing a linear function for variables called the constraints or restrictions
II. Linear scale is involved in LPP.
III. A feasible solution to the linear programming problem should satisfy the problem constraints.
IV. If no feasible region is obtained *via* plotting the constraints, then the linear programming problem has infeasible region.

Which of the above statements are correct?
(a) I, II, III, IV (b) I, II, IV
(c) I, IV (d) IV only

36. Two pipes can fill a tank in 20 minutes and 24 minutes respectively and a waste pipe can empty 3 gallons of water per minute. If all the three pipes working together can fill the tank in 15 minutes, find the capacity of the tank.
(a) 100 gallons (b) 110 gallons
(c) 120 gallons (d) 130 gallons

37. The rate of change of total cost (y) of a commodity per unit change of output (x) is called the marginal cost of the commodity. If $y = 3x\left(\frac{x+7}{x+5}\right)+5$, then the marginal cost will
(a) increase as the output increase
(b) not affect as the output increase
(c) decrease as the output increase
(b) increase as the input increase

38. At what rate of interest will the present value of a perpetuity of ₹750 payable at the end of every 6 months be ₹20000?
(a) 10% (b) 9% (c) 8.5% (d) 7.5%

39. If p_1 = Price of current year, q_1 = Quantity in the current year, p_0 = Price in the base year and q_0 = Quantity in the base year. Then, match the index number given in column I with relative formula in column II.

Column I		Column II	
(P)	Paasche's Index Number	(1)	$\sqrt{\frac{\Sigma p_1q_0}{\Sigma p_0q_0}\times\frac{\Sigma p_1q_1}{\Sigma p_0q_1}}\times 100$
(Q)	Fisher's Ideal Index Number	(2)	$\frac{\Sigma p_1q_1}{\Sigma p_0q_1}\times 100$
(R)	Laspeyre's Index Number	(3)	$\frac{\Sigma p_1q_0}{\Sigma p_0q_0}\times 100$

(a) (P) → (3), (Q) → (2), (R) → (1)
(b) (P) → (2), (Q) → (1), (R) → (3)
(c) (P) → (3), (Q) → (1), (R) → (2)
(d) (P) → (1), (Q) → (3), (R) → (2)

40. A cone is sitting on a semi globe. If the radius of globe is r and the cone height is h. Then the volume of solid is
(a) $\pi r^2(r+h)$ (b) $\frac{\pi r^3}{3}(r+h)$
(c) $\frac{\pi r^2}{3}(2r+h)$ (d) None of these

41. Which of the following factor does not affect EMI (equated monthly installment)
(a) Principal borrowed (b) Rate of interest
(c) Market trend (d) Tenure of the loan

42. Find the last three digits of the product 1234 × 5678
(a) 712 (b) 632 (c) 652 (d) 932

43. Consider the following statements:
Statement-I : When boat rows in the direction of the stream, it is called upstream.
Statement-II : When boat rows opposite to the direction of the stream, it is called downstream.
In the light of above statements, choose the correct answer from the options given below.
(a) Both statement I and II are correct.
(b) Both statement I and II are incorrect.
(c) Statement I is correct but statement II is incorrect.
(d) Statement II is correct but statement I is incorrect.

44. Find 2nd order derivative if $x = k(A - \sin A)$ and $y = k(1 - \cos A)$
(a) $-\frac{\cos ec^2 A/2}{2}\cdot\frac{1}{K(1-\cos A)}$
(b) $\frac{-\cos ec^2 A/2}{2\sin A}$
(c) $\frac{\cot A/2}{K(1-\cos A)}$
(d) None of these

45. If area enclosed by a door in a building is $A = 10x - \frac{1}{2}(\pi+4)x^2$, where $2x$ is the breadth of door then to allow maximum airflow inside the building, the width of the door is
(a) $\frac{10}{4+\pi}$ m (b) $\frac{20}{2+\pi}$ m
(c) $\frac{20}{4+\pi}$ m (d) $\frac{40}{4+\pi}$ m

SOLUTIONS

SECTION A

1. **(b) :** Let $I=\int\frac{x^2}{(ax+b)^2}dx$

Put $ax+b=t \Rightarrow dx=\frac{1}{a}dt$

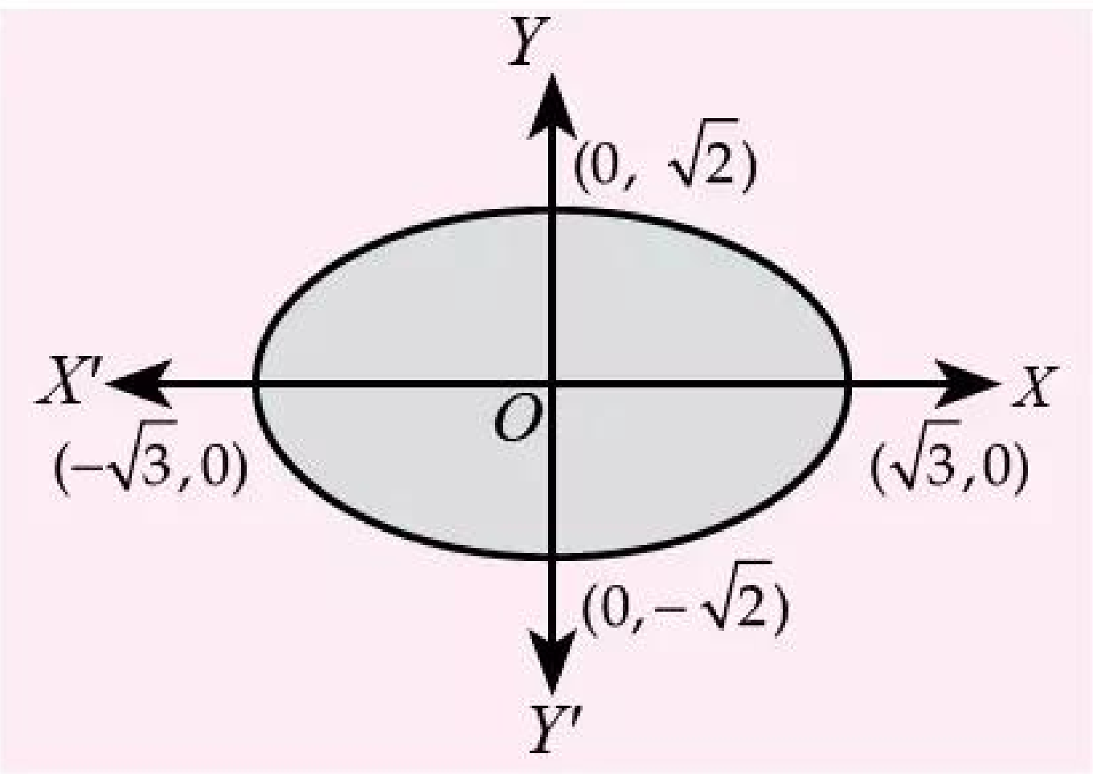
$$\therefore \quad I=\frac{1}{a^3}\int\frac{(t-b)^2}{t^2}dt=\frac{1}{a^3}\int\left(1+\frac{b^2}{t^2}-\frac{2b}{t}\right)dt$$

$$=\frac{1}{a^3}\left(t-\frac{b^2}{t}-2b\log t\right)+C$$

$$=\frac{1}{a^3}\left(ax+b-\frac{b^2}{ax+b}-2b\log(ax+b)\right)+C$$

2. **(a) :** From definition only IV and V are correct statements.

3. **(d) :**



Area of ellipse $\frac{x^2}{3}+\frac{y^2}{2}=1$ is given by

$$4\int_0^{\sqrt{3}}\sqrt{2\left(1-\frac{x^2}{3}\right)}\,dx=4\sqrt{\frac{2}{3}}\int_0^{\sqrt{3}}\sqrt{3-x^2}\,dx$$

$$=4\sqrt{\frac{2}{3}}\left[\frac{x}{2}\sqrt{3-x^2}+\frac{3}{2}\sin^{-1}\frac{x}{\sqrt{3}}\right]_0^{\sqrt{3}}$$

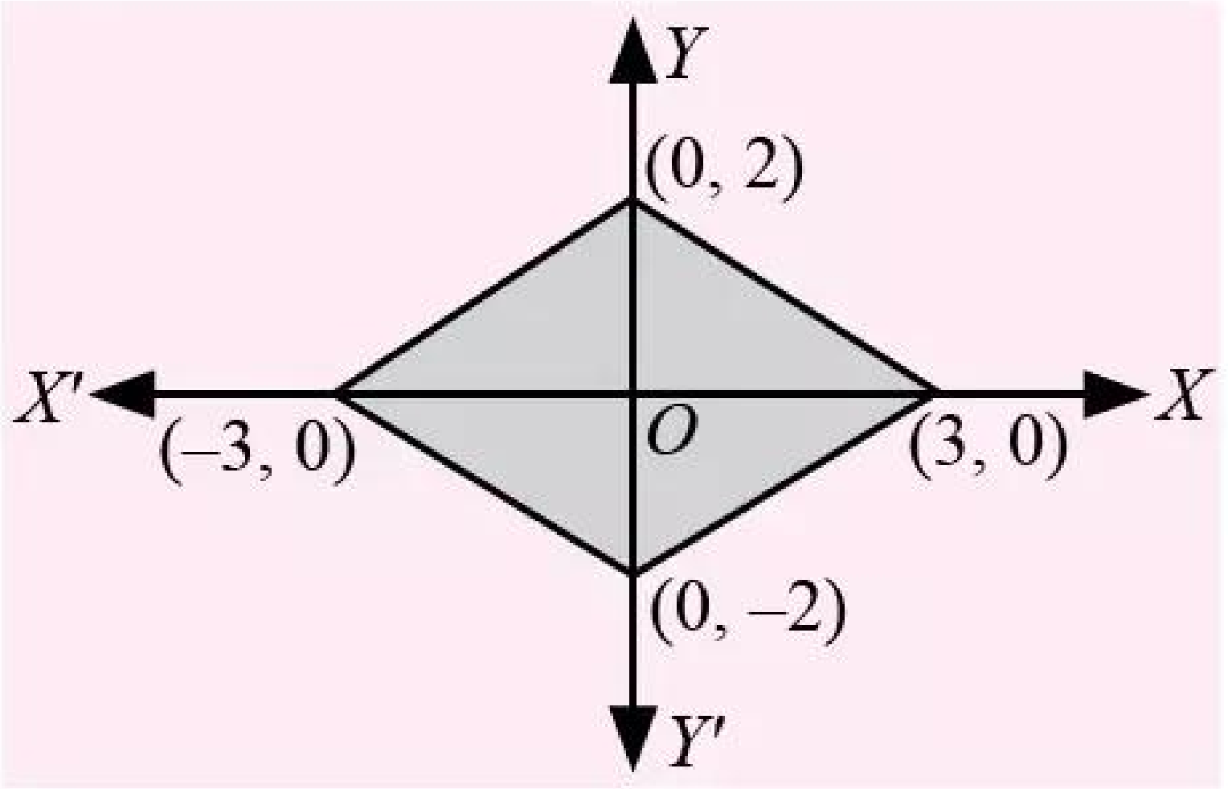
$$=4\sqrt{\frac{2}{3}}\times\frac{3}{2}\times\frac{\pi}{2}=\sqrt{\frac{2}{3}}\times3\pi=\sqrt{6}\,\pi \text{ sq. units}$$

$$=2\sqrt{6}\left(\frac{\pi}{2}\right)=7.69 \text{ (approx.)}$$

Area bounded by $2|x|+3|y|\le 6$

$$=4\int_0^3\left(\frac{6-2x}{3}\right)dx$$

$$=4\left[2x-\frac{x^2}{3}\right]_0^3$$



= 4 [6 – 3 – 0]

= 12 sq. units

and length of major axis of the ellipse = $2\sqrt{3}<3+3$

4. **(a) :** $\Delta=\begin{vmatrix}1 & 1 & 1\\ 1 & 1+\sin\theta & 1\\ 1+\cos\theta & 1 & 1\end{vmatrix}$

Applying $R_1\to R_1-R_3$ and $R_2\to R_2-R_3$, we get

$$\Delta=\begin{vmatrix}-\cos\theta & 0 & 0\\ -\cos\theta & \sin\theta & 0\\ 1+\cos\theta & 1 & 1\end{vmatrix}$$

Expanding along C_3, we get

$\Delta = 1\,(-\cos\theta\sin\theta-0)=-\cos\theta\sin\theta=\frac{-1}{2}\sin 2\theta$

We know, $-1\le\sin 2\theta\le 1$

$$\Rightarrow\quad -\frac{1}{2}\le\frac{1}{2}\sin 2\theta\le\frac{1}{2}\quad\Rightarrow\quad\frac{1}{2}\ge-\frac{1}{2}\sin 2\theta\ge\frac{-1}{2}$$

$$i.e.,\quad -\frac{1}{2}\le-\frac{1}{2}\sin 2\theta\le\frac{1}{2}\quad\Rightarrow\quad\frac{-1}{2}\le\Delta\le\frac{1}{2}$$

Hence, maximum value of Δ is $\frac{1}{2}$.

5. **(a)**

6. **(d) :** $P(E_i)=\frac{i}{1+i}\ (i=1, 2, ..., n)$

$$\therefore\quad P(E_1)=\frac{1}{2},\ P(E_2)=\frac{2}{3},\ P(E_3)=\frac{3}{4},\ ...,\ P(E_n)=\frac{n}{n+1}$$

$$P(\overline{E}_1\overline{E}_2\overline{E}_3\ldots\overline{E}_n)$$

$$\therefore\quad=\left(1-\frac{1}{2}\right)\times\left(1-\frac{2}{3}\right)\times\left(1-\frac{3}{4}\right)\times\ldots\times\left(1-\frac{n}{n+1}\right)$$

$$=\frac{1}{2}\times\frac{1}{3}\times\frac{1}{4}\times\ldots.\times\frac{1}{n+1}$$

∴ Statement-I is false but Statement-II is true.

7. **(a) :** If A is any square matrix, then

$AA^{-1}=I$ and $A^{-1}I=A^{-1}$

Since, $A^2-A+I=O \Rightarrow A^{-1}A^2-A^{-1}A+A^{-1}I=O$

$\Rightarrow\ (A^{-1}A)A-(A^{-1}A)+A^{-1}=O$

$\Rightarrow\ A-I+A^{-1}=O \Rightarrow A^{-1}=I-A$

8. **(d) :** Let X denotes the number of defective eggs drawn. Since, the drawing is done with replacement, the trials are Bernoulli trials. Clearly, X has the binomial distribution with $n=10$ and $p=\frac{10}{100}=\frac{1}{10}$

Therefore, $q=1-p=\frac{9}{10}$

Now, P(atleast one defective egg)

$$=P(X\ge 1)=1-P(X=0)=1-{}^{10}C_0\left(\frac{9}{10}\right)^{10}=1-\frac{9^{10}}{10^{10}}$$

9. **(a) :** According to question, we have

mean = 4 ⇒ $np=4$ ... (i)

and variance = 2

⇒ $npq=2$ ... (ii)

From (i) and (ii), we get

$4q=2\Rightarrow q=\frac{1}{2}\quad\therefore\quad p=1-q=1-\frac{1}{2}=\frac{1}{2}$

From (i), we get $n\cdot\frac{1}{2}=4\Rightarrow n=8$

Now, the probability of atleast 6 successes

$=P(6)+P(7)+P(8)$

$$= {}^8C_6\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^2 + {}^8C_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right) + {}^8C_8\left(\frac{1}{2}\right)^8$$

$$= \frac{1}{256}(28+8+1) = \frac{37}{256}$$

10. (c) : We have,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \text{ and } R_1 + R_2 = C \qquad \text{...(i)}$$

$$\Rightarrow \frac{1}{R} = \frac{R_1+R_2}{R_1R_2} = \frac{C}{R_1R_2} = \frac{C}{R_1(C-R_1)} \qquad [\because R_2 = C - R_1]$$

$$\Rightarrow R = \frac{R_1C - R_1^2}{C} = R_1 - \frac{R_1^2}{C}$$

$$\Rightarrow \frac{dR}{dR_1} = 1 - \frac{2R_1}{C} \text{ and } \frac{d^2R}{dR_1^2} = -\frac{2}{C}$$

For maximum or minimum, we have

$$\frac{dR}{dR_1} = 0 \Rightarrow 1 - \frac{2R_1}{C} = 0 \Rightarrow R_1 = \frac{C}{2}$$

Now, $\dfrac{d^2R}{dR_1^2} = -\dfrac{2}{C} < 0$ for all values of R_1

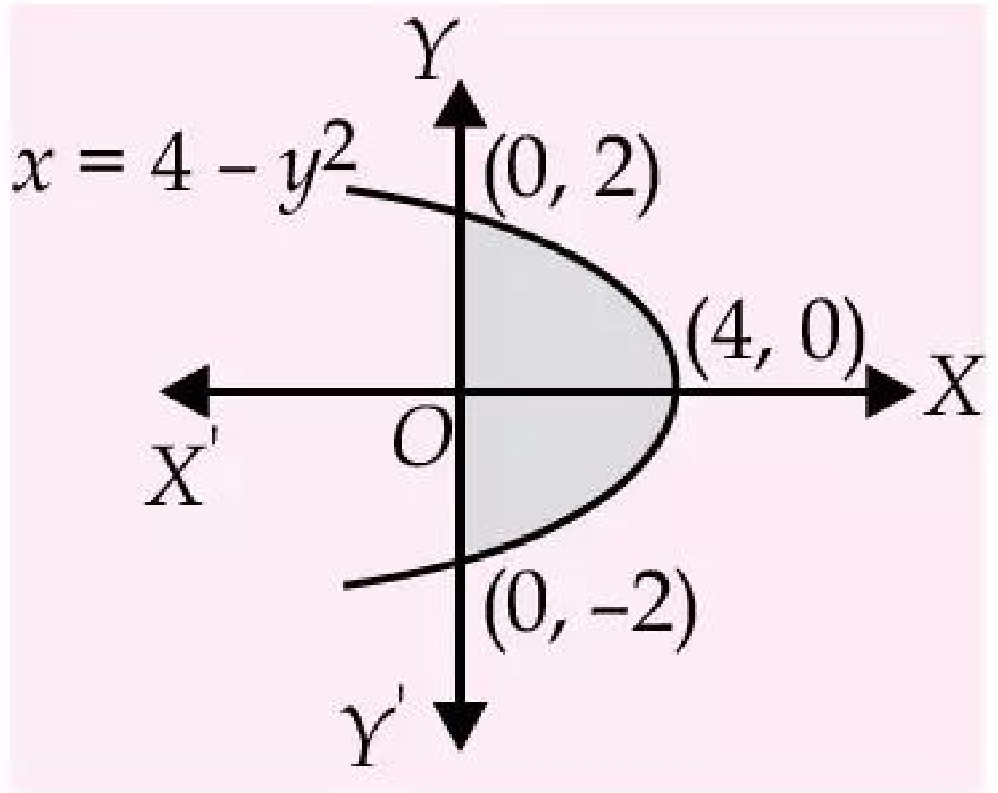
Thus, R is maximum when $R_1 = \dfrac{C}{2}$

Putting $R_1 = \dfrac{C}{2}$ in (i), we get $R_2 = C - \dfrac{C}{2} = \dfrac{C}{2}$

Hence, R is maximum when $R_1 = R_2 = \dfrac{C}{2}$

11. (a) : (P) Given curve is $y^2 = 4 - x$

$\therefore$ Required area



$$= 2\left|\int_0^4 \sqrt{4-x}\,dx\right|$$

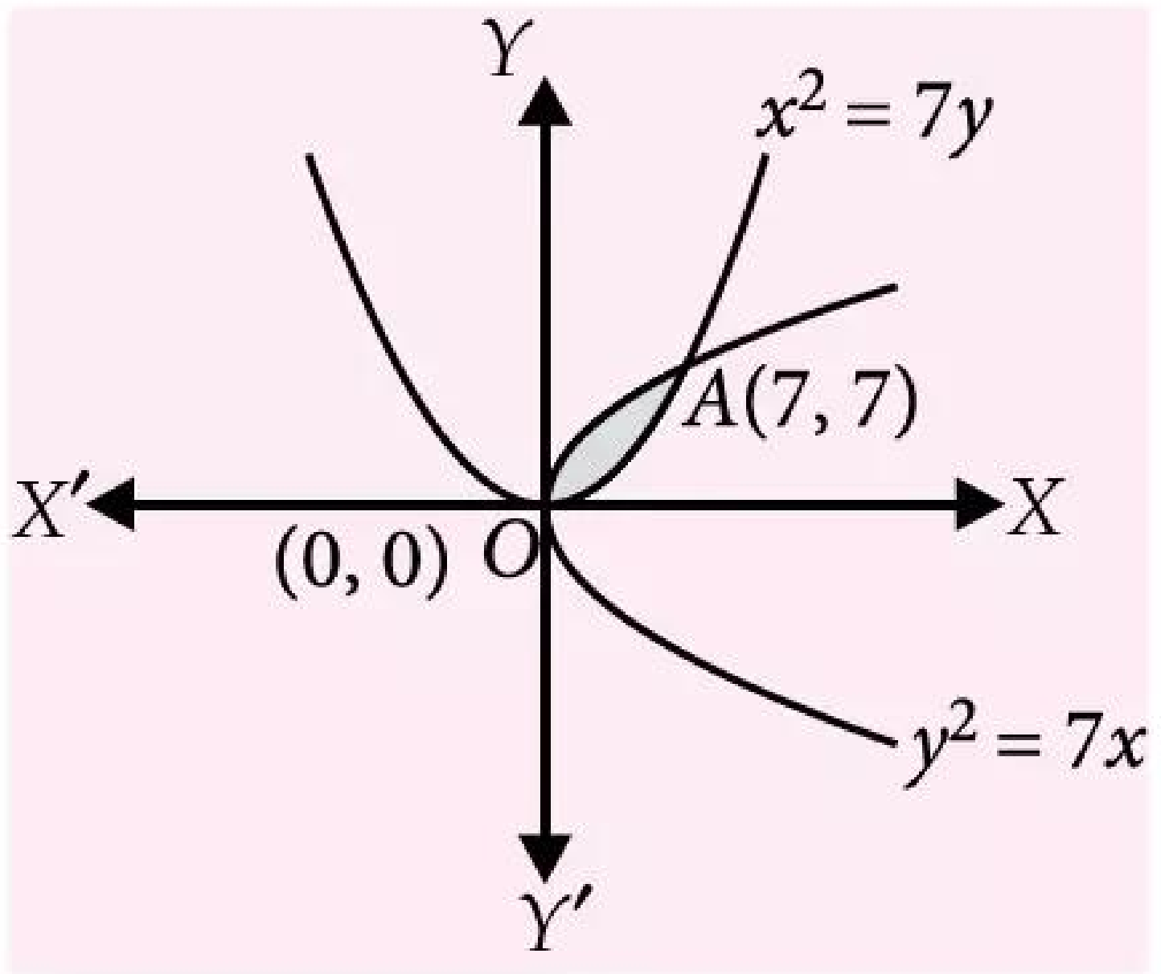
$$= 2\left|\left[\frac{(4-x)^{3/2}}{3/2}\right]_0^4\right|$$

$$= \frac{4}{3}(8) = \frac{32}{3} \text{ sq. units}$$

(Q) Given parabolas are $y^2 = 7x$ and $x^2 = 7y$

$\therefore$ Points of intersection are $O(0, 0)$ and $A(7, 7)$.

$\therefore$ Required area = Area of shaded region



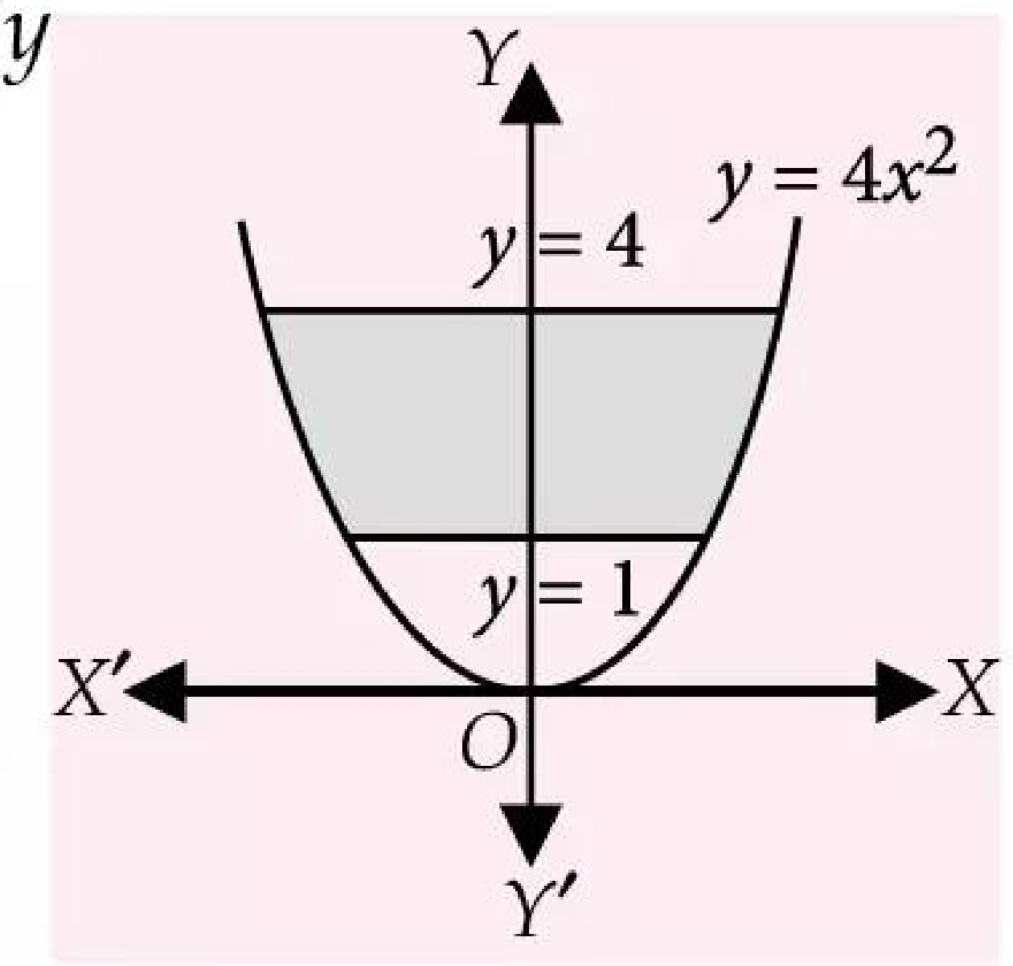
$$= \int_0^7 \left(\sqrt{7x} - \frac{x^2}{7}\right)dx$$

$$= \left[\sqrt{7}\frac{x^{3/2}}{3/2} - \frac{x^3}{21}\right]_0^7$$

$$= \left[\frac{2\sqrt{7}}{3} \times 7\sqrt{7} - \frac{343}{21}\right]$$

$$= \frac{98}{3} - \frac{49}{3} = \frac{49}{3} \text{ sq. units}$$

(R) The bounded region is as shown in figure.



$\therefore$ Required area $= 2\displaystyle\int_1^4 \frac{\sqrt{y}}{2}\,dy$

$$= \left[\frac{2}{3}y^{3/2}\right]_1^4$$

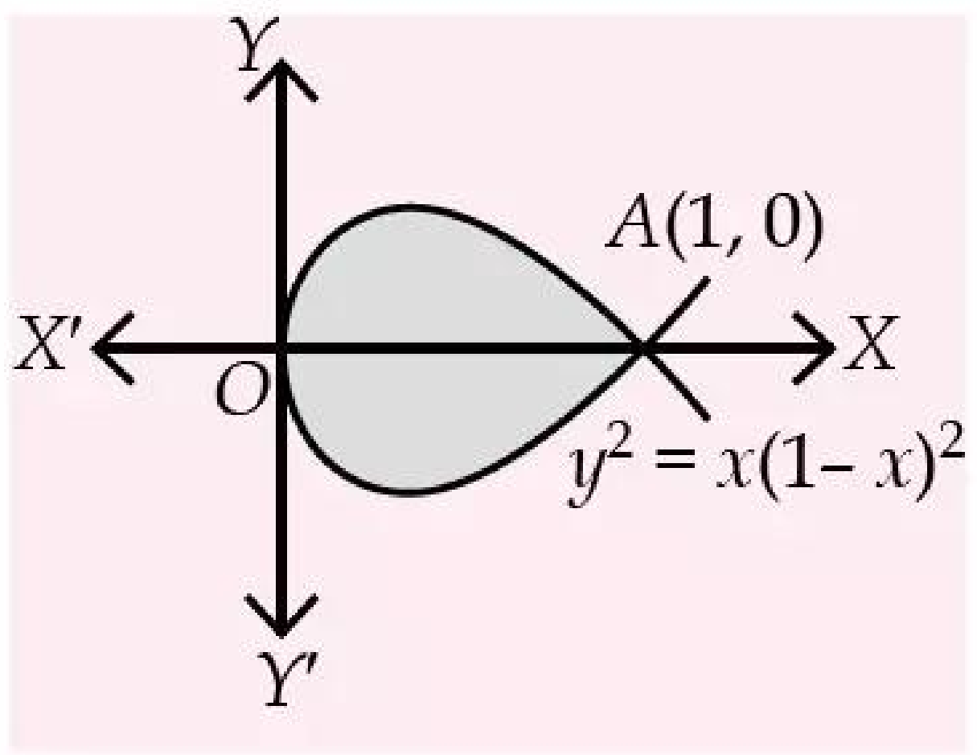
$$= \frac{2}{3}(8-1) = \frac{14}{3} \text{ sq. units}$$

(S) Given curve is $y^2 = x(1 - x)^2$

If $y = 0$, then $x(1 - x)^2 = 0$

$\Rightarrow$ $x = 0, x = 1 \Rightarrow A \equiv (1, 0)$

$\therefore$ Required area



$$= 2\int_0^1 \sqrt{x}\,(1-x)\,dx$$

$$= 2\left[\frac{x^{3/2}}{3/2} - \frac{x^{5/2}}{5/2}\right]_0^1$$

$$= 2\left[\frac{2}{3}(1) - \frac{2}{5}(1)\right] = \frac{8}{15} \text{ sq. unit}$$

12. (c) : (P) $y = ae^{3x} + be^{5x} \Rightarrow \dfrac{3dy}{dx} = 3ae^{3x} + 5be^{5x}$

$$= 3ae^{3x} + 3be^{5x} + 2be^{5x} = 3y + 2be^{5x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{3dy}{dx} + 10be^{5x} = \frac{3dy}{dx} + 5\left(\frac{dy}{dx} - 3y\right)$$

$$\left(\because 2be^{5x} = \frac{dy}{dx} - 3y\right)$$

$$\Rightarrow y'' - 8y' + 15y = 0$$

(Q) $ax^2 + by^2 = 1$

$$\Rightarrow ax + by\frac{dy}{dx} = 0 \qquad \text{[On differentiating]}$$

$$\Rightarrow a + b\left(\frac{dy}{dx}\right)^2 + by\frac{d^2y}{dx^2} = 0 \qquad \text{[Again differentiating]}$$

$$\Rightarrow a + b\left[\left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2}\right] = 0 \Rightarrow a + b[(y')^2 + yy''] = 0$$

(R) $xe^y - y^2 = 2 \qquad$ ...(i)

$$\Rightarrow xe^y\frac{dy}{dx} + e^y - 2y\frac{dy}{dx} = 0 \qquad \text{[On differentiating]}$$

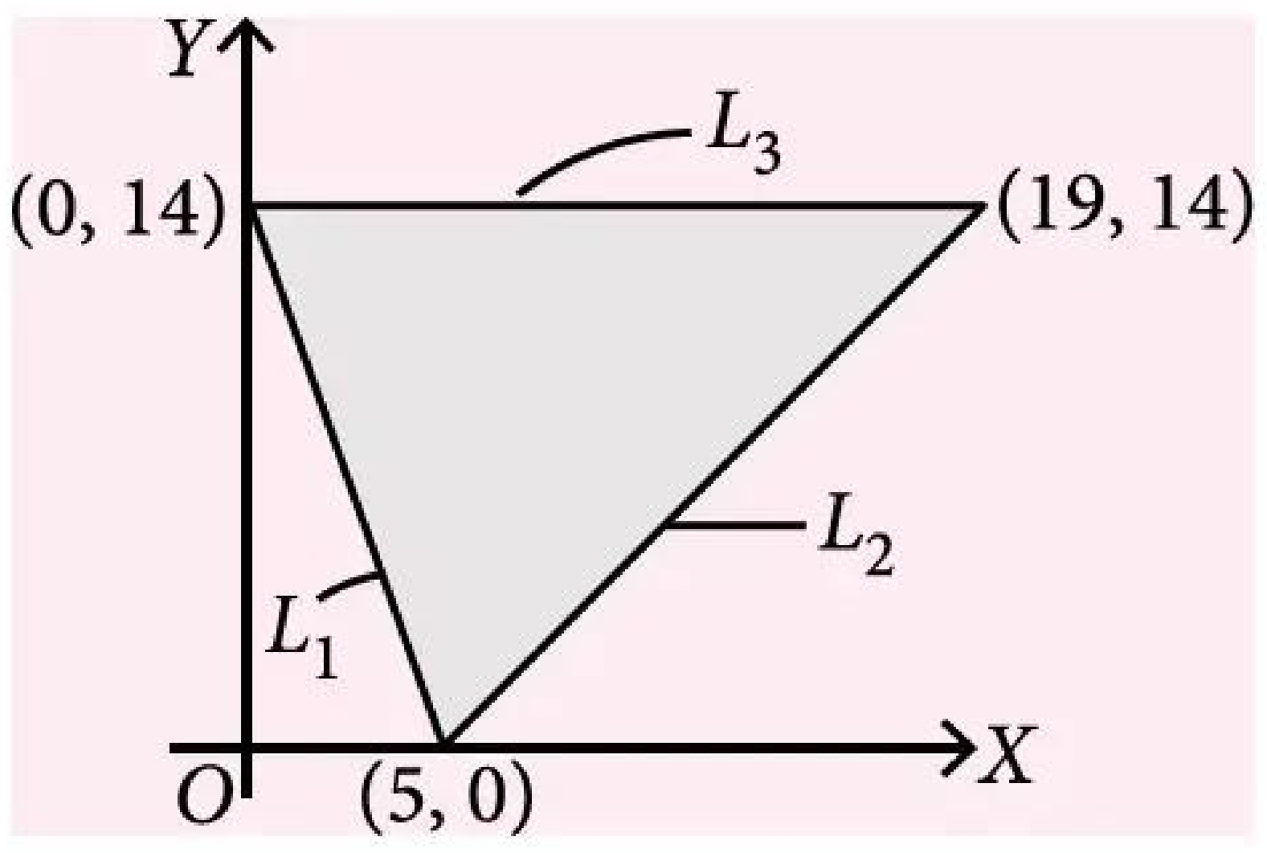
$$\Rightarrow (xe^y - 2y)\frac{dy}{dx} + e^y = 0$$

$$\Rightarrow (y^2 + 2 - 2y)\frac{dy}{dx} + e^y = 0 \qquad \text{[Using (i)]}$$

$$\Rightarrow (y^2 + 2 - 2y)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2(2y - 2) + e^y\frac{dy}{dx} = 0$$

[Again differentiating]

13. (c) :



Line (L_1) represent an equation $14x + 5y = 70$
Since, shaded region (L_1) does not contain origin.
$\therefore$ Its corresponding inequation is
$14x + 5y \geq 70$ ...(i)
Line (L_2) represent an equation $x - y = 5$
Since, shaded region (L_2) contains the origin.
$\therefore$ Its corresponding inequation is $x - y \leq 5$ ...(ii)
Line (L_3) represent an equation $y = 14$. Its shaded region contains the origin.
$\therefore$ Its corresponding inequation is $y \leq 14$ ...(iii)
$\therefore$ From (i), (ii) and (iii), we get
$14x + 5y \geq 70,\ x - y \leq 5$ and $y \leq 14$

14. (c)

15. (b) : Let $f(x) = x^4 - 62x^2 + ax + 9$
Then, $f'(x) = 4x^3 - 124x + a$
It is given that $f(x)$ attains its maximum at $x = 1$
$\therefore\ f'(1) = 0 \Rightarrow 4 - 124 + a = 0 \Rightarrow a = 120$

SECTION B1 (MATHEMATICS)

16. (d) :

$$f(x) = \frac{1}{\sqrt{\{\sin x\} + \{\sin(\pi + x)\}}} = \frac{1}{\sqrt{\{\sin x\} + \{(-\sin x)\}}}$$

Now, $\{\sin x\} + \{-\sin x\} = \begin{cases} 0, & \sin x \text{ is integer} \\ 1, & \sin x \text{ is not integer} \end{cases}$

For $f(x)$ to be defined, $\{\sin x\} + \{-\sin x\} \neq 0$
$\Rightarrow\ \sin x \neq \text{integer} \Rightarrow \sin x \neq \pm 1, 0 \Rightarrow x \neq \frac{n\pi}{2}$

Hence, domain is $R - \left\{\frac{n\pi}{2}, n \in I\right\}$.

17. (c) : Since, $0 \leq \cos^{-1}\left(\frac{x^2}{2} + \sqrt{1 - x^2}\sqrt{1 - \frac{x^2}{4}}\right) \leq \frac{\pi}{2}$

Because $\cos^{-1} x$ lies in $\left[0, \frac{\pi}{2}\right]$, when x is positive.

$\Rightarrow\ \cos^{-1}\frac{x}{2} - \cos^{-1} x \geq 0 \Rightarrow \cos^{-1}\frac{x}{2} \geq \cos^{-1} x$

$\Rightarrow\ \frac{x}{2} \leq x$ [$\because$ cos x is a decreasing function]

$\Rightarrow\ \frac{x}{2} \geq 0 \Rightarrow x \geq 0$

Also, $\left|\frac{x}{2}\right| \leq 1,\ |x| \leq 1 \Rightarrow |x| \leq 1$

Thus, $0 \leq x \leq 1$.

18. (a) : $A = \begin{bmatrix} 1 & \omega^2 & \omega \\ \omega^2 & \omega & 1 \\ \omega & 1 & \omega^2 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & \omega^2 & \omega \\ \omega^2 & \omega & 1 \\ \omega & 1 & \omega^2 \end{bmatrix} = A$

$\because\ A^T = A$. Hence, A is a symmetric matrix.

19. (b) : $M_{11} = \begin{vmatrix} b & f \\ h & g \end{vmatrix} = bg - hf \Rightarrow c_{11} = M_{11} = bg - hf$

$M_{21} = \begin{vmatrix} f & c \\ h & g \end{vmatrix} = fg - ch \Rightarrow c_{21} = -M_{21} = ch - fg$

$M_{31} = \begin{vmatrix} f & c \\ b & f \end{vmatrix} = f^2 - bc \Rightarrow c_{31} = M_{31} = f^2 - bc$

20. (b) : $f(x + y) = f(x) \cdot f(y) \Rightarrow f(0 + 5) = f(0) \cdot f(5)$

Also $f'(5) = \lim\limits_{h \to 0} \frac{f(5 + h) - f(5)}{h}$

$= \lim\limits_{h \to 0} \frac{f(5)f(h) - f(5 + 0)}{h} = \lim\limits_{h \to 0} \frac{f(5)f(h) - f(5)f(0)}{h}$

$= f(5)\lim\limits_{h \to 0} \frac{f(h) - f(0)}{h} = f(5)f'(0) = 2 \times 3 = 6$

21. (c)

22. (c) : I. Let $I = \int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}}dx$

$\Rightarrow\ I = \int \frac{(\sin x + \cos x)}{\sqrt{\sin^2 x + \cos^2 x + 2\sin x \cos x}}dx$

$= \int \frac{\sin x + \cos x}{\sqrt{(\sin x + \cos x)^2}}dx = \int 1\,dx = x + C$

II. Let $I = \int \frac{dx}{1 + \cos x} = \frac{1}{2}\int \frac{1}{\cos^2 \frac{x}{2}}dx$

$= \frac{1}{2}\int \sec^2 \frac{x}{2}dx = \frac{1}{2}\tan\frac{x}{2} \cdot 2 + C = \tan\frac{x}{2} + C$

III. Let $I = \int \frac{x^2 + 2}{x + 1}dx = \int\left(x - 1 + \frac{3}{x + 1}\right)dx$

$= \frac{x^2}{2} - x + 3\log|x + 1| + C$

IV. Let

$I = \int \frac{dx}{\sqrt{16 - 9x^2}} \Rightarrow I = \int \frac{dx}{\sqrt{(4)^2 - (3x)^2}} = \frac{1}{3}\sin^{-1}\left(\frac{3x}{4}\right) + C$

V. Let $I = \int \frac{x + 3}{(x + 4)^2}e^x dx$

$= \int e^x\left(\frac{1}{(x + 4)} - \frac{1}{(x + 4)^2}\right)dx = e^x\left(\frac{1}{x + 4}\right) + C$

$\left[\text{Using} \int e^x\{f(x) + f'(x)\}dx = e^x f(x) + C\right]$

23. (c)

24. (a) : (P) We have, $f(x) = -\tan^{-1} x$

Differentiating w.r.t. x, we get $f'(x) = \frac{-1}{1+x^2}$

$f'(x) < 0$ ∴ f is decreasing.

(Q) We have, $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Put $x = \tan\theta$, we get

$$f(x) = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta$$

$\therefore \quad f(x) = 2\tan^{-1} x \Rightarrow f'(x) = \frac{2}{1+x^2} > 0$

∴ f is increasing.

(R) We have, $2x = y^2$ ...(i) and $2xy = k$ ...(ii)

Differentiating (i) and (ii) w.r.t. x, we have

$$2 = 2y\frac{dy}{dx} \text{ and } 2\left(y + x\frac{dy}{dx}\right) = 0$$

$\Rightarrow \frac{dy}{dx} = \frac{1}{y}$ ...(iii) and $\frac{dy}{dx} = \frac{-y}{x}$ ...(iv)

The two curves cut at right angle if product of slopes of tangents = –1.

$\therefore \quad \frac{1}{y} \times \frac{-y}{x} = -1$ or $\frac{1}{x} = 1 \Rightarrow x = 1$

Put $x = 1$ in (i), we get $y^2 = 2 \Rightarrow y = \sqrt{2}$

Putting $x = 1$ and $y = \sqrt{2}$ in (ii) we get $2(1)(\sqrt{2}) = k$ or $k^2 = 8$

25. (d): If $\vec{a}, \vec{b}, \vec{c}$ are linearly dependent vectors, then $\vec{c}$ should be a linear combination of $\vec{a}$ and $\vec{b}$.

Let $\vec{c} = p\vec{a} + q\vec{b}$ i.e., $\hat{i} + \alpha\hat{j} + \beta\hat{k} = p(\hat{i}+\hat{j}+\hat{k}) + q(4\hat{i}+3\hat{j}+4\hat{k})$

Equating coefficients of $\hat{i}, \hat{j}, \hat{k}$ on both sides, we get

$1 = p + 4q, \alpha = p + 3q, \beta = p + 4q$

From first and third, $\beta = 1$

Now, $|\vec{c}| = \sqrt{3}$ [Given] $\Rightarrow 1 + \alpha^2 + \beta^2 = 3$

$\Rightarrow \quad 1 + \alpha^2 + 1 = 3 \quad \Rightarrow \quad \alpha = \pm 1$

Hence, $\alpha = \pm 1, \beta = 1$

26. (a) : Equation of the plane through the line of intersection of the planes $x + y + z = 6$ and $2x + 3y + 4z + 5 = 0$ is

$(x + y + z - 6) + \lambda(2x + 3y + 4z + 5) = 0$ ...(i)

Since, it passes through (4, 4, 4), then

$(4 + 4 + 4 - 6) + \lambda(8 + 12 + 16 + 5) = 0$

$\Rightarrow \quad 6 + 41\lambda = 0 \Rightarrow \lambda = \frac{-6}{41}$

Now, we get from (i), $41(x+y+z-6) - 6(2x+3y+4z+5) = 0$

$\Rightarrow \quad 29x + 23y + 17z = 276$

27. (b): When $x \ge 6$ and $y \ge 2$, then

$2x + y \ge 2 \times 6 + 2$, i.e., $2x + y \ge 14$

Hence, $x \ge 0$, $y \ge 0$ and $2x + y \ge 10$ are automatically satisfied by every point of the region

$\{(x, y) : x \ge 6\} \cap \{(x, y) : y \ge 2\}$

28. (b): We know that, $\Sigma p_i = 1$

$\therefore \quad a + 4a + 3a + 7a + 8a + 10a + 6a + 9a = 1$

$\Rightarrow \quad 48a = 1 \Rightarrow a = \frac{1}{48}$

Now, $P(X < 3) = P(0) + P(1) + P(2)$

$= a + 4a + 3a = 8a = 8 \cdot \frac{1}{48} = \frac{1}{6}$

$P(X \ge 4) = P(4) + P(5) + P(6) + P(7)$

$= 8a + 10a + 6a + 9a = 33a = \frac{33}{48}$

and $P(0 < X < 5) = P(1) + P(2) + P(3) + P(4)$

$= 4a + 3a + 7a + 8a = 22a = \frac{22}{48} = \frac{11}{24}$

29. (d): Let $y = f(x) = (ax^2 + b)^3 \Rightarrow ax^2 + b = y^{1/3}$

$$\Rightarrow \quad x^2 = \frac{y^{1/3} - b}{a} \quad \Rightarrow \quad x = \left(\frac{y^{1/3} - b}{a}\right)^{1/2} = f^{-1}(y)$$

Since, $f(g(x)) = g(f(x))$ ∴ We have, $g(x) = \left(\frac{x^{1/3} - b}{a}\right)^{1/2}$.

30. (b) : $\begin{vmatrix} 1 & a & -1 \\ 2 & -1 & a \\ a & 1 & 2 \end{vmatrix} = 0$

Applying $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - aR_1$, we get

$$\begin{vmatrix} 1 & a & -1 \\ 0 & -1-2a & a+2 \\ 0 & 1-a^2 & 2+a \end{vmatrix} = 0$$

Expanding along C_1, we get

$(a + 2)(a^2 - 2a - 2) = 0 \quad \Rightarrow \quad a = -2, 1 \pm \sqrt{3}$

31. (a) : $\vec{r} \cdot \vec{a} = \vec{r} \cdot \vec{b} = \vec{r} \cdot \vec{c} = 0$ i.e., $\vec{r} \perp \vec{a}, \vec{r} \perp \vec{b}$ and $\vec{r} \perp \vec{c}$ is possible only if $\vec{a}, \vec{b}$ and c are coplanar.

Hence, Statement-II is true.

Since, $\vec{r} \cdot \vec{a} = 0 \Rightarrow$ Either $\vec{r} = 0$ or $\vec{r} \perp \vec{a}$

Similarly, $\vec{r} \cdot \vec{b} = 0 \Rightarrow$ Either $\vec{r} = 0$ or $\vec{r} \perp \vec{b}$

Also, $\vec{r} \cdot \vec{c} = 0 \Rightarrow$ Either $\vec{r} = 0$ or $\vec{r} \perp \vec{c}$

But $\vec{r}$ cannot be perpendicular to $\vec{a}, \vec{b}, \vec{c}$ as $\vec{a}, \vec{b}$ and $\vec{c}$ are non coplanar.

$\therefore \quad \vec{r} = 0$ i.e., $\vec{r}$ is a null vector. ∴ Statement-I is true

32. (b): (i) The equation of the plane passing through the points (2, 1, 0), (3, -2, -2) and (3, 1, 7) is given by

$$\begin{vmatrix} x-2 & y-1 & z-0 \\ 3-2 & -2-1 & -2-0 \\ 3-2 & 1-1 & 7-0 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x-2 & y-1 & z \\ 1 & -3 & -2 \\ 1 & 0 & 7 \end{vmatrix} = 0$$

$\Rightarrow \quad (x - 2)(-21 + 0) - (y - 1)(7 + 2) + z(3) = 0$

$\Rightarrow \quad -21x - 9y + 3z = -51 \Rightarrow 7x + 3y - z = 17$

(ii) Given, $A \equiv (2, 3, 4)$ and $B \equiv (4, 5, 8)$

$\therefore \quad$ Midpoint of AB is $\left(\frac{2+4}{2}, \frac{3+5}{2}, \frac{4+8}{2}\right)$ i.e., (3, 4, 6)

Also, $\vec{n} = (4-2)\hat{i} + (5-3)\hat{j} + (8-4)\hat{k} = 2\hat{i} + 2\hat{j} + 4\hat{k}$

Hence, the required equation of the plane is

$(\vec{r} - \vec{a})\cdot\vec{n} = 0$, where $\vec{a} = 3\hat{i} + 4\hat{j} + 6\hat{k}$

$\Rightarrow\ [(x-3)\hat{i} + (y-4)\hat{j} + (z-6)\hat{k}]\cdot(2\hat{i} + 2\hat{j} + 4\hat{k}) = 0$

$\Rightarrow\ 2x - 6 + 2y - 8 + 4z - 24 = 0$

$\Rightarrow\ 2x + 2y + 4z = 38 \ \Rightarrow\ x + y + 2z = 19$

(iii) The equation of the plane passing through (2, 1, –1) is

$a(x-2) + b(y-1) + c(z+1) = 0$ ...(i)

Since, it passes through (–1, 3, 4)

$\therefore\ a(-1-2) + b(3-1) + c(4+1) = 0$

$\Rightarrow\ -3a + 2b + 5c = 0$ ...(ii)

Since, the plane (i) is perpendicular to the plane $x - 2y + 4z = 10$ $\therefore\ a - 2b + 4c = 0$ ...(iii)

On solving (ii) and (iii), we get

$$\frac{a}{18} = \frac{b}{17} = \frac{c}{4} = \lambda \ \Rightarrow\ a = 18\lambda,\ b = 17\lambda,\ c = 4\lambda$$

Hence, required equation of plane is

$18\lambda(x-2) + 17\lambda(y-1) + 4\lambda(z+1) = 0$

$\Rightarrow\ 18x - 36 + 17y - 17 + 4z + 4 = 0 \Rightarrow 18x + 17y + 4z = 49$

(iv) Since, the line drawn from the point (–2, –1, –3) meets a plane at right angle at the point (1, –3, 3). So, the plane passes through the point (1, –3, 3) and normal to plane $\vec{n}$ is

$(-2-1)\hat{i} + (-1+3)\hat{j} + (-3-3)\hat{k}$

$\Rightarrow\ \vec{n} = -3\hat{i} + 2\hat{j} - 6\hat{k}$ and $\vec{a} = \hat{i} - 3\hat{j} + 3\hat{k}$

Now, the required equation of plane is

$(\vec{r} - \vec{a})\cdot\vec{n} = 0$

$\Rightarrow\ [(x\hat{i} + y\hat{j} + z\hat{k}) - (\hat{i} - 3\hat{j} + 3\hat{k})]\cdot(-3\hat{i} + 2\hat{j} - 6\hat{k}) = 0$

$\Rightarrow\ [(x-1)\hat{i} + (y+3)\hat{j} + (z-3)\hat{k}]\cdot(-3\hat{i} + 2\hat{j} - 6\hat{k}) = 0$

$\Rightarrow\ -3x + 3 + 2y + 6 - 6z + 18 = 0$

$\Rightarrow\ -3x + 2y - 6z = -27 \ \therefore\ 3x - 2y + 6z - 27 = 0$

33. (b): I. We have, $x^2 + y^2 - 2x - 4y + 1 = 0$ ...(1)

let $P(x_1, y_1)$ be a required point on the given curve where the tangent is parallel to y-axis, then

Differentiating (1) w.r.t. x, we get

$$2x + 2y\frac{dy}{dx} - 2 - 4\frac{dy}{dx} + 0 = 0 \Rightarrow \frac{dy}{dx} = \frac{2(1-x)}{2(y-2)} = \frac{1-x}{y-2}$$

$\Rightarrow$ Slope of tangent at $P(x_1, y_1) = \dfrac{1-x_1}{y_1-2}$

$\because$ The tangent is parallel to Y-axis,

$\therefore\ y_1 - 2 = 0 \Rightarrow y_1 = 2.$

Substituting $x = x_1$ and $y_1 = 2$ in (1), we get

$x_1^2 + 4 - 2x_1 - 8 + 1 = 0 \Rightarrow x_1^2 - 2x_1 - 3 = 0$

$\Rightarrow\ (x_1 - 3)(x_1 + 1) = 0 \Rightarrow x_1 = 3, -1$

Hence, the required points are (3, 2) and (–1, 2).

II. Given function is $f(x) = \tan^{-1}(\sin x + \cos x)$

Differentiating w.r.t. x, we get

$$f'(x) = \frac{1}{1 + (\sin x + \cos x)^2}\frac{d}{dx}(\sin x + \cos x)$$

$$= \frac{\cos x - \sin x}{1 + (\sin x + \cos x)^2}$$

We know that for all $x \in \left(0, \frac{\pi}{4}\right)$, $\cos x > \sin x$

$\Rightarrow\ \cos x - \sin x > 0$ for all $x \in \left(0, \frac{\pi}{4}\right)$

$\Rightarrow\ f'(x) > 0$ for all $x \in \left(0, \frac{\pi}{4}\right)$.

Hence, f is strictly increasing on $\left(0, \frac{\pi}{4}\right)$.

III. $\sin x + \sqrt{3}\cos x = 2\left\{\frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x\right\}, x \in R$

$$= 2\left\{\sin x\cos\frac{\pi}{3} + \cos x\sin\frac{\pi}{3}\right\} = 2\left\{\sin\left(x + \frac{\pi}{3}\right)\right\},\ x \in R$$

Since maximum value of $\sin\left(x + \frac{\pi}{3}\right)$ is 1, therefore, maximum value of $\sin x + \sqrt{3}\cos x$ is 2 and it occurs when $\sin\left(x + \frac{\pi}{3}\right) = 1$,

i.e., when $x + \frac{\pi}{3} = n\pi + (-1)^n\frac{\pi}{2}, n \in I$,

i.e., $x = n\pi - \frac{\pi}{3} + (-1)^n\frac{\pi}{2}, n \in I$.

If we take $n = 1$, we find that $x = \frac{\pi}{6}$.

$\therefore$ Given function assumes maximum value at $\frac{\pi}{6}$.

34. (b): $\dfrac{xdy + ydx}{xy} = \tan\left(\dfrac{y}{x}\right)\dfrac{(xdy - ydx)}{x^2}$

$\Rightarrow\ \dfrac{d(xy)}{xy} = \tan\left(\dfrac{y}{x}\right)d\left(\dfrac{y}{x}\right)$

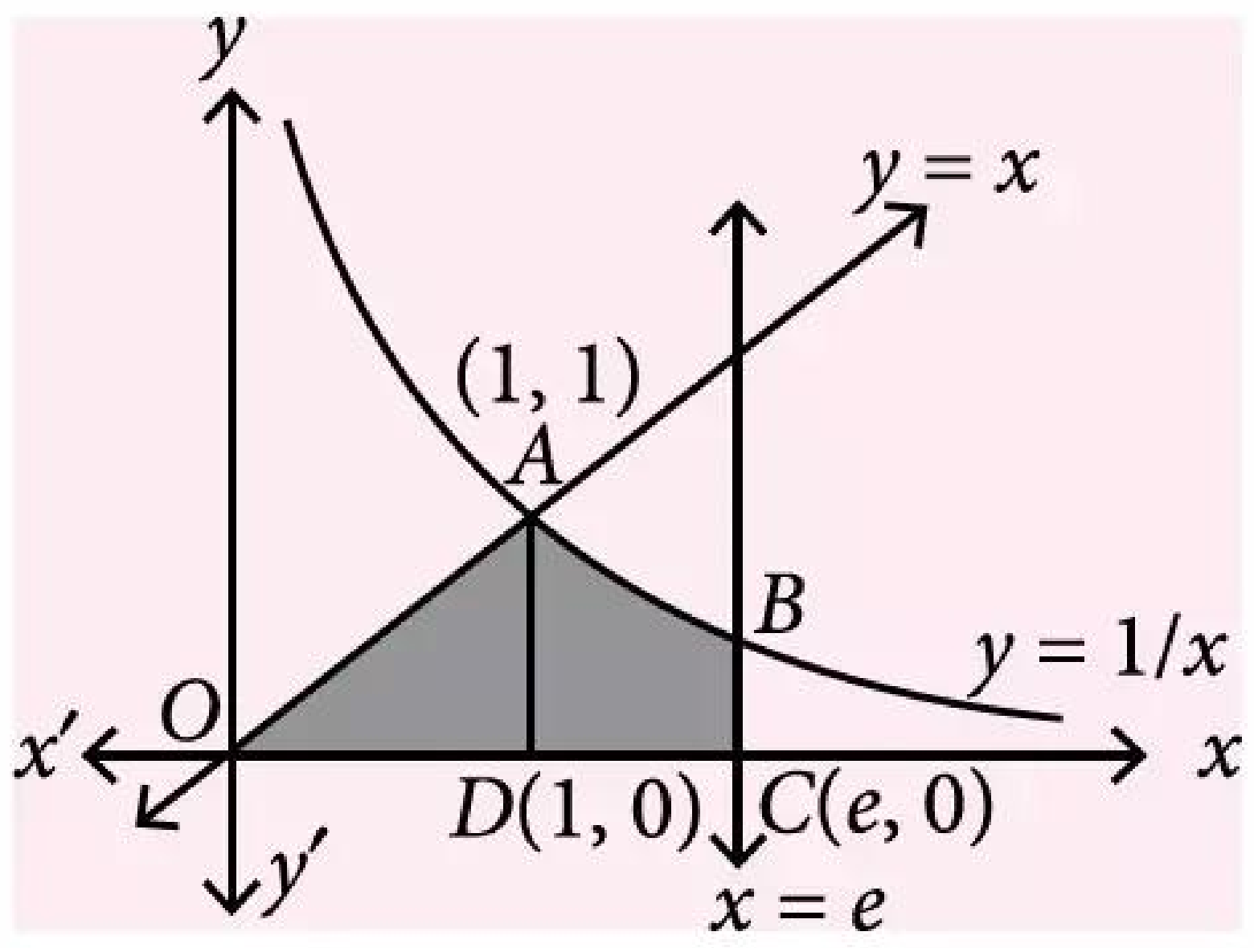
On integration, we get

$\ln xy = \ln\sec\left(\frac{y}{x}\right) - \ln c \Rightarrow \sec\left(\frac{y}{x}\right) = cxy$

35. (b): We have, $y = x$, $x = e$, $y = \frac{1}{x}$ and $x \geq 0$

$\therefore$ Required area = Area of ΔODA + Area of region $ABCD$

$$= \frac{1}{2}(1\times1) + \int_1^e \frac{1}{x}dx = \frac{1}{2} + \big[\log|x|\big]_1^e = \frac{1}{2} + 1 = \frac{3}{2} \text{ sq. units}$$



36. (b): Let $I=\int 2x\sin(x^2+1)dx$

Put $x^2+1=v \Rightarrow 2xdx=dv$ [Differentiating both sides]

Now, $I=\int \sin v\,dv = -\cos v + c = -\cos(x^2+1)+C$

37. (a)

38. (b): Let $I=\int\frac{\sin(2\tan^{-1}x)}{1+x^2}dx$

Put $\tan^{-1}x=t \Rightarrow \frac{1}{1+x^2}dx=dt$

$$\therefore\ I=\int\sin 2t\,dt=-\frac{\cos 2t}{2}+C=\frac{-[\cos 2(\tan^{-1}x)]}{2}+C$$

39. (a): Let $I=\int\frac{\sqrt{1+x^2}}{x^4}dx=\int\frac{\sqrt{1+x^2}}{x}\cdot\frac{1}{x^3}dx$

$$=\int\sqrt{\frac{1}{x^2}+1}\cdot\frac{1}{x^3}dx$$

Now, putting $1+\frac{1}{x^2}=t^2$

$$\Rightarrow \frac{-2}{x^3}dx=2tdt \Rightarrow -\frac{1}{x^3}dx=tdt$$

$$\therefore\quad I=\int -t^2dt=-\frac{t^3}{3}+C=-\frac{1}{3}\left(1+\frac{1}{x^2}\right)^{3/2}+C$$

40. (c) **41. (a)** **42. (b)**

43. (c) : The equation of the plane on which players of club B are seated is

$$\vec{r}\cdot(\hat{i}+3\hat{j}+2\hat{k})=8$$

$$\Rightarrow\quad (x\hat{i}+y\hat{j}+z\hat{k})\cdot(\hat{i}+3\hat{j}+2\hat{k})=8$$

$$\Rightarrow\quad x+3y+2z=8 \quad\Rightarrow\quad \frac{x}{8}+\frac{y}{8/3}+\frac{z}{4}=1$$

44. (d): The equation of the plane on which players of Club B are seated.

$x+3y+2z=8$ ...(i)

At point (1, 1, 2) plane Eq. (i).

$1+3\times1+2\times2=8$, which is satisfied.

45. (a): The length of the perpendicular from the origin to the plane eq. (i) is

$$\left|\frac{-8}{\sqrt{(1)^2+(3)^2+(2)^2}}\right|=\frac{8}{\sqrt{14}}\text{ units}$$

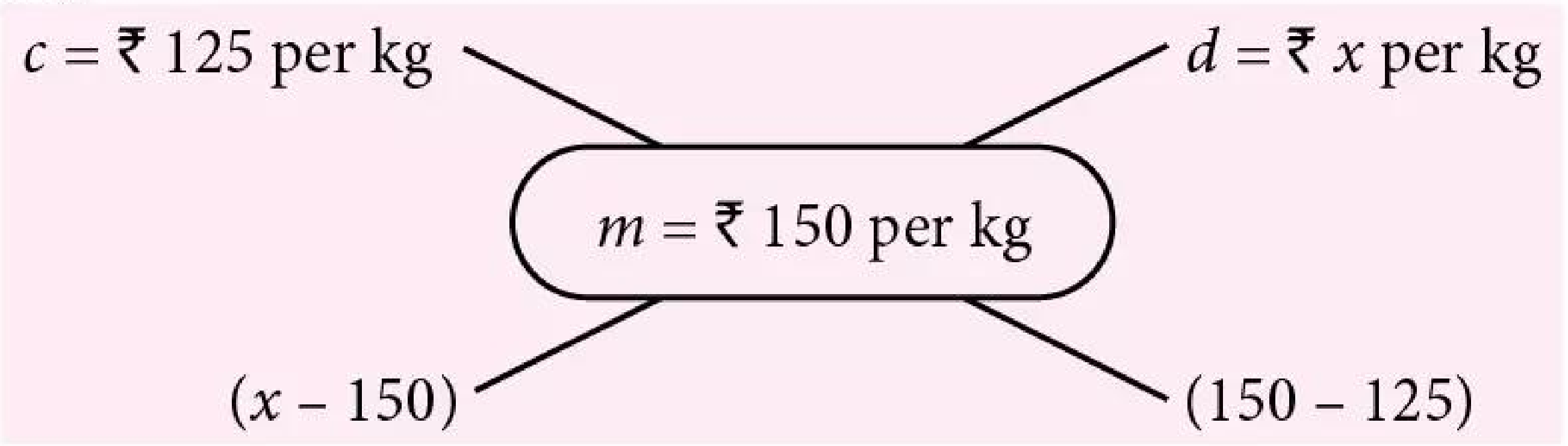
SECTION B2 (APPLIED MATHEMATICS)

16. (b): Since tea A and tea C are mixed in equal proportion, so their average price is $\frac{120+130}{2}$ *i.e.* ₹ 125 per kg.

As the price per kg of the mixture is ₹150 per kg which is more than the average price of tea A and tea C. So, the price per kg of tea B must be greater than ₹ 150. Let the price of tea B be ₹ x per kg.

(Tea A + tea C) and tea B are mixed in the ratio (1 + 1) : 2 *i.e.* 2 : 2.

Here,



$$\therefore\quad \frac{2}{2}=\frac{x-150}{150-125} \Rightarrow x-150=25 \Rightarrow x=175$$

Hence, the price of tea B is ₹175 per kg.

17. (a): Given $f(x)=2x^3-9x^2+12x-5$ ...(i)

Differentiating (i) w.r.t. x, we get

$f'(x)=2\cdot3x^2-9\cdot2x+12=6(x^2-3x+2)$

Now $f'(x)=0 \Rightarrow 6(x^2-3x+2)=0 \Rightarrow (x-1)(x-2)=0$

$\Rightarrow\quad x=1, 2$

Also 1, 2 both are in [0, 3], therefore, 1 and 2 both are turning points.

Further, $f(1)=2\cdot1^3-9\cdot1^2+12\cdot1-5=2-9+12-5=0$,

$f(2)=2\cdot2^3-9\cdot2^2+12\cdot2-5=16-36+24-5=-1$,

$f(0)=-5$

and $f(3)=2\cdot3^3-9\cdot3^2+12\cdot3-5=54-81+36-5=4$.

Therefore, the absolute maximum value = 4

18. (c)

19. (a): Given, $P=$ ₹1000000, $i=\frac{9}{12\times100}=0.0075$

and $n=12\times10=120$

$$\text{So, EMI}=\frac{P\times i\times(1+i)^n}{(1+i)^n-1}=\frac{1000000\times0.0075\times(1.0075)^{120}}{(1.0075)^{120}-1}$$

$$=\frac{7500\times2.4514}{2.4514-1}=\frac{7500\times2.4514}{1.4514}=12667.42\approx12667$$

Hence, EMI = ₹ 12667

20. (d): We know that $E(X)=\sum X_iP(X_i)$

$=-4\times0.1+(-3)\times0.2+(-2)\times0.3+(-1)\times0.2+0\times0.2$

$=-0.4-0.6-0.6-0.2=-1.8$

21. (b): (P) 21 mod (2) ≡ 1 mod (2)

∴ Remainder = 1

(Q) (8 × 7) mod 6 ≡ 2 mod 6

So, remainder = 2.

(R) 12 mod 4 ≡ 0 mod 4

So, remainder = 0.

(S) (9 + 23)mod 12 = 8 mod 12

So, remainder = 8.

22. (c) : We solve each inequality separately.

$4x-5<11 \Rightarrow 4x<16 \Rightarrow x<4$ ...(i)

and $-3x-4\geq 8 \Rightarrow -3x\geq12 \Rightarrow x\leq-4$ ...(ii)

From (i) and (ii), we find that the values of x which satisfy both the given inequalities are given by $x\leq-4$.

Hence, the solution set is $[x : x\in \mathbf{R}, x\leq-4]$ *i.e.* $(-\infty, -4]$.

23. (d) : Average cost (AC) = $\frac{\text{Total cost }(C(x))}{x}$

$$= \frac{\frac{x^3}{4}+9x^2-7}{x} = \frac{x^2}{4}+9x-\frac{7}{x}$$

24. (d) : The term population also refers to the group of objects, events, experiments, observations, procedure etc.

25. (a) : Given, $F = ₹10000$, $r = 10\%$ per annum, $N = 5$ years, $d = 8\%$

$$\Rightarrow \quad i = \frac{8}{100} = 0.08$$

So, coupon payment $C = ₹10000 \times \frac{10}{100} = ₹1000$

Now, P.V. $= \frac{C[1-(1+i)^{-N}]}{i} + F(1+i)^{-N}$

$$= \frac{1000[1-(1.08)^{-5}]}{0.08} + 10000(1.08)^{-5}$$

$$\therefore \quad \text{P.V.} = \frac{1000[1-0.6808)]}{0.08} + 10000 \times 0.6808$$

$= 3990 + 6808 = 10798$

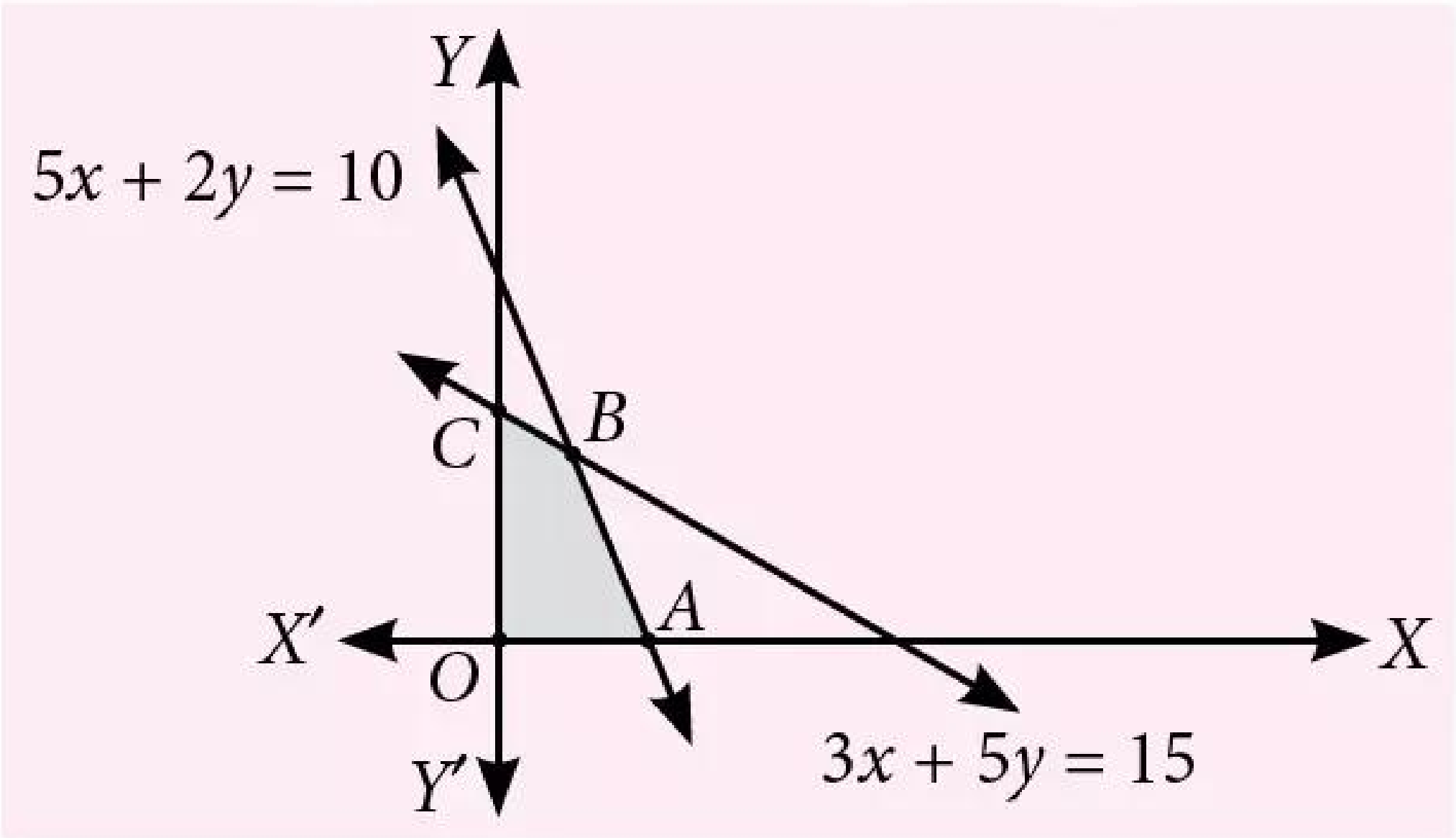
Hence, the fair value of bond is ₹10798.

26. (b) : Given, $t_1 = 4h$, $t_2 = 8h$ and $y = 2$ km/h

Speed of man in still water $= y\left(\frac{t_2+t_1}{t_2-t_1}\right)$

$$= 2\left(\frac{8+4}{8-4}\right) = 2\left(\frac{12}{4}\right) = 2\times 3 = 6 \text{ km/h}$$

27. (a) : We draw the lines $3x + 5y = 15$ and $5x + 2y = 10$, and shade the region satisfied by the given inequalities.



The shaded region in the adjoining figure shows the feasible region determined by the given constraints. We observe that region $OABC$ is a convex polygon and bounded, and has corner points O(0, 0), $A(2, 0)$, $B\left(\frac{20}{19}, \frac{45}{19}\right)$ and $C(0, 3)$.

Using corner point method, we see that at (0, 0), $Z = 0$; at $A(2, 0)$, $Z = 10$; at $B\left(\frac{20}{19}, \frac{45}{19}\right)$, $Z = 10$; at $C(0, 3)$, $Z = 6$.

Thus, Z is maximum at all points on segement AB Thus, we have multiple optimal solutions.

28. (d) : Given $f(x) = x^3 - 3x^2 + 3x$, $D_f = \mathbf{R}$.

Differentiating it w.r.t. x, we get

$f'(x) = 3x^2 - 6x + 3 = 3(x^2 - 2x + 1)$

$= 3(x-1)^2 \geq 0$ for all $x \in \mathbf{R}$

Therefore, the function f is increasing on $\mathbf{R}$.

29. (c) : First let us find the respective share of profit of Ramesh and Mohan.

Required ratio $= \frac{\text{Ramesh's investment}}{\text{Mohan's investment}} = \frac{3500000}{4200000} = 5:6$

Profit earned = 220000

Ramesh's share of profit $= \frac{5}{11} \times 220000 = 100000$

30. (b)

31. (c) : Annual depreciation = ₹12000

Scrap value of scooter = ₹55000

Useful life = 10 years

Let original cost of scooter = ₹x

Annual depreciation $= \frac{\text{original cost - scrap value}}{\text{useful life}}$

$$\Rightarrow \quad 12000 = \frac{x-55000}{10}$$

$\Rightarrow \quad x - 55000 = 120000 \Rightarrow x = 175000$

Hence, original cost of scooter was ₹175000.

32. (d) : Given $P(X = 0) = P(X = 1) = p$ and $P(X = 2) = P(X = 3) = k$(say)

We know that $\Sigma p_i = 1$

$\Rightarrow \quad p + p + k + k = 1 \Rightarrow 2p + 2k = 1$

$\Rightarrow \quad p + k = \frac{1}{2} \Rightarrow k = \frac{1}{2} - p$

x_i	p_i	p_ix_i	$p_ix_i^2$
0	p	0	0
1	p	p	p
2	$\frac{1}{2}-p$	$1-2p$	$2-4p$
3	$\frac{1}{2}-p$	$\frac{3}{2}-3p$	$\frac{9}{2}-9p$
Total		$\frac{5}{2}-4p$	$\frac{13}{2}-12p$

Given, $\Sigma p_i x_i^2 = 2\Sigma p_i x_i$

$$\Rightarrow \quad \frac{13}{2} - 12p = 2\left(\frac{5}{2} - 4p\right)$$

$$\Rightarrow \quad \frac{13}{2} - 12p = 5 - 8p \Rightarrow -4p = -\frac{3}{2} \Rightarrow p = \frac{3}{8}$$

33. (a) : We have, $\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$

$$\therefore \quad A^T = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$$

Given, $A^TA = I_3$

$$\therefore \quad \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow \quad 2x^2 = 1, 6y^2 = 1, 3z^2 = 1$

$$\Rightarrow \quad x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{6}}, z = \pm\frac{1}{\sqrt{3}}$$

Now, $2x^2 + 6y^2 + 3z^2 = 1 + 1 + 1 = 3$

and $x^2 + y^2 + z^2 = \frac{1}{2} + \frac{1}{6} + \frac{1}{3} = \frac{3+1+2}{6} = 1$

$\therefore$ Statements II, III and V are true.

34. (b) **35. (c)**

36. (c) : Let the waste pipe take t minutes to empty the full tank.

According to given,

$$\frac{1}{20} + \frac{1}{24} - \frac{1}{t} = \frac{1}{15} \Rightarrow \frac{1}{t} = \frac{1}{20} + \frac{1}{24} - \frac{1}{15}$$

$$\Rightarrow \frac{1}{t} = \frac{6+5-8}{120} \Rightarrow \frac{1}{t} = \frac{3}{120} \Rightarrow t = 40$$

$\therefore$ Volume of water emptied by the waste pipe in 1 minute = 3 gallons

So, volume of water emptied by the waste pipe in 40 minutes = 3 × 40 = 120 gallons.

$\therefore$ The capacity of the tank = 120 gallons.

37. (c) : Here total cost,

$$y = 3x\left(\frac{x+7}{x+5}\right) + 5 = 3\left(\frac{x^2+7x}{x+5}\right) + 5$$

$\therefore$ Marginal cost (MC),

$$= \frac{dy}{dx} = 3\left[\frac{(x+5)(2x+7) - (x^2+7x)}{(x+5)^2}\right]$$

$$= 3\left[\frac{x^2+10x+35}{(x+5)^2}\right] = 3\left[\frac{(x+5)^2+10}{(x+5)^2}\right] = 3\left[1 + \frac{10}{(x+5)^2}\right].$$

To check whether MC falls continuously with increase in x or not, we will check that $\frac{d}{dx}(\text{MC}) < 0$ for all values of x or not.

Here $\frac{d}{dx}(\text{MC}) = \frac{-60}{(x+5)^3} < 0$, for all (positive) values of x.

Hence, the marginal cost falls continuously as the output increase.

38. (d): Let the rate of interest be r% per annum, then $i = \frac{r}{200}$.

Given, R = ₹750 and P = ₹20000

$$\text{Using, } P = \frac{R}{i} \Rightarrow i = \frac{R}{P} \Rightarrow \frac{r}{200} = \frac{750}{20000} \Rightarrow r = 7.5$$

39. (b)

40. (c) : Volume of the semi globe (hemisphere) $= \frac{2\pi r^3}{3}$

Volume of cone $= \frac{\pi r^2 h}{3}$

$$\therefore \quad \text{Volume of the solid} = \frac{2\pi r^3}{3} + \frac{\pi r^2 h}{3} = \frac{\pi r^2}{3}(2r + h)$$

41. (c) : Market trend does not affect EMI

42. (c) : To find the last three digits of the product 1234 × 5678, we find 1234 × 5678 (mod 1000).

Since 1234 = 234 (mod 1000) and 5678 = 678 (mod 1000)

So, 1234 × 5678 = 234 × 678 (mod 1000) = 652 (mod 1000).

Hence, the last three digits of the product 1234 × 5678 are 652.

43. (b)

44. (a) : $x = k(A - \sin A), y = k(1 - \cos A)$

$$X'(A) = \frac{dx}{dA} = K(1 - \cos A)$$

$$Y'(A) = \frac{dy}{dA} = K\sin A$$

$$\frac{dy}{dx} = \frac{k\sin A}{k(1-\cos A)} = \frac{2\sin A/2 \cos A/2}{2\sin^2 A/2} = \frac{\cos A/2}{\sin A/2} = \cot A/2$$

$$\text{Now, } \frac{d^2y}{dx^2} = \frac{d}{dA}\left(\frac{dy}{dx}\right) \cdot \frac{dA}{dx}$$

$$= \frac{1}{2}\left(-\operatorname{cosec}^2\frac{A}{2}\right) \cdot \frac{1}{K(1-\cos A)}$$

45. (c) : To allow maximum airflow inside the building A should be maximum.

$$A = 10x - \frac{1}{2}(\pi + 4)x^2$$

$$\Rightarrow \quad \frac{dA}{dx} = 10 - (\pi + 4)x \text{ and } \frac{d^2A}{dx^2} = -(\pi + 4)$$

$$\text{Now, } \frac{dA}{dx} = 0 \Rightarrow 10 - (\pi + 4)x = 0 \Rightarrow x = \frac{10}{\pi + 4}$$

$$\text{At } x = \frac{10}{\pi + 4}, \frac{d^2A}{dx^2} = -(\pi + 4) < 0$$

$\Rightarrow$ A is maximum when $x = \frac{10}{\pi + 4}$ m

Width of door $= 2x = \frac{20}{\pi + 4}$ m.

JEE WORK OUTS

Single Option Correct Type

1. Let $f : (-1, 1) \to B$, be a function defined by $f(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, then f is both one-one and onto, when B is the interval

(a) $\left[0, \frac{\pi}{2}\right)$ (b) $\left(0, \frac{\pi}{2}\right)$

(c) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (d) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

2. Let three matrices $A = \begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}$; $B = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$, then $tr(A) + tr\left(\frac{ABC}{2}\right) +$

$$tr\left(\frac{A(BC)^2}{4}\right) + tr\left(\frac{A(BC)^3}{8}\right) + \ldots\ldots. + \infty =$$

(a) 6 (b) 9

(c) 12 (d) None of these

3. A circular cylinder is inscribed in a sphere of radius r. If the volume of the cylinder is maximum, then its height is

(a) $\frac{2r}{\sqrt{3}}$ (b) r (c) $\frac{3r}{2}$ (d) $\sqrt{2}r$

4. A swimmer S is in the sea at a distance 4 km from the closest point A on a straight shore. His house B is on the shore at a distance 5 km from A. He can swim at a speed of 3 km per hour and walk at a speed of 5 km per hour. The distance of the point on the shore from B so that he reaches his home in the minimum time is

(a) 3 (b) 4 (c) 2 (d) 1

5. If $x\frac{dy}{dx} + 2y = \ln x$, then $e^2 y(e) - y(1) =$

(a) $\frac{e^2+1}{2}$ (b) $\frac{e^2+1}{3}$ (c) $\frac{e^2+1}{4}$ (d) $e^2 + 1$

6. If $\lim\limits_{x \to 0} \frac{1 - \cos x\sqrt{\cos 2x}}{x^2} = \frac{k_1}{k_2}$, then the value of $k_1^2 - k_2^2$ equals

(a) 1 (b) 2 (c) 4 (d) 5

7. The solution of the system of equations $x + y = \frac{\pi}{2}$, $\sin x + \sin y = \sqrt{2}$ is $\{x, y\}$, then

(a) $x = 2k\pi + \frac{\pi}{4}$, $y = \frac{\pi}{4} - 2k\pi$

(b) $x = 2k\pi - \frac{\pi}{2}$, $y = \frac{\pi}{2} + x$

(c) $x = 2k\pi + \frac{\pi}{3}$, $y = \frac{\pi}{3} - 2k\pi$

(d) $x = k\pi + \frac{\pi}{4}$, $y = \frac{\pi}{4} - k\pi$ $(k \in I)$

8. If ω is cube root of unity, then the linear factors of $x^3 + y^3$ in complex number is

(a) $(x - y)(x - \omega y)(x - \omega^2 y)$
(b) $(x - y)(x + \omega y)(x + \omega^2 y)$
(c) $(x + y)(x + \omega y)(x + \omega^2 y)$
(d) $(x + y)(x - \omega y)(x - \omega^2 y)$

9. If $\frac{dy}{dx} = \frac{y}{x}\left(\frac{x\cos\frac{y}{x} + y\sin\frac{y}{x}}{y\sin\frac{y}{x} - x\cos\frac{y}{x}}\right)$, then

(a) $x \cos\frac{y}{x} = cy$ (b) $x \sec\frac{y}{x} = cy$

(c) $\cos\frac{y}{x} = cxy$ (d) $\sec\frac{y}{x} = cxy$

10. The value of $\frac{^{40}C_0}{1} + \frac{^{40}C_2}{3} + \frac{^{40}C_4}{5} + \ldots.. + \frac{^{40}C_{40}}{41}$ is

(a) $\frac{2^{41}-1}{40}$ (b) $\frac{2^{40}}{41}$ (c) $\frac{2^{40}-1}{40}$ (d) $\frac{2^{41}}{41}$

11. Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}, \vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$. If $|\vec{c}|=1$ and $(\vec{a}\times\vec{b})\times\vec{c}=\vec{0}$, then $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2 =$

(a) 0 (b) 1

(c) $|\vec{a}|^2\,|\vec{b}|^2$ (d) $|\vec{a}\times\vec{b}|^2$

12. If $a, b, c, \ldots, k$ are roots of the equation $f(x) = 0$, then the value of $\frac{f(x)}{x-a}+\frac{f(x)}{x-b}+\ldots+\frac{f(x)}{x-k}$ is

(a) 2 (b) 0

(c) 1 (d) None of these

13. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively , is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is green, the probability that the original signal was green is

(a) $\frac{3}{5}$ (b) $\frac{6}{7}$ (c) $\frac{12}{13}$ (d) $\frac{9}{20}$

14. Let a_m be the m^{th} term of an A.P. with common difference 'd'. If $\lambda_1 = \sum_{r=1}^{2017} a_{2r}$ and $\lambda_2 = \sum_{r=1}^{2017} a_{2r-1}$, then for $d = 1$, the value of $\lambda_1 - \lambda_2$ equals

(a) 2017 (b) 2016 (c) 2015 (d) 2018

15. Let $I=\int\frac{e^x dx}{e^{4x}+e^{2x}+1}, J=\int\frac{e^{-x}dx}{e^{-4x}+e^{-2x}+1}$.

Then for an arbitrary constant c, $J - I =$

(a) $\frac{1}{2}\ln\left(\frac{e^{4x}-e^{2x}+1}{e^{4x}+e^{2x}+1}\right)+c$ (b) $\frac{1}{2}\ln\left(\frac{e^{2x}+e^{x}+1}{e^{2x}-e^{x}+1}\right)+c$

(c) $\frac{1}{2}\ln\left(\frac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}\right)+c$ (d) $\frac{1}{2}\ln\left(\frac{e^{4x}+e^{2x}+1}{e^{4x}-e^{2x}+1}\right)+c$

One or More Than One Option(s) Correct Type

16. Let x_1, x_2, x_3, x_4 be four non-zero numbers satisfying the equation $\tan^{-1}\frac{a}{x}+\tan^{-1}\frac{b}{x}+\tan^{-1}\frac{c}{x}+\tan^{-1}\frac{d}{x}=\frac{\pi}{2}$. Then

(a) $x_1 + x_2 + x_3 + x_4 = a + b + c + d$

(b) $\frac{1}{x_1}+\frac{1}{x_2}+\frac{1}{x_3}+\frac{1}{x_4}\neq 0$

(c) $x_1x_2x_3x_4 = abcd$

(d) $(x_2 + x_3 + x_4)(x_3 + x_4 + x_1)(x_4 + x_1 + x_2)(x_1 + x_2 + x_3) = abcd$

17. The determinant $\begin{vmatrix} a & b & a\alpha+b \\ b & c & b\alpha+c \\ a\alpha+b & b\alpha+c & 0 \end{vmatrix} = 0$ if

(a) a, b, c are in A.P. (b) a, b, c are in G.P.

(c) α is a root of $ax^2 + 2bx + c = 0$

(d) $(x - \alpha)$ is a factor of $ax^2 + 2bx + c = 0$

18. The plane $ax + by = 0$ is rotated through an angle α about its line of intersection with the plane $z = 0$. Then the equation to the plane in new position is/are

(a) $ax+by+z\sqrt{a^2+b^2}\tan\alpha=0$

(b) $ax+by-z\sqrt{a^2+b^2}\tan\alpha=0$

(c) $ax+by+z\sqrt{a^2+b^2}\cot\alpha=0$

(d) $ax+by-z\sqrt{a^2+b^2}\cot\alpha=0$

19. If A, B are two events such that $P(A\cup B) \geq 3/4$ and $\frac{1}{8}\leq P(A\cap B)\leq\frac{3}{8}$ then

(a) $P(A)+P(B)\leq\frac{11}{8}$ (b) $P(A)\cdot P(B)\leq\frac{3}{8}$

(c) $P(A)+P(B)\geq\frac{7}{8}$ (d) None of these

20. The variable x takes two values x_1 and x_2 with frequencies f_1 and f_2, respectively. If σ denotes the standard deviation of x, then

(a) $\sigma^2=\frac{f_1x_1^2+f_2x_2^2}{f_1+f_2}-\left(\frac{f_1x_1+f_2x_2}{f_1+f_2}\right)^2$

(b) $\sigma^2=\frac{f_1f_2}{(f_1+f_2)^2}(x_1-x_2)^2$

(c) $\sigma^2=\frac{(x_1-x_2)^2}{(f_1+f_2)^2}$ (d) None of these

21. A circle C of unit radius lies in the first quadrant and touches both the axes. The circle C_1 touches both the axes and intersects C such that the common chord is longest. Its radius is

(a) 2 (b) 3 (c) 1/2 (d) 1/3

22. Which of the following is/are discontinuous at $x=1$?

(a) $f(x)=\frac{1}{1+2^{\tan x}}$

(b) $g(x)=\lim\limits_{n\to\infty}\dfrac{1}{1+n\sin^2(\pi x)}$

(c) $h(x)=2^{-2^{\left(\frac{1}{1-x}\right)}}, x\neq 1$ and $h(1)=1$

(d) $\phi(x)=\dfrac{x-1}{|x-1|+2(x-1)^2}, x\neq 1$ and $\phi(1)=1$

23. If $I=\int_0^{\pi/2}\sqrt{\sin x}\,dx$ and $J=\int_0^{\pi/4}\sqrt{\tan x}\,dx$, then

(a) $I>\dfrac{\pi}{3}$ (b) $J>\dfrac{\pi}{6}$

(c) $I<\dfrac{\pi^{3/2}}{3\sqrt{2}}$ (d) $J<\dfrac{\pi^{3/2}}{12}$

24. Match the following :

Column-I		Column-II	
(P)	The area bounded by the curves $y^2=9x$ and $y=3x$ is	(1)	1
(Q)	The area bounded by the curve $y=\dfrac{x}{\sqrt{2x^2+1}}$, x-axis and $x=0, 2$ is	(2)	$\dfrac{\pi}{6}$
(R)	The area bounded by one arc of the curve $y=\sin^2 3x$ and the x-axis is	(3)	$\dfrac{1}{2}$

(a) (P)–(3), (Q)–(1), (R)–(2)
(b) (P)–(2), (Q)–(3), (R)–(1)
(c) (P)–(2), (Q)–(1), (R)–(3)
(d) (P)–(1), (Q)–(2), (R)–(3)

25. Match the following.

Column-I		Column-II	
(P)	The probability that length of a randomly selected chord of a circle lies between 1/4 and 3/4 of its diameter is	(1)	1/30
(Q)	Three numbers are selected from the set {1, 2, 3, …, 10}. The probability that they form a G.P. is	(2)	1/2
(R)	If x and y are selected from the set {1, 2, 3, …, 10}, the probability, that x^3+y^3 is a multiple of 3, is	(3)	1/4
(S)	A card is lost from a pack of 52 playing cards. If a card is drawn from the remaining cards, the probability that it is a spade is	(4)	1/3

(a) (P)–(1), (Q)–(2), (R)–(3), (S)–(4)
(b) (P)–(2), (Q)–(3), (R)–(1), (S)–(4)
(c) (P)–(1), (Q)–(4), (R)–(3), (S)–(2)
(d) (P)–(2), (Q)–(1), (R)–(4), (S)–(3)

Numerical Answer Type

26. The tangents drawn from a point P to the ellipse $9x^2+16y^2=144$ make angles θ_1, θ_2 with the major axis. If $\theta_1+\theta_2=2\alpha$, then the locus of P is $x^2-2xy\cot 2\alpha-y^2=\lambda$, where λ is ________ .

27. From a point P, tangents are drawn to the ellipse $x^2+2y^2=2$. If the chord of contact is a normal chord of the ellipse, then the locus of P is $\dfrac{8}{x^2}+\dfrac{1}{y^2}=\lambda$, where λ is ________ .

28. Suppose a matrix A satisfies $A^2-5A+7I=O$. If $A^5=aA+bI$, then the value of $a+\dfrac{3b}{7}+21$ must be ________ .

29. A biquadratic function $y=ax^4+bx^3+cx^2+dx+e$ $(a\neq 0)$ touches $y=px+q$ at $x=\alpha, \beta$ $(\alpha<\beta)$. Area of the region bounded by these graphs is $\dfrac{a(\beta-\alpha)^5}{\lambda}$ then the value of $\lambda/5$ is ________ .

30. If the planes $kx+4y+z=0$, $4x+ky+2z=0$ and $2x+2y+z=0$ intersect along a straight line, then the number of value of k is ________ .

SOLUTIONS

1. (c) : For $x\in(-1, 1)$, we have

$$f(x)=\tan^{-1}\left(\frac{2x}{1-x^2}\right)$$

$$\therefore f(\tan\theta)=\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)\quad(\text{By } x=\tan\theta)$$

$$=\tan^{-1}(\tan 2\theta)\Rightarrow f(x)=2\tan^{-1}x$$

$$\Rightarrow -\frac{\pi}{2}<\tan^{-1}\left(\frac{2x}{1-x^2}\right)<\frac{\pi}{2}.$$

2. (a): $BC=\begin{bmatrix}3&4\\2&3\end{bmatrix}\begin{bmatrix}3&-4\\-2&3\end{bmatrix}\Rightarrow BC=\begin{bmatrix}1&0\\0&1\end{bmatrix}=I_2$

$\therefore$ Given expression

$$=tr(A)+tr\left(\frac{A}{2}\right)+tr\left(\frac{A}{2^2}\right)+\ldots+\infty$$

$$=tr(A)+\frac{1}{2}tr(A)+\frac{1}{2^2}tr(A)+\ldots+\infty$$

$$=\frac{tr(A)}{1-(1/2)}=2tr(A)=2(2+1)=6$$

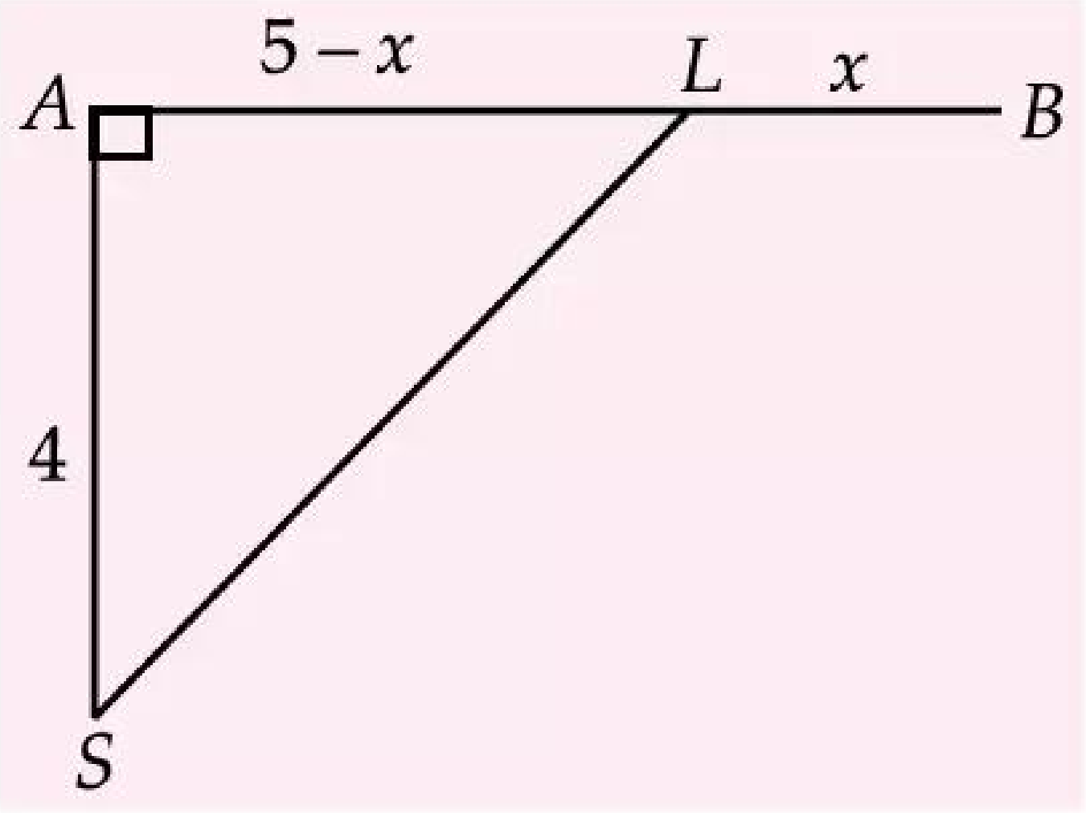
3. (a)

4. (c) : T = Total time

$= \frac{SL}{3}+\frac{LB}{5}$

$= \frac{\sqrt{(5-x)^2+16}}{3}+\frac{x}{5}$

$= \frac{\sqrt{x^2-10x+41}}{3}+\frac{x}{5}$



Now, $\frac{dT}{dx}=\frac{x-5}{3\sqrt{x^2-10x+41}}+\frac{1}{5}=0$

$\Rightarrow$ $x^2 - 10x + 16 = 0 \Rightarrow x = 2.$

5. (c) **6. (d)**

7. (a) : $x + y = \frac{\pi}{2}$ ...(i)

$\sin x + \sin y = \sqrt{2}$

$\Rightarrow\ 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}=\sqrt{2}\ \Rightarrow\ \cos\frac{x-y}{2}=1$ (using (i))

$\Rightarrow$ $x - y = 2.2\,k\pi = 4\,k\pi$

$$\Rightarrow \begin{cases} x+y = \frac{\pi}{2} \\ x-y = 4\pi k \end{cases} \quad (k\in I)$$

$\Rightarrow$ $x = 2k\pi + \frac{\pi}{4},\ y = \frac{\pi}{4} - 2k\pi,\ k \in I$

8. (c) **9. (d)**

10. (b) : Consider

$(1+x)^{40} = {}^{40}C_0 + {}^{40}C_1x + {}^{40}C_2x^2 + {}^{40}C_3x^3 + \ldots + {}^{40}C_{40}x^{40}$

$(1-x)^{40} = {}^{40}C_0 - {}^{40}C_1x + {}^{40}C_2x^2 - {}^{40}C_3x^3 + \ldots + {}^{40}C_{40}x^{40}$

On adding, we get

$(1+x)^{40} + (1-x)^{40} = 2\,[{}^{40}C_0 + {}^{40}C_2x^2 + {}^{40}C_4x^4 + \ldots + {}^{40}C_{40}x^{40}]$

Integrating from 0 to 1 on both sides, with respect to x, we get

$$\int_0^1\left((1+x)^{40}+(1-x)^{40}\right)dx$$

$$=2\left[\frac{{}^{40}C_0}{1}+\frac{{}^{40}C_2}{3}+\frac{{}^{40}C_4}{5}+\ldots..+\frac{{}^{40}C_{40}}{41}\right]$$

$$\therefore\quad \frac{{}^{40}C_0}{1}+\frac{{}^{40}C_2}{3}+\frac{{}^{40}C_4}{5}+\ldots..+\frac{{}^{40}C_{40}}{41}$$

$$=\frac{1}{2}\left[\left(\frac{(1+x)^{41}}{41}-\frac{(1-x)^{41}}{41}\right)\right]_0^1$$

$$=\frac{1}{2}\left[\left(\frac{2^{41}}{41}-0\right)-\left(\frac{1}{41}-\frac{1}{41}\right)\right]=\frac{2^{40}}{41}$$

11. (d): $(\vec{a}\times\vec{b})\times\vec{c}=\vec{0}\Rightarrow\vec{c}$ is parallel to $\vec{a}\times\vec{b}$,

$|\vec{c}\,|=1\Rightarrow\vec{c}=\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}\,|}$

$\therefore\quad (\vec{a}\times\vec{b})\cdot\vec{c}=\frac{|\vec{a}\times\vec{b}\,|^2}{|\vec{a}\times\vec{b}\,|}=|\vec{a}\times\vec{b}\,|$

$$\therefore\quad \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2 = |\vec{a}\cdot\,\vec{b}\,|^2$$

12. (d) : As $a, b, c, \ldots, k$ are roots of $f(x) = 0$

$\therefore$ $f(x) = (x - a)\,(x - b)\,(x - c)\ \ldots\ (x - k)$

$\Rightarrow$ $\log f(x) = \log\,(x - a) + \log\,(x - b) + \ldots + \log\,(x - k)$

$\Rightarrow$ $\frac{1}{f(x)}f'(x)=\frac{1}{x-a}+\frac{1}{x-b}+\ldots..+\frac{1}{x-k}$

$\Rightarrow$ $f'(x)=\frac{f(x)}{x-a}+\frac{f(x)}{x-b}+\ldots..+\frac{f(x)}{x-k}$

13. (c) :

Original colour	Colour at A	Colour at B	Probability
G	G	G	$\frac{4}{5}\cdot\frac{3}{4}\cdot\frac{3}{4}=\frac{36}{80}$
G	R	G	$\frac{4}{5}\cdot\frac{1}{4}\cdot\frac{3}{4}=\frac{12}{80}$
R	G	G	$\frac{1}{5}\cdot\frac{1}{4}\cdot\frac{1}{4}=\frac{1}{80}$
R	R	G	$\frac{1}{5}\cdot\frac{3}{4}\cdot\frac{1}{4}=\frac{3}{80}$

The probability that signal received at B is green, is $\frac{36+12+1+3}{80}=\frac{52}{80}$ of these 4 rows, the first two rows correspond to the case of original signal was green. The probability $=\frac{36+12}{80}=\frac{3}{5}$

$\therefore$ The required probability is $\frac{3}{5}\div\frac{52}{80}=\frac{12}{13}.$

14. (a): $a_m = a + (m - 1)d$, where 'a' is first term of A.P.

Now $\lambda_1-\lambda_2=\sum_{r=1}^{2017}(a_{2r}-a_{2r-1})$ but a_{2r} and a_{2r-1} are consecutive terms having common difference d.

$\therefore\ \lambda_1-\lambda_2=\sum_{r=1}^{2017}d=2017d=2017$ (as $d=1$)

15. (c)

16. (c,d): $\tan^{-1}\frac{a}{x}+\tan^{-1}\frac{b}{x}=\frac{\pi}{2}-\left(\tan^{-1}\frac{c}{x}+\tan^{-1}\frac{d}{x}\right)$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{a}{x}+\frac{b}{x}}{1-\frac{ab}{x^2}}\right)=\frac{\pi}{2}-\tan^{-1}\left(\frac{\frac{c}{x}+\frac{d}{x}}{1-\frac{cd}{x^2}}\right)$$

$$\Rightarrow \tan^{-1}\frac{(a+b)x}{x^2-ab}=\cot^{-1}\frac{(c+d)x}{x^2-cd}$$

$$\Rightarrow \frac{(a+b)x}{x^2-ab}=\frac{x^2-cd}{(c+d)x}$$

$$\Rightarrow x^4-(ab+ac+ad+bc+bd+cd)x^2+abcd=0$$

Its roots are x_1, x_2, x_3, x_4

$\therefore \sum x_1 = 0, \sum x_1x_2 = -\sum ab, \sum x_1x_2x_3 = 0$

and $x_1x_2x_3x_4 = abcd$

$\therefore \sum \frac{1}{x_1} = 0$ and $(x_2+x_3+x_4)(x_3+x_4+x_1)(x_4+x_1+x_2)$

$(x_1 + x_2 + x_3) = x_1x_2x_3x_4 = abcd$

17. (b, c, d)

18. (a, b): Given planes are $ax + by = 0$ ...(i)

and $z = 0$ ...(ii)

$\therefore$ Equation of any plane passing through the line of intersection of planes (i) and (ii) may be taken as

$ax + by + kz = 0$ ...(iii)

The direction cosines of a normal to the plane (iii) are

$$\frac{a}{\sqrt{a^2+b^2+k^2}}, \frac{b}{\sqrt{a^2+b^2+k^2}}, \frac{k}{\sqrt{a^2+b^2+k^2}}$$

The direction cosines of a normal to the plane (i) are

$$\frac{a}{\sqrt{a^2+b^2}}, \frac{b}{\sqrt{a^2+b^2}}, 0$$

Since the angle between the planes (i) and (ii) is α

$$\therefore \cos\alpha=\frac{a\cdot a+b\cdot b+k\cdot 0}{\sqrt{a^2+b^2+k^2}\sqrt{a^2+b^2}}=\sqrt{\frac{a^2+b^2}{a^2+b^2+k^2}}$$

$$\Rightarrow k^2\cos^2\alpha = a^2(1-\cos^2\alpha)+b^2(1-\cos^2\alpha)$$

$$\Rightarrow k^2=\frac{(a^2+b^2)\sin^2\alpha}{\cos^2\alpha}\Rightarrow k=\pm\sqrt{a^2+b^2}\tan\alpha$$

putting in (iii) we get equation of plane as;

$$ax+by\pm z\sqrt{a^2+b^2}\tan\alpha=0$$

19. (a, c) : $P(A\cup B)\ge\frac{3}{4}$ ...(i), $\frac{1}{8}\le P(A\cap B)\le\frac{3}{8}$ ...(ii)

Let $P(A) + P(B)$ be x.

From (i), $x-P(A\cap B)\ge\frac{3}{4}$

$\Rightarrow x-\frac{3}{4}\ge P(A\cap B)\ge\frac{1}{8}$ [From (ii)]

$\Rightarrow x\ge\frac{7}{8}$ $\because P(A\cup B)\le 1\Rightarrow x-P(A\cap B)\le 1$ [From (ii)]

$$\Rightarrow x-1\le P(A\cap B)\le\frac{3}{8}\Rightarrow x\le\frac{11}{8}$$

20. (a,b) : Since, $\sigma^2=\left\{\frac{f_1x_1^2+f_2x_2^2}{f_1+f_2}-\left(\frac{f_1x_1+f_2x_2}{f_1+f_2}\right)^2\right\}$

$$=\frac{1}{(f_1+f_2)}\left\{f_1x_1^2+f_2x_2^2-\frac{(f_1x_1+f_2x_2)^2}{f_1+f_2}\right\}$$

$$=\frac{1}{(f_1+f_2)^2}(f_1^2x_1^2+f_1f_2x_1^2+f_2f_1x_2^2+f_2^2x_2^2-f_1^2x_1^2 -f_2^2x_2^2-2f_1f_2x_1x_2)$$

$$=\frac{f_1f_2}{(f_1+f_2)^2}(x_1-x_2)^2$$

21. (b, d) : $C : x^2 + y^2 - 2(x + y) + 1 = 0,$

$C_1 : x^2 + y^2 - 2r(x + y) + r^2 = 0$

The common chord is $C - C_1 = 0$

$\Rightarrow 2(r-1)(x+y) = r^2 - 1$ or $2(x+y) = 1 + r$ ...(i)

If $r < 1$, (r, r) lies on (i)

$\therefore 2(r+r) = 1 + r \Rightarrow r = \frac{1}{3}$

If $r > 1$, $(1, 1)$ lies on (i)

$\therefore 2(1+1) = 1 + r \Rightarrow r = 3.$

22. (b, c, d) : (a) $f(x)$ is continuous at $x = 1$

(b) $g(1^+) = 0, g(1) = 1 \Rightarrow g(x)$ is discontinuous at $x = 1$

(c) $\left.\begin{array}{l}h(1^+)=1\\ h(1^-)=0\end{array}\right]\Rightarrow h(x)$ is discontinuous at $x = 1$

(d) L.H.L. ≠ R.H.L.

$\Rightarrow \phi(x)$ is discontinuous at $x = 1$

23. (a, c) : The curve $y = \sin x$ lies between the lines $y = x$ and $y=\frac{2x}{\pi}, x\in\left[0,\frac{\pi}{2}\right]$

$$\therefore \frac{2x}{\pi}<\sin x<x\Rightarrow\sqrt{\frac{2}{\pi}}\sqrt{x}<\sqrt{\sin x}<\sqrt{x}$$

$$\therefore \sqrt{\frac{2}{\pi}}\int_0^{\pi/2}\sqrt{x}\,dx<I<\int_0^{\pi/2}\sqrt{x}\,dx\Rightarrow\frac{\pi}{3}<I<\frac{\pi^{3/2}}{3\sqrt{2}}$$

The curve $y = \tan x$ lies between the lines $y = x$ and $y=\frac{4x}{\pi}, x\in\left[0,\frac{\pi}{4}\right]$

$$\therefore \frac{4x}{\pi}>\tan x>x\Rightarrow\frac{2}{\sqrt{\pi}}\sqrt{x}>\sqrt{\tan x}>\sqrt{x}$$

$$\frac{2}{\sqrt{\pi}}\int_0^{\pi/4}\sqrt{x}\,dx>J>\int_0^{\pi/4}\sqrt{x}\,dx\ \therefore\ \frac{\pi}{6}>J>\frac{\pi^{3/2}}{12}.$$

24. (a) **(P)** $y^2 = 9x, y = 3x \Rightarrow x = 0, 1$

$$\text{Area}=\int_0^1\left(3\sqrt{x}-3x\right)dx=3\left(\frac{2}{3}-\frac{1}{2}\right)=\frac{1}{2}$$

(Q) $\text{Area}=\int_0^2\frac{x}{\sqrt{2x^2+1}}dx=\left[\frac{1}{2}\sqrt{2x^2+1}\right]_0^2=\frac{3-1}{2}=1$

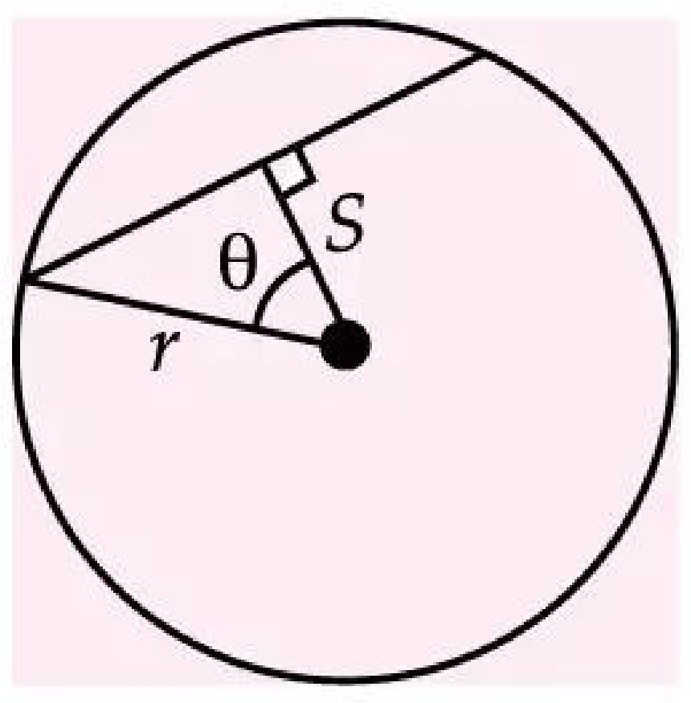
(R) Area $= \int_0^{\frac{\pi}{3}} \sin^2 3x\,dx$

Substituting $3x = t$, we get,

$$\text{Area} = \frac{1}{3}\int_0^{\pi} \sin^2 t\,dt = \frac{1}{3}\cdot\frac{1}{2}\cdot\pi = \frac{\pi}{6}$$

25. (d): (P) Let P be the length of the chord, S be the distance of the mid point of the chord from the centre and r the radius of the given circle then

$S = r\cos\theta$, $P = 2r\sin\theta$



Now, $\frac{1}{4}(2r) < 2r\sin\theta < \frac{3}{4}\,2r$

$$\Rightarrow \quad \frac{1}{4} < \sqrt{1-\cos^2\theta} < \frac{3}{4}$$

$$\Rightarrow \quad \frac{\sqrt{7}}{4}r < S < \frac{\sqrt{15}}{4}r$$

Let $R_1 = \frac{\sqrt{7}}{4}r$, $R_2 = \frac{\sqrt{15}}{4}r$

$\therefore$ Given condition is satisfied if mid point of the chord lies with in region between concentric circles of radii $\frac{\sqrt{7}}{4}r$ and $\frac{\sqrt{15}}{4}r$.

$$\therefore \text{Required probability} = \frac{\text{Area of the circular annulus}}{\text{Area of given circle}}$$

$$= \frac{\pi(R_2{}^2 - R_1{}^2)}{\pi r^2} = \left(\frac{15}{16} - \frac{7}{16}\right) = \frac{1}{2}$$

(Q) 1, 2, 4; 4, 6, 9; 2, 4, 8; 1, 3, 9 are 4 G.P.s

$$\text{Probability} = \frac{4}{\binom{10}{3}} = \frac{4}{120} = \frac{1}{30}$$

(R) 1, 4, 7, 10; 2, 5, 8; 3, 6, 9

$x^3 + y^3$ is divisible by $3 \Rightarrow x + y$ is divisible by 3.

$\therefore$ x in the first row above and y in the second row or x in the second row and y in the first row or both x and y are in the third row.

$$\text{Required probability} = \frac{4\times 3+3}{\binom{10}{2}} = \frac{15}{45} = \frac{1}{3}$$

(S) SS stand for spade lost and spade drawn, NS stand for nonspade lost and spade drawn

$\therefore$ Required probability $= P(SS) + P(NS)$

$$= \frac{1}{4}\cdot\frac{12}{51} + \frac{3}{4}\cdot\frac{13}{51} = \frac{1}{4}$$

26. (7): The tangent with slope m to

$\frac{x^2}{16} + \frac{y^2}{9} = 1$ is $y = mx \pm \sqrt{16m^2+9}$

$\Rightarrow \quad (mx - y)^2 = 16m^2 + 9$

$\Rightarrow \quad (16 - x^2)m^2 + 2xym + 9 - y^2 = 0$

$$\tan\theta_1 + \tan\theta_2 = \frac{-2xy}{16-x^2}, \quad \tan\theta_1\cdot\tan\theta_2 = \frac{9-y^2}{16-x^2}$$

Now, $\theta_1 + \theta_2 = 2\alpha \Rightarrow \tan 2\alpha = \frac{-2xy}{7-x^2+y^2}$

$\Rightarrow \quad 7 - x^2 + y^2 + 2xy\cot 2\alpha = 0$

$\Rightarrow \quad x^2 - 2xy\cot 2\alpha - y^2 = 7$. $\therefore \lambda = 7$

27. (1): The chord of contact of tangents from $P(x_1, y_1)$ is $xx_1 + 2yy_1 = 2$ ...(i)

The normal chord at $(\sqrt{2}\cos\theta, \sin\theta)$ is $\frac{\sqrt{2}x}{\cos\theta} - \frac{y}{\sin\theta} = 1$ ...(ii)

(i), (ii) are identical

$$\therefore \quad \frac{x_1\cos\theta}{\sqrt{2}} = \frac{2y_1\sin\theta}{-1} = 2, \ \cos\theta = \frac{2\sqrt{2}}{x_1}, \ \sin\theta = -\frac{1}{y_1}$$

The locus of P is $\frac{8}{x^2} + \frac{1}{y^2} = 1$.

$\therefore \quad \lambda = 1$

28. (5)

29. (6): For $a > 0$, $f(x) = ax^4 + bx^3 + cx^2 + dx + e$

Since line $y = px + q$ touches $y = f(x)$ at two points $x = \alpha, \beta$

$\therefore |f(x) - g(x)| = a(x-\alpha)^2(x-\beta)^2$

$$\therefore \quad \text{Area} = \int_\alpha^\beta |f(x) - g(x)|\,dx = a\int_\alpha^\beta (x-\alpha)^2(x-\beta)^2\,dx$$

$$= a\left\{\left[(x-\alpha)^2\frac{(x-\beta)^3}{3}\right]_\alpha^\beta - \frac{2}{3}\int_\alpha^\beta (x-\alpha)(x-\beta)^3\,dx\right\}$$

$$= -\frac{2a}{3}\left\{\left[(x-\alpha)\frac{(x-\beta)^4}{4}\right]_\alpha^\beta - \frac{1}{4}\int_\alpha^\beta (x-\beta)^4\,dx\right\}$$

$$= \frac{a}{6}\int_\alpha^\beta (x-\beta)^4\,dx = \left[\frac{a(x-\beta)^5}{30}\right]_\alpha^\beta = \frac{a(\beta-\alpha)^5}{30}$$

30. (2): The three planes pass through the origin. For unique solution, the determinant of coefficient matrix is non zero. If they intersect along a line, the number of common points is infinity. The coefficient matrix is singular.

$$\begin{vmatrix} k & 4 & 1 \\ 4 & k & 2 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$k(k - 4) + 8 - 2k = 0$ or $k^2 - 6k + 8 = 0$, $k = 2, 4$.

◆

Quiz CLUB

Unlock Your Knowledge!

1. If $f(x)=x^3-\frac{1}{x^3}$, then find the value of $\left(f(x)+f\left(\frac{1}{x}\right)\right)$.
2. If $\tan A=\frac{1}{2}$ and $\tan B=\frac{1}{3}$, then find the value of $A + B$.
3. If a complex number Z lies in the interior or on the boundary of a circle of radius 3 units and centre $(-4, 0)$, then find the greatest value of $|z + 1|$.
4. If ${}^{n}P_5 = 60\ {}^{n-1}P_3$, then find the value of n.
5. If the third terms of G.P is 4, then find the product of first 5 terms.
6. Find the equation of the directrix of the parabola $y^2 + 4y + 4x + 2 = 0$.
7. If $f(x) = x \sin x$, then find $f'\left(\frac{\pi}{2}\right)$.
8. Find the median of 36, 72, 46, 42, 60, 45, 53, 46, 51, 49.
9. What is the total number of sample spaces when a dice is thrown 2 times?
10. What is the coefficient of the middle term in the expansion of $(2 + 3x)^4$?
11. If $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$, then find the value of $\cos^{-1}x + \cos^{-1}y$.
12. Write the value of k, for which $A=\begin{bmatrix} K & 8 \\ 4 & 2K \end{bmatrix}$ is a singular matrix.
13. Write the value of c in Rolle's thorem for the function, $f(x)=\sin 2x$ in $\left(0,\frac{\pi}{2}\right)$.
14. Find the area (in sq. units) bounded by the curves $y^2 = 4x$ and $y = x$.
15. If $\frac{d}{dx}f(x)=g(x)$, then what is the anti-derivative of $g(x)$?
16. Find the degree of the differential equation $\left(1+\frac{dy}{dx}\right)^3=\left(\frac{dy}{dx}\right)^2$.
17. What is the magnitude of vector $\vec{v}=\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}$?
18. What is represented by the equation $x^2 - x - 2 = 0$ in 3D space?
19. If $P(A) = 0.8$, $P(B) = 0.5$ and $P(B/A) = 0.4$, then what is the value of $P(A \cap B)$?
20. The side of an equilateral triangle is increasing at the rate of 2 cm/s. What is the rate at which area increases, when the side is 10 cm?

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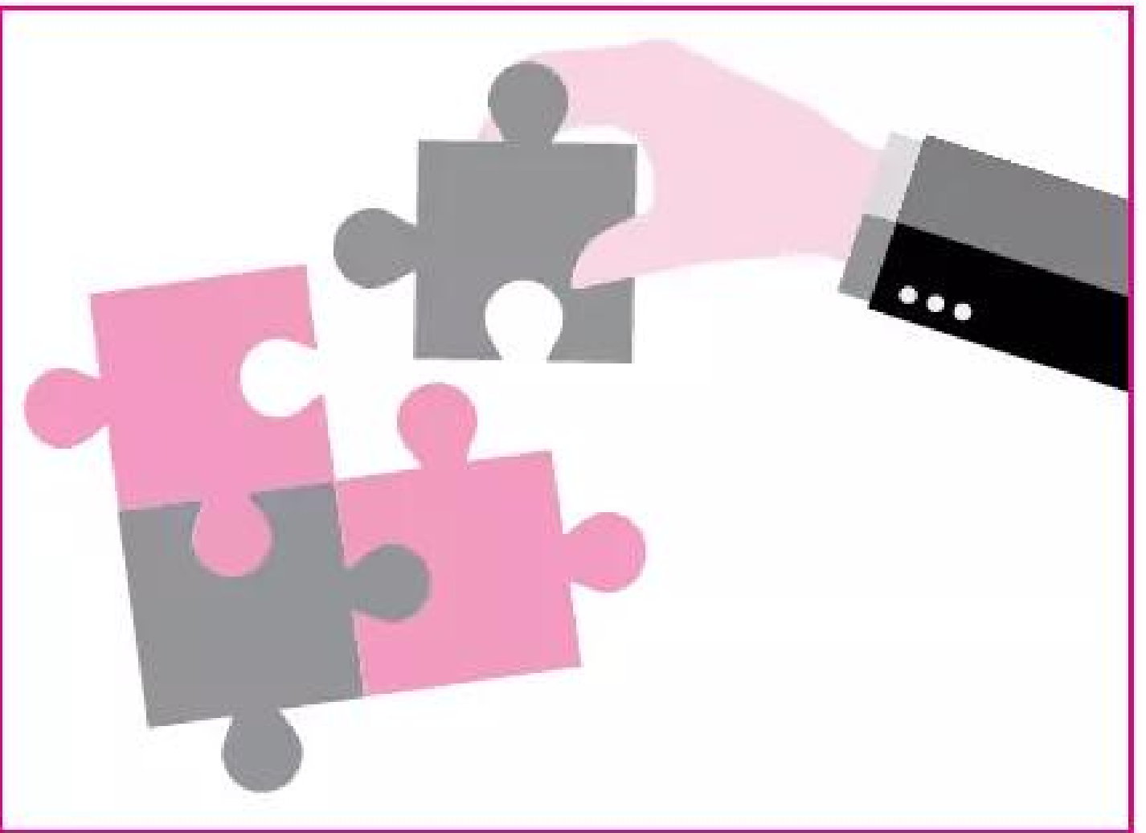
Readers can send their responses at editor@mtg.in or post us with complete address by 10th of every month. Winners' names and answers will be published in next issue.

PUZZLE CORNER

ANSWER · MARCH 2023



3– 2	5	1– 4	4– 1	2/ 3	6
6* 6	2– 4	3	5	2/ 2	1
1	2	2– 5	3	7+ 6	1– 4
7+ 4	3	3+ 2	6 6	1	5
11+ 5	6	1	2– 2	4	1– 3
3/ 3	1	15+ 6	4	5	2

Are You Ready *for* JEE?

Top Weightage Topics for JEE

In this article we will discuss about some of the most important topics of mathematics in respect to JEE Main and Advanced. It is based on the analysis of previous years' questions of JEE Main and Advanced.

SERIES 2

Statistics

ARITHMETIC MEAN

For Ungrouped Data (Raw or discrete data)

Direct Method : Let $x_1, x_2, \ldots, x_n$ be n values of the variable x, then A.M. ($\overline{x}$) is defined as

$$\overline{x} = \frac{x_1 + x_2 + \ldots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Shortcut Method : Let A be the assumed mean and $d_i = x_i - A$ be the deviation of each value of the variable from the assumed mean, then

$$\overline{x} = A + \frac{1}{n}\sum_{i=1}^{n} d_i$$

For Grouped Data

For a discrete frequency distribution

A.M. ($\overline{x}$), $\dfrac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} = \dfrac{\Sigma\, fx}{N}$, where $\Sigma f = N$ = sum of all frequencies of the variables x_i.

For a continuous frequency distribution

$$\overline{x} = A + \sum_{i=1}^{n} f_i d_i; \quad (\text{where } N = \Sigma f)$$

where, A = assumed mean, $d_i = \dfrac{x_i - A}{h}$, h is width of the class intervals.

GEOMETRIC MEAN

For Ungrouped Data

If the variable assumes the values $x_1, x_2, \ldots, x_n$ all positive, then their G.M. is defined as the n^{th} root of their products

i.e., $\text{G.M.} = \sqrt[n]{x_1 x_2 x_3 \ldots x_n} = (x_1 x_2 x_3 \ldots x_n)^{1/n}$

$$\therefore \quad \log(\text{G.M.}) = \frac{1}{n}\sum_{i=1}^{n} \log x_i$$

$$\Rightarrow \quad \text{G.M.} = \text{antilog}\left[\frac{1}{n}\sum_{i=1}^{n} \log x_i\right]$$

For Grouped Data or Continuous Series

Let $x = x_1, x_2, \ldots, x_n$ and $f = f_1, f_2, \ldots, f_n$, then

$$\text{G.M.} = \left[x_1^{f_1} \cdot x_2^{f_2}, \ldots, x_n^{f_n}\right]^{\frac{1}{N}}$$

$$= \text{antilog}\left[\frac{1}{N}\sum_{i=1}^{n} f_i \log x_i\right],$$

where $N = \Sigma f$ = total number of observations.

Range

It is the difference between two extreme observations of the given distribution.

Range = Maximum Value – Minimum Value

Empirical relation between mean, median and mode

Mode = 3 median – 2 mean

MEAN DEVIATION

For Ungrouped Data

Let $x_1, x_2, \ldots.., x_n$ be n observations, then mean deviation,

About mean, $\bar{x}$	$\frac{1}{n}\sum_{i=1}^{n}\lvert x_i-\bar{x}\rvert$
About median, M	$\frac{1}{n}\sum_{i=1}^{n}\lvert x_i-M\rvert$

For Grouped Data

Let $x_1, x_2, \ldots.., x_n$ be a set of n observations occurring with frequencies $f_1, f_2, \ldots, f_n$ respectively, then mean deviation,

About mean, $\bar{x}$	$\frac{1}{N}\sum_{i=1}^{n}f_i\lvert x_i-\bar{x}\rvert$
About median, M	$\frac{1}{N}\sum_{i=1}^{n}f_i\lvert x_i-M\rvert$

VARIANCE

For Ungrouped Data

$$\sigma^2=\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$$

For Grouped Data

$$\sigma^2=\frac{1}{N}\sum_{i=1}^{n}f_i\,(x_i-\bar{x})^2$$

STANDARD DEVIATION

For Ungrouped Data

$$\sigma=\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2}$$

For Grouped Data

$$\sigma=\sqrt{\frac{1}{N}\sum_{i=1}^{n}f_i\,(x_i-\bar{x})^2}$$

Shortcut Method for Finding Variance and Standard Deviation

- $\operatorname{var}(\sigma^2)=h^2\left[\frac{1}{N}\sum_{i=1}^{n}f_i\,u_i^2-\left\{\frac{1}{N}\sum_{i=1}^{n}f_i\,u_i\right\}^2\right]$
- $\text{S.D. }(\sigma)=\frac{h}{N}\left[\sqrt{N\sum_{i=1}^{n}f_i\,u_i^2-\left(\sum_{i=1}^{n}f_i\,u_i\right)^2}\right]$

where h = width of the class-interval

N = Sum of frequencies, $u_i=\frac{x_i-a}{h}$,

a = assumed mean

Combined Variance (σ^2) for two series

Let $\bar{x}_1$ and $\bar{x}_2$ be means of two groups with S.Ds. σ_1, σ_2 and number of elements n_1 and n_2, then combined S.D. is given by

$$\text{S.D. }(\sigma)=\sqrt{\frac{n_1\left(\sigma_1^2+d_1^2\right)+n_2\left(\sigma_2^2+d_2^2\right)}{n_1+n_2}}$$

$$\operatorname{var}(\sigma^2)=(\text{S.D.})^2=\frac{n_1\left(\sigma_1^2+d_1^2\right)+n_2\left(\sigma_2^2+d_2^2\right)}{n_1+n_2},$$

where $d_1=\bar{x}_1-\bar{x}$, $d_2=\bar{x}_2-\bar{x}$ and $\bar{x}=\frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2}$

Properties of Standard Deviation

- S.D. is independent of change of origin.
- S.D. is not independent of change of scale.

Coefficient of Variation

$$\text{C.V.} = \frac{\text{S.D.}}{\text{Mean}} \times 100 \text{ i.e., } \frac{\sigma}{\bar{x}} \times 100, \quad \bar{x} \neq 0$$

The data whose C.V. is less is said to be more consistent.

Probability

CONDITIONAL PROBABILITY

Let A and B be certain events. The probability of occurrence of A if it is known that event B has already happened, is called conditional probability of happening of A. It is denoted by $P(A|B)$ and defined as

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \; P(B) \neq 0$$

Similarly, $P(B|A) = \dfrac{P(A \cap B)}{P(A)}, \; P(A) \neq 0$

Properties of Conditional Probability

Let A, B and E be the events of sample space S of random experiment, then we have

- $P(S \mid E) = P(E \mid E) = 1$
- $P((A \cup B) \mid E) = P(A \mid E) + P(B \mid E)$ $- P((A \cap B) \mid E)$, provided $P(E) \neq 0$
- $P(A' \mid B) = 1 - P(A \mid B)$

Note : If A and B are disjoint events, then $P((A \cup B) \mid E) = P(A \mid E) + P(B \mid E)$.

ADDITION THEOREM

For two events A and B

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A \cup B) = P(A) + P(B)$ if events A and B are mutually exclusive.
- $P(A \cap B') = P(A) - P(A \cap B)$
- $P(B \cap A') = P(B) - P(A \cap B)$
- P(exactly one of them) $= P$(only one out of two) $= P(A) + P(B) - 2P(A \cap B)$

  $= P(A) - P(A \cap B) + P(B) - P(A \cap B)$

  $= P(A \cap B') + P(B \cap A')$
- $P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$

For three Events A, B and C

- P(at least one of A, B, C)

  $= \Sigma P(A) - \Sigma P(A \cap B) + P(A \cap B \cap C)$
- Probability of at least two of A, B, C *i.e.*, P(at least two of A, B, C)

  $= P(A \cap B) + P(B \cap C) + P(C \cap A) - 2P(A \cap B \cap C)$
- P(exactly two of A, B, C)

  $= P(A \cap B) + P(B \cap C) + P(C \cap A) - 3P(A \cap B \cap C)$
- P(exactly one of A, B, C)

  $= P(A) + P(B) + P(C) - 2P(A \cap B) - 2P(B \cap C) - 2P(C \cap A) + 3P(A \cap B \cap C)$

MULTIPLICATION THEOREM

For two events A and B

$$P(A \cap B) = \begin{cases} P(A)\,P(B \mid A), \text{if } P(A) \neq 0 \\ P(B)\,P(A \mid B), \text{if } P(B) \neq 0 \end{cases}$$

For n events $A_1, A_2, \ldots, A_n$

$$P(A_1 \cap A_2 \cap \ldots\ldots \cap A_n) = P(A_1)\,P(A_2 \mid A_1)\,P(A_3 \mid A_1 \cap A_2) \ldots\ldots P(A_n \mid A_1 \cap A_2 \cap \ldots \cap A_{n-1})$$

INDEPENDENT EVENTS

Events are said to be independent if the probability of occurrence or non occurrence of one is not affected by the occurrence or non-occurrence of the other.

Properties of Independent Events

If A and B are two independent events associated with a random experiment, then

- $P(A \cap B) = P(A)\, P(B)$
- $P(A|B) = P(A)$, provided $P(B) \neq 0$ and $P(B|A) = P(B)$, provided $P(A) \neq 0$

If A, B and C are said to be mutually independent events associated with a random experiment, if

- $P(A \cap B) = P(A)\, P(B)$
- $P(B \cap C) = P(B)\, P(C)$
- $P(A \cap C) = P(A)\, P(C)$
- $P(A \cap B \cap C) = P(A)\, P(B)\, P(C)$

TOTAL PROBABILITY THEOREM

Let $A_1, A_2, A_3, ..., A_n$ be the set of n mutually exclusive and exhaustive event and E be the some event which occurs with A_1 or A_2 ...or A_n then, $P(E)$

$$= P(A_1)\, P(E/A_1) + P(A_2)\, P(E/A_2) + ... + P(A_n)\, P(E/A_n)$$

$$= \sum_{i=1}^{n} P(A_i)\ P(E/A_i)$$

BAYES' THEOREM

If $A_1, A_2, ..., A_n$ are set of n M.E. and exhaustive events of the sample space S. If E be any arbitrary event which occurs with $A_1, A_2, ..., A_n$, then

$$P(A_i/E) = \frac{P(A_i)\ P(E/A_i)}{\sum_{i=1}^{n} P(A_i)\ P(E/A_i)}$$

RANDOM VARIABLES AND ITS PROBABILITY DISTRIBUTIONS

Let X is a discrete random variable which can assume values $x_1, x_2, x_3, ..., x_n$ (arranged in increasing order of magnitude) with probabilities $p_1, p_2, p_3, ..., p_n$, then the probability distribution of X is the system of numbers, shown in the table.

X	x_1	x_2	– – –	x_n
$P(X = x)$	p_1	p_2	– – –	p_n

Note:

- $P(x_k)$ *i.e.*, $P(X = x_k)$ lies between 0 and 1 for $k = 1, 2, ..., n$
- $\sum_{i=1}^{n} p_i = 1$
- $P(X \leq x_i) = p_1 + p_2 + ... + p_i$,

  $P(X \geq x_i) = p_i + p_{i+1} + ... + p_n$,

  $P(X \leq x_i) = P(X < x_i) + P(X = x_i)$ etc.

Mean

$$\text{Mean } (\mu) = \sum_{i=1}^{n} p_i x_i = p_1x_1 + p_2x_2 + ... + p_nx_n$$

Variance

$$\text{Variance } (\sigma^2) = \sum_{i=1}^{n} p_i\, (x_i - \mu)^2$$

or $$Var(X) = \sum_{i=1}^{n} x_i^2\, p(x_i) - \left[\sum_{i=1}^{n} x_i\, p(x_i)\right]^2$$

or $$Var(X) = E(X^2) - \{E(X)\}^2$$

Standard Deviation

$$\text{Standard Deviation, } (\sigma) = \sqrt{Var(X)}$$

BINOMIAL DISTRIBUTION

$P(X=r)={}^nC_r p^r q^{n-r}$, where $r = 0, 1, 2, \ldots, n$, and p, q are probabilities of success and failure respectively.

Also, $p + q = 1$

Recurrence Relation for Binomial Distribution

$$P(r+1)=\left(\frac{n-r}{r+1}\right)\left(\frac{p}{q}\right)P(r),\quad r\in\{0, 1, 2, \ldots, n-1\}$$

Mean

$\bar{X} = E(x) = np$

Variance

$\sigma^2 = npq$

Standard Variance

$\sigma = \sqrt{npq}$

BERNOULLI'S TRIALS

Trials of a random experiment are called bernoulli's trials, if

- they are finite in number.
- they are independent of each other.
- each trial has exactly two outcomes *i.e.*, success or failure.
- the probability of success or failure remains same in each trial.

Trigonometry

SOME IMPORTANT TRIGONOMETRIC FORMULAE

Sum and difference of two angles

- $\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$
- $\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$
- $\tan(\theta \pm \phi) = \dfrac{\tan\theta \pm \tan\theta}{1 \mp \tan\theta \cdot \tan\phi}$
- $\cot(\theta \pm \phi) = \dfrac{\cot\theta \cdot \cot\phi \mp 1}{\cot\phi \pm \cot\theta}$

Product to sum

- $2\sin\theta\cos\phi = \sin(\theta+\phi) + \sin(\theta-\phi)$
- $2\cos\theta\sin\phi = \sin(\theta+\phi) - \sin(\theta-\phi)$
- $2\cos\theta\cos\phi = \cos(\theta+\phi) + \cos(\theta-\phi)$
- $2\sin\theta\sin\phi = \cos(\theta-\phi) - \cos(\theta+\phi)$

Multiple angles

- $\sin 2\theta = 2\sin\theta\cos\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$
- $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$

  $= 1 - 2\sin^2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$
- $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$
- $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$; $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$
- $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

Sum to product

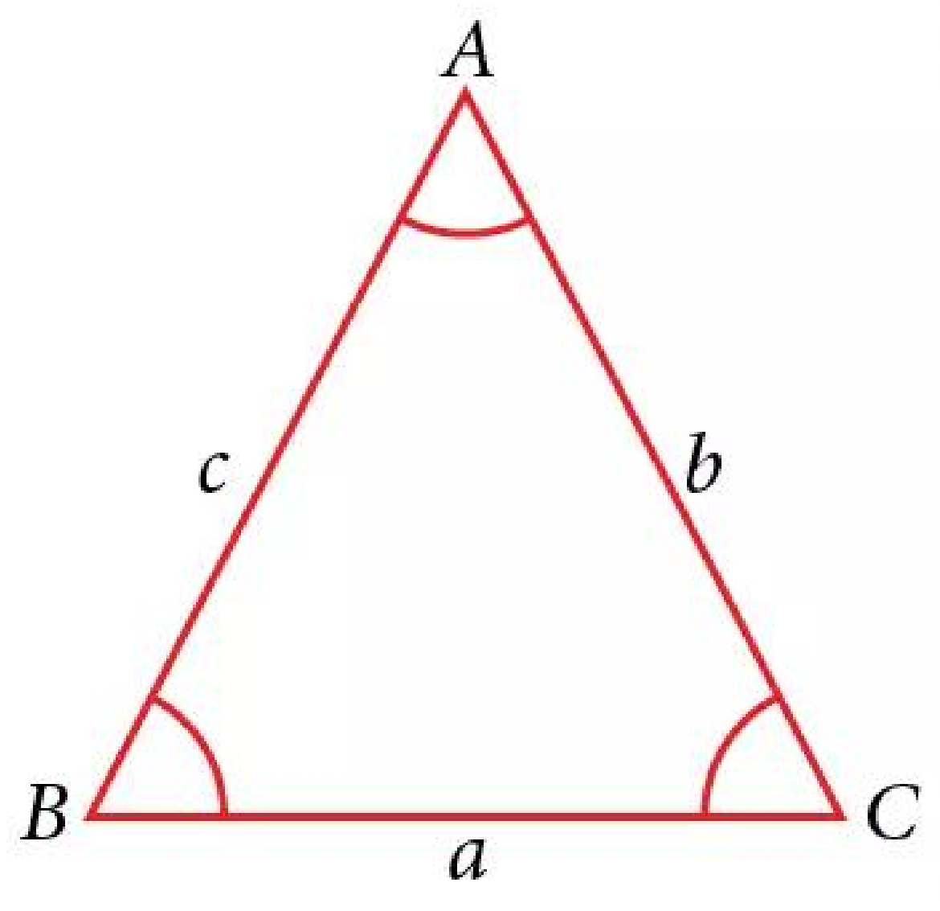
- $\sin\theta + \sin\phi = 2\sin\left(\dfrac{\theta+\phi}{2}\right)\cos\left(\dfrac{\theta-\phi}{2}\right)$
- $\sin\theta - \sin\phi = 2\cos\left(\dfrac{\theta+\phi}{2}\right)\sin\left(\dfrac{\theta-\phi}{2}\right)$
- $\cos\theta + \cos\phi = 2\cos\left(\dfrac{\theta+\phi}{2}\right)\cos\left(\dfrac{\theta-\phi}{2}\right)$
- $\cos\theta - \cos\phi = -2\sin\left(\dfrac{\theta+\phi}{2}\right)\sin\left(\dfrac{\theta-\phi}{2}\right)$

PROPERTIES OF TRIANGLES

Sine Rule

In any triangle, sides are proportional to the sines of the opposite angles, i.e., in a triangle ABC

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



Napier's Rule

- $\tan\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{b+c}\right)\cot\frac{A}{2}$
- $\tan\left(\frac{C-A}{2}\right) = \left(\frac{c-a}{c+a}\right)\cot\frac{B}{2}$
- $\tan\left(\frac{A-B}{2}\right) = \left(\frac{a-b}{a+b}\right)\cot\frac{C}{2}$

Conditional Identities

If A, B and C are angles of a triangle or $A + B + C = \pi$

- $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$
- $\cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C$
- $\sin A + \sin B + \sin C = 4 \cos\frac{A}{2} \cos\frac{B}{2} \cos\frac{C}{2}$
- $\cos A + \cos B + \cos C = 1 + 4 \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$
- $\sin B + \cos C - \sin A = 4 \cos\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$
- $\cos B + \cos C - \cos A = -1 + 4 \sin\frac{A}{2} \cos\frac{B}{2} \cos\frac{C}{2}$
- $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

Cosine Rule

Let A, B and C be angles of a triangle and a, b and c be lengths of sides opposite to angles A, B and C respectively, then

- $a^2 = b^2 + c^2 - 2bc \cos A \Rightarrow \cos A = \frac{b^2 + c^2 - a^2}{2bc}$
- $b^2 = c^2 + a^2 - 2ca \cos B \Rightarrow \cos B = \frac{c^2 + a^2 - b^2}{2ac}$
- $c^2 = a^2 + b^2 - 2ab \cos C \Rightarrow \cos C = \frac{a^2 + b^2 - c^2}{2ab}$

Area of a Triangle

$$\Delta = \frac{1}{2}bc \sin A = \frac{1}{2}ca \sin B = \frac{1}{2}ab \sin C$$

$$= \sqrt{s(s-a)(s-b)(s-c)} = \frac{abc}{4R} = rs,$$

where R and r are the radii of the circumcircle and the incircle of the ΔABC, respectively.

Escribed Circles

Let r_1, r_2, r_3 are ex-radii and a, b, c are sides of ΔABC, then

- $r_1 = \frac{\Delta}{s-a} = s \tan\frac{A}{2} = 4R \sin\frac{A}{2} \cos\frac{B}{2} \cos\frac{C}{2}$
- $r_2 = \frac{\Delta}{s-b} = s \tan\frac{B}{2} = 4R \sin\frac{B}{2} \cos\frac{C}{2} \cos\frac{A}{2}$
- $r_3 = \frac{\Delta}{s-c} = s \tan\frac{C}{2} = 4R \sin\frac{C}{2} \cos\frac{A}{2} \cos\frac{B}{2}$

Incircle

The circle touching the three sides of the triangle internally is called the inscribed or the incircle of the triangle. Its radius r is called the in-radius of the circle. In triangle ABC,

- $r = \frac{\Delta}{s} = (s-a)\tan\frac{A}{2} = (s-b)\tan\frac{B}{2}$
  $= (s-c)\tan\frac{C}{2} = 4R \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$

GENERAL SOLUTIONS OF TRIGONOMETRIC EQUATIONS

General solution of sin *x*

- $\sin x = 0,\ x = \pi k,\ k \in I$ (integer)
- $\sin x = 1,\ x = \dfrac{\pi}{2} + 2k\pi,\ k \in I$
- $\sin x = -1,\ x = -\dfrac{\pi}{2} + 2k\pi,\ k \in I$
- $\sin x = a,\ |a| > 1$ having no solution because $-1 \le \sin x \le 1$.

General solution of tan *x*

$\tan x = 0,\ x = \pi k,\ k \in I$

General solution of cos *x*

- $\cos x = 0,\ x = \dfrac{\pi}{2} + \pi k,\ k \in I$
- $\cos x = 1,\ x = 2k\pi\ \forall\ k \in I$
- $\cos x = -1,\ x = \pi + 2k\pi,\ k \in I$
- $\cos x = a,\ |a| > 1$ having no solution.

Trigonometric ratios

- $\sin x = \sin \alpha \ \Rightarrow\ x = \pi k + (-1)^k \alpha,\ k \in I$
- $\cos x = \cos \alpha \ \Rightarrow\ x = 2k\pi \pm \alpha,\ k \in I$
- $\tan x = \tan \alpha \ \Rightarrow\ x = k\pi + \alpha,\ k \in I$
- $\sin^2 x = \sin^2 \alpha,\ \cos^2 x = \cos^2 \alpha$ and $\tan^2 x = \tan^2 \alpha$ having same solution given by $x = k\pi \pm \alpha,\ k \in I$

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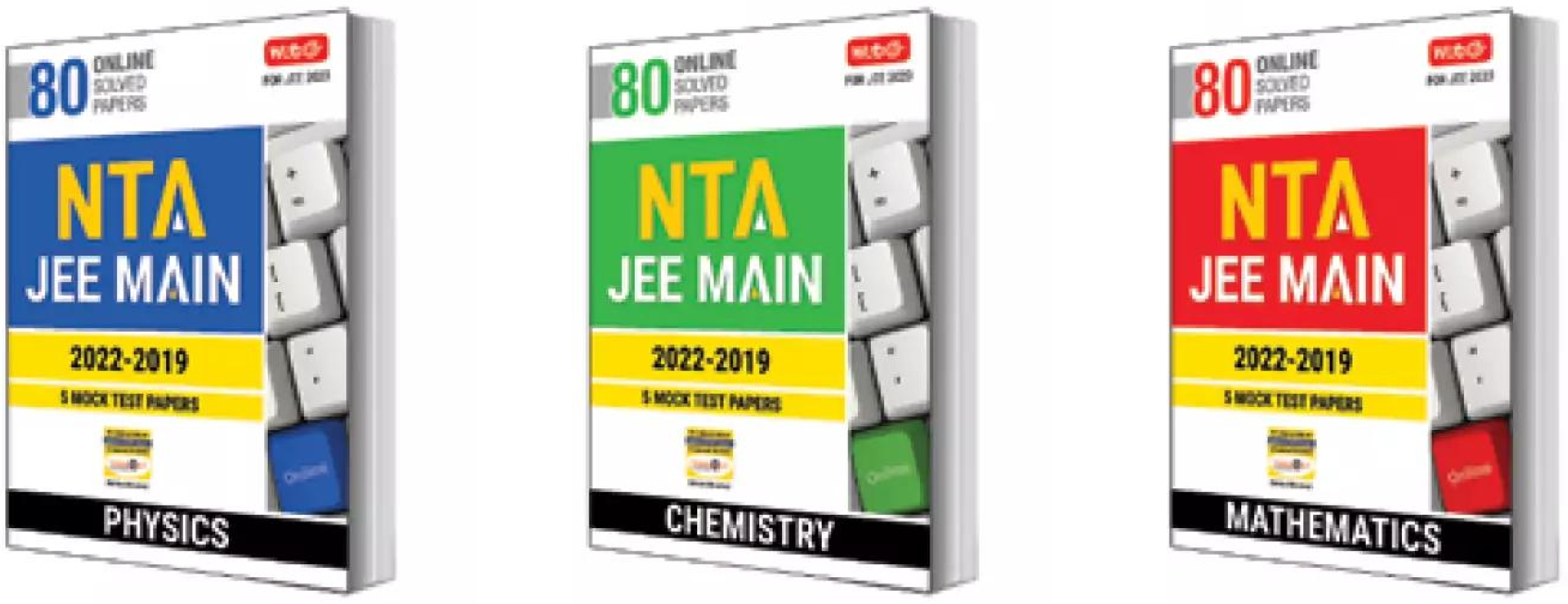

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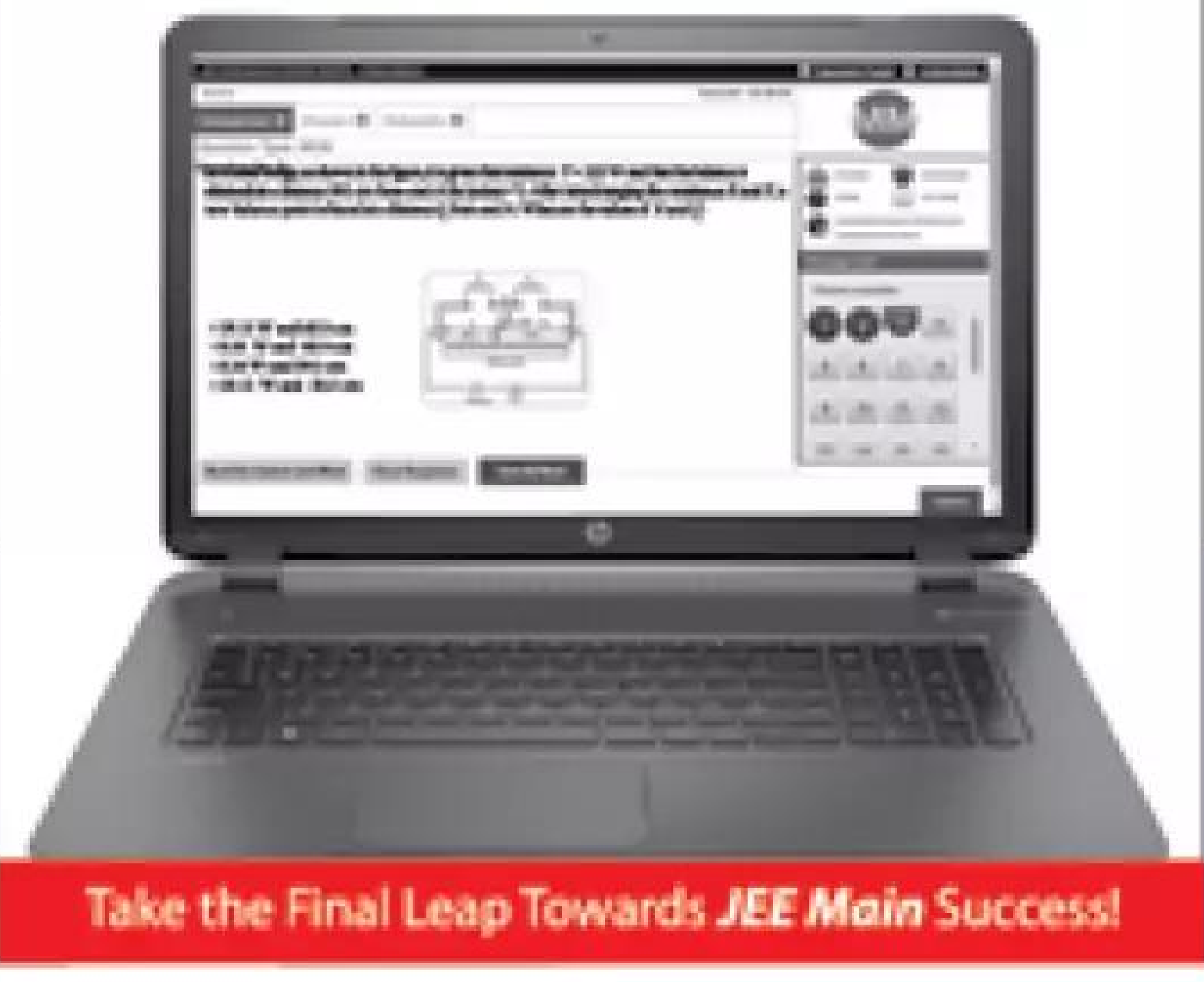

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SOME IMPORTANT FORMULAE OR SPECIAL FORMS

$\sin^{-1}(\sin x) = x, -\pi/2 \le x \le \pi/2$

$\cos^{-1}(\cos x) = x, 0 \le x \le \pi$

$\tan^{-1}(\tan x) = x, -\pi/2 < x < \pi/2$

$\sin(\sin^{-1}x) = x, -1 \le x \le 1$

$\cos(\cos^{-1}x) = x, -1 \le x \le 1$

$\tan(\tan^{-1}x) = x, x \in R$

$\sin(\sin^{-1}x) = x$ or $\sin^{-1}(\sin x) = x$

$\sin^{-1} 1/x = \mathrm{cosec}^{-1} x, x \ge 1$ or $x \le -1$

$\cos^{-1} 1/x = \sec^{-1} x, x \ge 1$ or $x \le -1$

$\tan^{-1} 1/x = \cot^{-1} x, x > 0$

$\sin^{-1}(-x) = -\sin^{-1} x, x \in [-1, 1]$

$\cos^{-1}(-x) = \pi - \cos^{-1} x, x \in [-1, 1]$

$\tan^{-1}(-x) = -\tan^{-1} x, x \in R$

$\cot^{-1}(-x) = \pi - \cot^{-1} x, x \in R$

$\mathrm{cosec}^{-1}(-x) = -\mathrm{cosec}^{-1} x, |x| \ge 1$

$\sec^{-1}(-x) = \pi - \sec^{-1} x, |x| \ge 1$

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2} \,\forall -1 \le x \le 1$$

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2} \,\forall\, x \in R$$

$$\sec^{-1} x + \mathrm{cosec}^{-1} x = \frac{\pi}{2} \,\forall\, |x| \ge 1$$

$$2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right), -1 < x < 1$$

$$= \sin^{-1}\left(\frac{2x}{1+x^2}\right), |x| \le 1 = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), x \ge 0$$

$$2\sin^{-1} x = \begin{cases} \sin^{-1}(2x\sqrt{1-x^2}), & |x| \le \dfrac{1}{\sqrt{2}}, \\ \pi - \sin^{-1}(2x\sqrt{1-x^2}), & \dfrac{1}{\sqrt{2}} < x \le 1 \end{cases}$$

$$2\cos^{-1} x = \begin{cases} \cos^{-1}(2x^2-1), & 0 \le x \le 1, \\ 2\pi - \cos^{-1}(2x^2-1), & -1 \le x \le 0 \end{cases}$$

$$3\sin^{-1} x = \sin^{-1}(3x - 4x^3), \text{ if } -\frac{1}{2} \le x \le \frac{1}{2}$$

$$= \pi - \sin^{-1}(3x - 4x^3), \text{ if } \frac{1}{2} < x \le 1$$

$$= -\pi - \sin^{-1}(3x - 4x^3), \text{ if } -1 \le x < -\frac{1}{2}$$

$$3\cos^{-1} x = \cos^{-1}(4x^3 - 3x), \text{ if } \frac{1}{2} \le x \le 1$$

$$= 2\pi - \cos^{-1}(4x^3 - 3x), \text{ if } -\frac{1}{2} \le x \le \frac{1}{2}$$

$$= 2\pi + \cos^{-1}(4x^3 - 3x), \text{ if } -1 \le x \le -\frac{1}{2}$$

$$3\tan^{-1} x = \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), \text{ if } -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$$

$$= \pi + \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), \text{ if } x > \frac{1}{\sqrt{3}}$$

$$= -\pi + \tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right), \text{ if } x < -\frac{1}{\sqrt{3}}$$

$$\sin^{-1} x + \sin^{-1} y = \begin{cases} \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\}; \text{ if } -1 \le x, y \le 1 \text{ and } x^2 + y^2 \le 1 \\ \pi - \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\}; \text{ if } 0 < x, y \le 1 \text{ and } x^2 + y^2 > 1 \end{cases}$$

$$\sin^{-1} x - \sin^{-1} y = \begin{cases} \sin^{-1}\left\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\right\}; \text{ if } -1 \le x, y \le 1 \text{ and } x^2 + y^2 \le 1 \\ \pi - \sin^{-1}\left\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\right\}; \text{ if } 0 < x \le 1, -1 \le y \le 0 \text{ and } x^2 + y^2 > 1 \end{cases}$$

$$\cos^{-1} x + \cos^{-1} y = \begin{cases} \cos^{-1}\left\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\right\}; \text{ if } -1 \le x, y \le 1 \text{ and } x + y \ge 0 \\ 2\pi - \cos^{-1}\left\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\right\}; \text{ if } -1 \le x, y \le 1 \text{ and } x + y \le 0 \end{cases}$$

$$\cos^{-1} x - \cos^{-1} y = \begin{cases} \cos^{-1}\left\{xy + \sqrt{1-x^2}\sqrt{1-y^2}\right\}; \text{ if } -1 \le x, y \le 1 \text{ and } x \le y \\ -\cos^{-1}\left\{xy + \sqrt{1-x^2}\sqrt{1-y^2}\right\}; \text{ if } -1 \le y \le 0, 0 < x \le 1 \text{ and } x \ge y \end{cases}$$

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\frac{x-y}{1+xy}\right), xy > -1; \quad \tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right), xy < 1$$

Exam on 30th April 2023

WB JEE

PRACTICE PAPER 2023

Category-I (Q.1 to 50)

(Carry 1 mark each. Only one option is correct. Negative marks : ¼)

1. Which of the following is a singleton set?

(a) $\{x : |x| = 5, x \in N\}$
(b) $\{x : |x| = 6, x \in Z\}$
(c) $\{x : x^2 + 2x + 1 = 0, x \in N\}$
(d) $\{x : x^2 = 7, x \in N\}$

2. If $A=\begin{bmatrix}1 & 3\\ 3 & 4\end{bmatrix}$ and $A^2 - kA - 5I = O$, then $k =$

(a) 5 (b) 3
(c) 7 (d) None of these

3. The function $f(x)=\sec\left[\log\left(x+\sqrt{1+x^2}\right)\right]$ is

(a) odd (b) even
(c) neither odd nor even (d) constant

4. The arg $\left(\dfrac{3+i}{2-i}+\dfrac{3-i}{2+i}\right)$ is equal to

(a) $\dfrac{\pi}{2}$ (b) 0 (c) $\dfrac{\pi}{4}$ (d) $-\dfrac{\pi}{4}$

5. The point in the interval $[0, 2\pi]$, where $f(x) = e^x \sin x$ has maximum slope, is

(a) $\pi/4$ (b) $\pi/2$ (c) π (d) $3\pi/2$

6. Find the transformed equation of the straight line $2x - 3y + 5 = 0$, when the origin is shifted to the point $(3, -1)$ after translation of axes.

(a) $2x - 3y + 14 = 0$ (b) $2x - 5y + 14 = 0$
(c) $2x - 3y + 15 = 0$ (d) $x - 4y + 14 = 0$

7. The value of $\dfrac{\log_8 17}{\log_9 23}-\dfrac{\log_{2\sqrt{2}} 17}{\log_3 23}$ is equal to

(a) 0 (b) 1 (c) $\dfrac{17}{8}$ (d) $\dfrac{23}{17}$

8. A circle passes through the points $(2, -2)$, $(3, 4)$ and has its centre on the line $2x + 2y = 7$. Find its centre and radius.

(a) $\left(1,\dfrac{5}{2}\right),\dfrac{\sqrt{37}}{2}$ (b) $(5, 1), \sqrt{37}$
(c) $(1, 5), \sqrt{37}$ (d) $\left(\dfrac{5}{2},1\right),\dfrac{\sqrt{37}}{2}$

9. The number of real roots of $\left(x+\dfrac{1}{x}\right)^3+\left(x+\dfrac{1}{x}\right)=0$ is

(a) 0 (b) 2 (c) 4 (d) 6

10. The distance between the point $(1, 1)$ and the tangent to the curve $y = e^{2x} + x^2$ drawn at the point $x = 0$ is

(a) $\dfrac{1}{\sqrt{5}}$ (b) $\dfrac{-1}{\sqrt{5}}$ (c) $\dfrac{2}{\sqrt{5}}$ (d) $\dfrac{-2}{\sqrt{5}}$

11. The domain of the function

$f(x)=\sqrt{\cos^{-1}\left(\dfrac{1-|x|}{2}\right)}$ is

(a) $(-3, 3)$ (b) $[-3, 3]$
(c) $(-\infty, -3) \cup (3, \infty)$ (d) $(-\infty, -3] \cup [3, \infty)$

12. Find perpendicular distance of the line joining the points $(\cos\theta, \sin\theta)$ and $(\cos\phi, \sin\phi)$ from the origin.

(a) $\left|\cos\left(\dfrac{\theta-\phi}{2}\right)\right|$ (b) $\left|\cos\left(\dfrac{\theta+\phi}{2}\right)\right|$
(c) $\left|\sin\left(\dfrac{\theta+\phi}{2}\right)\right|$ (d) None of these

13. If $\log_{10} 2 = 0.3010$, then the value of $\log_8 25$ is

(a) 1.5 (b) 1.2
(c) 1.34 (d) None of these

14. One of the equation of the straight lines which passes through the point $(3, 2)$ and are inclined to $x - 2y = 3$ at an angle of 45°, is

(a) $3x - y - 9 = 0$ (b) $3x - y - 7 = 0$
(c) $x + 3y - 9 = 0$ (d) Both (b) and (c)

15. The line $(x + g)\cos\theta + (y + f)\sin\theta = k$ touches the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ only if

(a) $g^2 + f^2 = c + k^2$ (b) $g^2 + f^2 = c^2 + k^2$
(c) $g^2 + f^2 = c^2 - k^2$ (d) $g^2 + f^2 = c - k^2$

16. The argument of $\dfrac{1-i\sqrt{3}}{1+i\sqrt{3}}$ is

(a) 210° (b) 180° (c) 240° (d) 60°

17. $\lim\limits_{n\to\infty} \dfrac{[x]+[2x]+\ldots+[nx]}{n^2}$, where $[\cdot]$ denotes the greatest integer function, is equal to

(a) $\dfrac{x}{2}$ (b) x
(c) $2x$ (d) None of these

18. Function $f(x)=\dfrac{\lambda\sin x+6\cos x}{2\sin x+3\cos x}$ is increasing, if

(a) $\lambda > 1$ (b) $\lambda < 1$ (c) $\lambda < 4$ (d) $\lambda > 4$

19. If the area of the circle $4x^2 + 4y^2 - 8x + 16y + k = 0$ is 9π sq. units, then $k =$

(a) 4 (b) 16 (c) ± 16 (d) – 16

20. The condition that one root of the equation $ax^2 + bx + c = 0$ may be square of the other, is

(a) $a^2c + ac^2 + b^3 - 3abc = 0$
(b) $a^2c^2 + ac^2 + b^2 + 3abc = 0$
(c) $ac^2 + ac - b^3 - 3abc = 0$
(d) $a^2c + ac^2 - b^3 - 3abc = 0$

21. $\lim\limits_{x\to\frac{\pi}{4}} (\tan x)^{\tan 2x}$ equals

(a) e^1 (b) e^2 (c) e^{-1} (d) e^{-2}

22. Point $R(h, k)$ divides a line segment between the axes in the ratio 1 : 2. Find the equation of the line.

(a) $2kx + hy = 3hk$ (b) $2kx + hy = 2hk$
(c) $2kx - hy = 3hk$ (d) None of these

23. If $\lim\limits_{x\to\infty}\left(1+\dfrac{a}{x}+\dfrac{b}{x^2}\right)^{2x} = e^2$, then

(a) $a = 2, b = 1$ (b) $a = 1, b = 2$
(c) $a = 1, b \in R$ (d) $a = b = 1$

24. $\cot^{-1}\left(\dfrac{ab+1}{a-b}\right)+\cot^{-1}\left(\dfrac{bc+1}{b-c}\right)+\cot^{-1}\left(\dfrac{ca+1}{c-a}\right)$ is equal to

(a) 0 (b) $\dfrac{\pi}{4}$ (c) 1 (d) 5

25. The equations $ax^2 + bx + a = 0$ $(a, b \in R)$ and $x^3 - 2x^2 + 2x - 1 = 0$ have 2 roots common. Then $a + b$ must be equal to

(a) 1 (b) –1
(c) 0 (d) None of these

26. The sum of the minimum distance and the maximum distance from the point (4, – 3) to the circle $x^2 + y^2 + 4x - 10y - 7 = 0$ is

(a) 20 (b) 12 (c) 10 (d) 16

27. The equation of the chord joining two points (x_1, y_1) and (x_2, y_2) on the hyperbola $xy = c^2$ is

(a) $\dfrac{x}{x_1-x_2}+\dfrac{y}{y_1-y_2}=1$ (b) $\dfrac{x}{y_1-y_2}+\dfrac{y}{x_1-x_2}=1$
(c) $\dfrac{x}{x_1+x_2}+\dfrac{y}{y_1+y_2}=1$ (d) $\dfrac{x}{y_1+y_2}+\dfrac{y}{x_1+x_2}=1$

28. The function $y = \dfrac{x}{1+|x|}$, $x \in R, y \in R$ is

(a) One-one onto function (b) Onto but not one-one
(c) One-one but not onto (d) None of these

29. $\lim\limits_{m\to\infty}\left(\cos\dfrac{x}{m}\right)^m$ is equal to

(a) 0 (b) e (c) $\dfrac{1}{e}$ (d) 1

30. The set of values of 'h' for which the number of distinct common normals to $(x - 2)^2 = 4(y - 3)$ and $x^2 + y^2 - 2x - hy - c = 0$ $(c > 0)$ is 3, is

(a) $(2, \infty)$ (b) $(4, \infty)$
(c) $(2, 4)$ (d) None of these

31. The eccentricity of the conic $3x^2 + 4y^2 - 6x - 8y + 4 = 0$ is

(a) $\dfrac{1}{2}$ (b) $\dfrac{1}{\sqrt{2}}$
(c) $\sqrt{2}$ (d) None of these

32. Let $f(x) = \dfrac{\alpha x}{x+1}$, $x \neq -1$. Then, for what value of α is $f(f(x)) = x$?

(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) 1 (d) –1

33. The probability for a randomly chosen month to have its 10th day as Sunday is

(a) $\dfrac{1}{84}$ (b) $\dfrac{10}{12}$ (c) $\dfrac{10}{84}$ (d) $\dfrac{1}{7}$

34. If $P=\begin{bmatrix} i & 0 & -i \\ 0 & -i & i \\ -i & i & 0 \end{bmatrix}$ and $Q=\begin{bmatrix} -i & i \\ 0 & 0 \\ i & -i \end{bmatrix}$, then PQ is equal to

(a) $\begin{bmatrix} -2 & 2 \\ 1 & -1 \\ 1 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -2 \\ -1 & 1 \\ -1 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & -2 \\ -1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

35. If the system of equations $2x + 3y + 5 = 0$, $x + ky + 5 = 0$, $kx - 12y - 14 = 0$ has non-trivial solution, then the value of k is

(a) $-2, \frac{12}{5}$ (b) $-1, \frac{1}{5}$ (c) $-6, \frac{17}{5}$ (d) $6, -\frac{12}{5}$

36. Given $n(U) = 20$, $n(A) = 12$, $n(B) = 9$, $n(A \cap B)=4$, where U is the universal set, A and B are subsets of U, then $n[(A \cup B)^c]$ equals to

(a) 17 (b) 9 (c) 11 (d) 3

37. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors, then $|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2$ does not exceed

(a) 4 (b) 9 (c) 8 (d) 6

38. In the interval $\left(0, \frac{\pi}{2}\right)$ area (in sq. units) lying between the curves $y = \tan x$, $y = \cot x$ and x-axis is

(a) 3 log 2 (b) 2 log 2

(c) log 2 (d) $\frac{1}{2}\log 2$

39. If $|\vec{a}-\vec{b}| = |\vec{a}| = |\vec{b}| = 1$, then the angle between $\vec{a}$ and $\vec{b}$ is

(a) $\frac{\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{2}$ (d) 0

40. A triangle has its three sides equal to p, q and r. If the coordinates of its vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, then $\begin{vmatrix} x_1 & y_1 & 2 \\ x_2 & y_2 & 2 \\ x_3 & y_3 & 2 \end{vmatrix}^2$ is equal to

(a) $(p + q + r)$

(b) $(p + q - r)^2$

(c) $(p + q + r)(q + r - p)(r + p - q)(p + q - r)$

(d) $(p - q + r)(q + r - p)(p + q - r)$

41. $\lim\limits_{x \to 0} \frac{8}{\sin^8 x}\left(1 - \cos\frac{x^2}{2} - \cos\frac{x^2}{4} + \cos\frac{x^2}{2}\cos\frac{x^2}{4}\right) =$

(a) $\frac{1}{16}$ (b) $\frac{1}{32}$ (c) $\frac{1}{64}$ (d) $\frac{1}{8}$

42. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is

(a) $\frac{5}{2}$ (b) $\frac{11}{2}$ (c) 6 (d) $\frac{13}{2}$

43. If $\int\left[\log(\log x) + \frac{1}{(\log x)^2}\right]dx = x\left[f(x) - g(x)\right] + C$, then

(a) $f(x) = \log(\log x), g(x) = \frac{1}{\log x}$

(b) $f(x) = \log x,\ g(x) = \frac{1}{\log x}$

(c) $f(x) = \frac{1}{\log x}, g(x) = \frac{1}{\log x}$

(d) $f(x) = \frac{1}{x\log x}, g(x) = \frac{1}{\log x}$

44. The number of ways in which four letters of the word MATHEMATICS can be arranged, is given by

(a) $\frac{8!}{4!4!}$ (b) 2454 (c) 2464 (d) 2474

45. If the system of equations $2x + 3y = 8$, $7x - 5y + 3 = 0$, $4x - 6y + \lambda = 0$ has no solution, then λ

(a) $= 4$ (b) $= 8$ (c) $\neq 8$ (d) $\neq 4$

46. Four candidates A, B, C and D have applied for the assignment to coach a school cricket team. If A is twice as likely to be selected as B and B and C are given about the same chance of being selected, while C is twice as likely to be selected as D, what is the probability that A will not be selected?

(a) $\frac{4}{9}$ (b) $\frac{5}{9}$ (c) $\frac{9}{4}$ (d) $\frac{9}{5}$

47. If $5\cos 2\theta + 2\cos^2\frac{\theta}{2} + 1 = 0$, $-\pi < \theta < \pi$, then $\theta =$

(a) $\pi/3$ (b) $\pi/3$, $\cos^{-1}(3/5)$

(c) $\cos^{-1}(3/5)$ (d) $\pi/3$, $\pi - \cos^{-1}(3/5)$

48. If $x^2 + y^2 = 4$, then $y\frac{dy}{dx} + x =$

(a) 4 (b) 0 (c) 1 (d) -1

49. Area of the region enclosed by $y = x^2$ and $y = \sqrt{x}$ is

(a) 1 sq. unit (b) $\frac{1}{2}$ sq. unit

(c) $\frac{1}{3}$ sq. unit (d) cannot be determined

50. If $y = (1 + x)(1 + x^2)(1 + x^4) \ldots (1 + x^{2n})$, then the value of $\left(\frac{dy}{dx}\right)$ at $x = 0$ is

(a) 0 (b) -1 (c) 1 (d) 2

Category-II (Q.51 to 65)

(Carry 2 marks each. Only one option is correct. Negative marks : ½)

51. A and B are two students. Their chances of solving a problem correctly are $\frac{1}{3}$ and $\frac{1}{4}$, respectively. If the probability of their making a common error is $\frac{1}{20}$ and they obtain the same answer, then the probability of their answer to be correct is

(a) $\frac{1}{12}$ (b) $\frac{1}{40}$ (c) $\frac{13}{120}$ (d) $\frac{10}{13}$

52. If $5x + 9 = 0$ is the directrix of the hyperbola $16x^2 - 9y^2 = 144$, then its corresponding focus is :

(a) $\left(-\frac{5}{3}, 0\right)$ (b) $(-5, 0)$

(c) $(5, 0)$ (d) $\left(\frac{5}{3}, 0\right)$

53. If in an A.P., $S_n = qn^2$ and $S_m = qm^2$, where S_r denotes the sum of r terms of the A.P., then S_q equals

(a) $\frac{q^3}{2}$ (b) mnq (c) q^3 (d) $(m + n)q^2$

54. The plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1}(\alpha)$ with x-axis. The value of α is equal to

(a) $\frac{\sqrt{3}}{2}$ (b) $\frac{\sqrt{2}}{3}$ (c) $\frac{2}{7}$ (d) $\frac{3}{7}$

55. Evaluate : $\int_0^1 \frac{x \tan^{-1} x}{(1+x^2)^{3/2}} \, dx$

(a) $\frac{4-\pi}{2\sqrt{2}}$ (b) $\frac{4+\pi}{2\sqrt{2}}$

(c) $\frac{4-\pi}{4\sqrt{2}}$ (d) None of these

56. Let $n(A) = m$, and $n(B) = n$. Then the total number of non-empty relations that can be defined from A to B is

(a) m^n (b) $n^m - 1$

(c) $mn - 1$ (d) $2^{mn} - 1$

57. The sum of the series

$1 + \frac{1}{2}\,{}^nC_1 + \frac{1}{3}\,{}^nC_2 + \ldots\ldots.. + \frac{1}{n+1}\,{}^nC_n$ is equal to

(a) $\frac{2^{n+1} - 1}{n+1}$ (b) $\frac{3(2^n - 1)}{2n}$

(c) $\frac{2^n + 1}{n+1}$ (d) $\frac{2^n + 1}{2n}$

58. If $f(x) = \begin{cases} \dfrac{(5^x - 1)^3}{\sin\left(\dfrac{x}{a}\right)\log\left(1 + \dfrac{x^2}{3}\right)}, & x \neq 0 \\ 9(\log_e 5)^3, & x = 0 \end{cases}$

is continuous at $x = 0$, then value of 'a' equals

(a) 3 (b) 2

(c) 1 (d) None of these

59. If A is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is equal to

(a) A (b) $I - A$ (c) $I + A$ (d) $3A$

60. $\theta = \tan^{-1}(2 \tan^2 \theta) - \tan^{-1}((1/3) \tan \theta)$, if $\tan \theta$ is equal to

(a) -2 (b) -1 (c) $2/3$ (d) 2

61. If two parallel chords of a circle, having diameter 4 units, lie on the opposite sides of the centre and subtend angles $\cos^{-1}\left(\frac{1}{7}\right)$ and $\sec^{-1}(7)$ at the centre respectively, then the distance between these chords, is

(a) $\frac{16}{7}$ (b) $\frac{8}{\sqrt{7}}$ (c) $\frac{8}{7}$ (d) $\frac{4}{\sqrt{7}}$

62. The number of vectors of unit length perpendicular to the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ is

(a) one (b) two (c) three (d) infinite

63. The following information relates to a sample of size 60; $\Sigma x^2 = 18000$, $\Sigma x = 960$. The variance of the data is

(a) 6.63 (b) 16 (c) 22 (d) 44

64. If the distance between the foci of an ellipse is half the length of its latus rectum, then the eccentricity of the ellipse is

(a) $\frac{1}{2}$ (b) $\frac{2\sqrt{2} - 1}{2}$ (c) $\sqrt{2} - 1$ (d) $\frac{\sqrt{2} - 1}{2}$

65. If $y = mx + c$ is the normal at a point on the parabola $y^2 = 8x$ whose focal distance is 8 units, then $|c|$ is equal to

(a) $8\sqrt{3}$ (b) $2\sqrt{3}$ (c) $16\sqrt{3}$ (d) $10\sqrt{3}$

CATEGORY - III (Q.66 TO 75)

(Carry 2 marks each. One or more options are correct. No negative marks)

66. For $0 < \theta < \frac{\pi}{2}$, if $x = \sum_{n=0}^{\infty} \cos^{2n}\theta$, $y = \sum_{n=0}^{\infty} \sin^{2n}\theta$,

$z = \sum_{n=0}^{\infty} \cos^{2n}\theta \sin^{2n}\theta$, then

(a) $xyz = zx + y$ (b) $xyz = xy + z$

(c) $xyz = x + y + z$ (d) $xyz = yz + x$

67. The solution of differential equation $\frac{dy}{dx} = \frac{x - y}{x + y}$ is

(a) $x^2 - y^2 + 2xy + c = 0$

(b) $x^2 - y^2 - xy + c = 0$

(c) $x^2 - y^2 + xy + c = 0$

(d) $x^2 - y^2 - 2xy + c = 0$

68. The equation $\frac{x^2}{14-a}+\frac{y^2}{9-a}=1$ represent

(a) an ellipse if $a < 9$
(b) a hyperbola if $9 < a < 14$
(c) a hyperbola if $a > 14$
(d) an ellipse $a > 9$

69. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 3, 5, 7, 9\}$. Which of the following is/are relation from A to B?

(a) $R_1 = \{(a, b) \mid b = 2 + a, a \in A, b \in B\}$
(b) $R_2 = \{(1, 1), (2, 1), (3, 3), (4, 3), (5, 5)\}$
(c) $R_3 = \{(1, 1), (1, 3), (3, 5), (3, 7), (5, 7)\}$
(d) $R_4 = \{(1, 3), (2, 5), (2, 4), (7, 9)\}$

70. Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the vertex O of the parabola. If P lies in the first quadrant and the area of the triangle OPQ is $3\sqrt{2}$, then which of the following is (are) the coordinates of P?

(a) $(4, 2\sqrt{2})$ (b) $(9, 3\sqrt{2})$
(c) $\left(\frac{1}{4}, \frac{1}{\sqrt{2}}\right)$ (d) $(1, \sqrt{2})$

71. The equations of common tangents to the hyperbolas $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$ are

(a) $x + y = \sqrt{a^2 - b^2}$ (b) $x - y = \sqrt{a^2 - b^2}$
(c) $x + y = -\sqrt{a^2 - b^2}$ (d) $x - y = -\sqrt{a^2 - b^2}$

72. If α, β, γ are the cube roots of p, $p < 0$, then for any x, y and z the value of $\frac{x\alpha+y\beta+z\gamma}{x\beta+y\gamma+z\alpha}$ is

(a) ω (b) $-\omega$ (c) ω^2 (d) $-\omega^2$

73. The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar, if k is

(a) 0 (b) 1 (c) –2 (d) –3

74. If $AB = A$ and $BA = B$, then

(a) $A^2 = A$ (b) $B^2 = B$
(c) $A^2 = -A$ (d) $B^2 = -B$

75. Solution of differential equation

$$\frac{dt}{dx}=\frac{t\left(\frac{d\,g(x)}{dx}\right)-t^2}{g(x)}$$ is

(a) $t=\frac{g(x)+c}{x}$ (b) $t=\frac{g(x)}{x}+c$
(c) $t=\frac{g(x)}{x+c}$ (d) $t = g(x) + x + c$

SOLUTIONS

1. (a) : (a) $|x| = 5 \Rightarrow x = 5$ $(\because x \in N)$

$\therefore$ Given set is singleton.

(b) $|x| = 6 \Rightarrow x = -6, 6$ $(\because x \in Z)$

$\therefore$ Given set is not singleton.

(c) $x^2 + 2x + 1 = 0 \Rightarrow (x + 1)^2 = 0 \Rightarrow x = -1, -1.$

Since $-1 \notin N$, $\therefore$ Given set $= \phi$

(d) $x^2 = 7 \Rightarrow x=\pm\sqrt{7}$

Since $\sqrt{7} \notin N$, $\therefore$ Given set $= \phi$

2. (a)

3. (b) : $\because$ $f(x)=\sec[\log(x+\sqrt{1+x^2})]$

$\therefore$ $f(-x)=\sec[\log\{-x+\sqrt{1+(-x)^2}\}]$

$$=\sec\left[\log\frac{(\sqrt{1+x^2}-x)(\sqrt{1+x^2}+x)}{(\sqrt{1+x^2}+x)}\right]$$

$$=\sec\left[\log\left(\frac{1+x^2-x^2}{x+\sqrt{1+x^2}}\right)\right]=\sec\log(x+\sqrt{1+x^2})^{-1}$$

$$=\sec[-\log(x+\sqrt{1+x^2})]=\sec[\log(x+\sqrt{1+x^2})]=f(x)$$

$\therefore$ $f(x)$ is an even function.

4. (b) : $\arg\left(\frac{3+i}{2-i}+\frac{3-i}{2+i}\right)=\arg\left(\frac{6+5i+i^2+6-5i+i^2}{5}\right)$

$$\Rightarrow \arg\left(\frac{10}{5}\right)=\arg(2)=0$$

5. (b)

6. (a) : Let the coordinates (x, y) changes to (x', y') in new coordinates system, where origin has the coordinates $h = 3$, $k = -1$. Then, we can write the transformation relation as $x = x' + 3$ and $y = y' - 1$. Substituting, these values in the given equation of the straight line, we get

$2(x' + 3) - 3(y' - 1) + 5 = 0 \Rightarrow 2x' - 3y' + 14 = 0$

Therefore, the equation of the straight line in new system is $2x - 3y + 14 = 0$.

7. (a) : Consider, $\frac{\log_{2^3} 17}{\log_{3^2} 23}-\frac{\log_{2^{3/2}} 17}{\log_3 23}$

$$=\frac{\frac{1}{3}\log_2 17}{\frac{1}{2}\log_3 23}-\frac{1/(3/2)\log_2 17}{\log_3 23}=0$$

8. (d) : Let the equation of the circle be

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(i)

As (2, –2), (3, 4) lie on the circle, we have

$4 + 4 + 4g - 4f + c = 0 \Rightarrow 4g - 4f + c + 8 = 0$ ...(ii)

and $9 + 16 + 6g + 8f + c = 0 \Rightarrow 6g + 8f + c + 25 = 0$ ...(iii)

Since the centre $(-g, -f)$ lies on $2x + 2y - 7 = 0$, therefore we get

$-2g - 2f - 7 = 0 \Rightarrow 2g + 2f + 7 = 0$ ...(iv)

Subtracting (ii) from (iii), we get

$2g + 12f + 17 = 0$ ...(v)

Subtracting (iv) from (v), we get $10f + 10 = 0 \Rightarrow f = -1$

Putting value of f in (iv), we get $2g + 5 = 0 \Rightarrow g = -\frac{5}{2}$, and then (ii) gives $c = -2$.

Also, we note that $g^2 + f^2 - c = \frac{25}{4} + 1 + 2 = \frac{37}{4}$, which is positive.

Its centre is $\left(\frac{5}{2}, 1\right)$ and radius $= \sqrt{\frac{25}{4} + 1 + 2} = \frac{\sqrt{37}}{2}$.

9. (a) : We have, $\left(x + \frac{1}{x}\right)^3 + \left(x + \frac{1}{x}\right) = 0$

$$\Rightarrow \left(x + \frac{1}{x}\right)\left[\left(x + \frac{1}{x}\right)^2 + 1\right] = 0$$

$$\Rightarrow \text{Either}, x + \frac{1}{x} = 0 \Rightarrow x^2 = -1 \Rightarrow x = \pm i$$

$$\text{or} \quad \left(x + \frac{1}{x}\right)^2 + 1 = 0 \Rightarrow x^2 + \frac{1}{x^2} + 3 = 0$$

$\Rightarrow \quad x^4 + 3x^2 + 1 = 0$

$$\Rightarrow \quad x^2 = \frac{-3 \pm \sqrt{9 - 4}}{2} = \frac{-3 \pm \sqrt{5}}{2} < 0$$

$\therefore$ There is no real root.

10. (c) : Putting $x = 0$ in $y = e^{2x} + x^2$, we get $y = 1$

$\therefore$ The given point is $P(0, 1)$

$y = e^{2x} + x^2$ ...(i)

$$\Rightarrow \frac{dy}{dx} = 2e^{2x} + 2x \Rightarrow \left[\frac{dy}{dx}\right]_P = 2$$

$\therefore$ Equation of tangent at P to (i) is

$y - 1 = 2(x - 0) \Rightarrow 2x - y + 1 = 0$ ...(ii)

$\therefore$ Required distance = Length of ⊥ from (1, 1) to (ii)

$$= \frac{2 - 1 + 1}{\sqrt{4 + 1}} = \frac{2}{\sqrt{5}}.$$

11. (b) : For $f(x)$ to be defined,

$$-1 \le \frac{1 - |x|}{2} \le 1 \ \& \ \cos^{-1}\left(\frac{1 - |x|}{2}\right) \ge 0$$

$[\because 0 \le \cos^{-1} y \le \pi \text{ always}]$

$\Rightarrow \ -2 \le 1 - |x| \le 2 \ \Rightarrow \ -3 \le -|x| \le 1$

$\Rightarrow \ -1 \le |x| \le 3 \ \Rightarrow \ |x| \le 3 \qquad [\because |x| \ge 0]$

$\therefore \ x \in [-3, 3]$.

12. (a)

13. (a)

14. (d) : Equation of straight line passing through (3, 2) and having slope m is $y - 2 = m(x - 3)$ ...(i)

and slope of line $x - 2y = 3$ is $\frac{1}{2}$

$$\therefore \ \tan 45° = \left|\frac{m - \frac{1}{2}}{1 + \frac{m}{2}}\right| \Rightarrow \pm 1 = \frac{m - \frac{1}{2}}{1 + \frac{m}{2}} \Rightarrow m = 3, -\frac{1}{3}$$

Now, from (i), the equation of straight lines are

$y - 2 = 3(x - 3) \ \Rightarrow \ 3x - y - 7 = 0$

and $y - 2 = -\frac{1}{3}(x - 3) \ \Rightarrow \ x + 3y - 9 = 0$

15. (a) : Given, equation of line is $x \cos \theta + y \sin \theta + g \cos \theta + f \sin \theta - k = 0$. This line will touch the circle if distance from $(-g, -f)$ to the line will be equal to the radius of the circle

$$i.e, \ \sqrt{g^2 + f^2 - c} = \frac{|-g \cos\theta - f \sin\theta + g \cos\theta + f \sin\theta - k|}{\sqrt{\sin^2\theta + \cos^2\theta}}$$

$\Rightarrow \ g^2 + f^2 - c = k^2 \ \Rightarrow \ g^2 + f^2 = k^2 + c$

16. (c) : Let $z = \frac{1 - i\sqrt{3}}{1 + i\sqrt{3}} = \frac{1 - i\sqrt{3}}{1 + i\sqrt{3}} \times \frac{1 - i\sqrt{3}}{1 - i\sqrt{3}}$

$$= \frac{1 - 3 - 2\sqrt{3}\,i}{1 + 3} = \frac{-2 - 2\sqrt{3}\,i}{4} = \frac{-1}{2} - \frac{\sqrt{3}}{2}i$$

$\because \ x < 0, y < 0 \qquad \therefore \ z$ lies in 3rd Quadrant

$$\therefore \ \tan\theta = \frac{-\sqrt{3}/2}{-1/2} = \sqrt{3} = \tan 60°$$

$= \ \tan(180° + 60°) = \tan 240° \ \Rightarrow \ \theta = 240°$

17. (a)

18. (d) : Given, $f(x) = \frac{\lambda \sin x + 6 \cos x}{2 \sin x + 3 \cos x}$

Differentiating w.r.t. x, we get

$$f'(x) = \frac{\begin{bmatrix}(2 \sin x + 3 \cos x)(\lambda \cos x - 6 \sin x) \\ -(\lambda \sin x + 6 \cos x)(2 \cos x - 3 \sin x)\end{bmatrix}}{(2 \sin x + 3 \cos x)^2}$$

The function is increasing, if $f'(x) > 0$

$\Rightarrow \ 3\lambda(\sin^2 x + \cos^2 x) - 12(\sin^2 x + \cos^2 x) > 0$

$\Rightarrow \ 3\lambda - 12 > 0 \ \Rightarrow \ \lambda > 4$

19. (d)

20. (a) : Let one root be α, then other root will be α^2.
Given equation is $ax^2 + bx + c = 0$

$\because$ α is root of this equation

So, $a\alpha^2 + b\alpha + c = 0$

Now, sum of roots $= \alpha + \alpha^2 = \frac{-b}{a}$ ...(i)

Product of roots $= \alpha \cdot \alpha^2 = \frac{c}{a}$ ...(ii)

Taking cube of equation (i), we get

$$(\alpha+\alpha^2)^3 = \frac{-b^3}{a^3} \Rightarrow \alpha^3 + (\alpha^2)^3 + 3\alpha\alpha^2(\alpha+\alpha^2) = \frac{-b^3}{a^3}$$

$$\Rightarrow \frac{c}{a} + \left(\frac{c}{a}\right)^2 + 3\frac{c}{a}\left(\frac{-b}{a}\right) = \frac{-b^3}{a^3} \quad \text{(Using (i) and (ii))}$$

$$\Rightarrow \frac{c}{a} + \frac{c^2}{a^2} - \frac{3bc}{a^2} = \frac{-b^3}{a^3} \Rightarrow ca^2 + c^2a - 3abc = -b^3$$

$$\Rightarrow a^2c + ac^2 + b^3 - 3abc = 0$$

21. (c) : $\lim\limits_{x \to \frac{\pi}{4}} (\tan x)^{\tan 2x}$ (1^∞ form)

$$= e^{\lim\limits_{x \to \pi/4} (\tan x - 1)\tan 2x}$$

$$= e^{\lim\limits_{x \to \pi/4} (1-\tan x)\frac{-2\tan x}{(1+\tan x)(1-\tan x)}} = e^{\lim\limits_{x \to \pi/4} \frac{-2\tan x}{1+\tan x}} = e^{-1}$$

22. (a)

23. (c) : Given, $\lim\limits_{x \to \infty}\left(1 + \frac{a}{x} + \frac{b}{x^2}\right)^{2x} = e^2$

$$\Rightarrow e^{\lim\limits_{x \to \infty}\left(\frac{a}{x}+\frac{b}{x^2}\right)2x} = e^2 \Rightarrow e^{\lim\limits_{x \to \infty}\left(a+\frac{b}{x}\right)\cdot 2} = e^2$$

$\therefore$ $2a = 2 \Rightarrow a = 1$

$\therefore$ $a = 1$ and b can take any real value.

24. (a)

25. (c) : We have, $x^3 - 2x^2 + 2x - 1 = 0$ ...(i)

$\Rightarrow$ $(x - 1)(x^2 - x + 1) = 0$

$\Rightarrow$ $x = 1, \frac{1 \pm \sqrt{3}i}{2}$

Since $ax^2 + bx + a = 0$ and eq. (i) have 2 common roots, therefore one of the roots of $ax^2 + bx + a = 0$ must be complex.

But complex roots occur in conjugate pair.

$\therefore$ Both roots of $ax^2 + bx + a = 0$ will be complex.

Thus, both roots of $ax^2 + bx + a = 0$ are $\frac{1 \pm \sqrt{3}i}{2}$.

$\therefore$ $ax^2 + bx + a = x^2 - x + 1$

On comparing the like powers of x, we get $a = 1$ and $b = -1$.

$\Rightarrow$ $a + b = 0$

26. (a)

27. (c)

28. (c) : Let $y = \frac{x}{1+|x|} = f(x)\ \forall\ x \in R$, y has some value means for each value of x there is a unique value of y.

$\therefore$ $y = \frac{x}{1+|x|}$ is one-one function.

Hence range of $f(x)$ is $R - \{-1, 1\}$.

But y can not have the values $-1, 1$ for some values of x.

$\therefore$ y is not an onto function.

29. (d) : $\lim\limits_{m\to\infty}\left(\cos\frac{x}{m}\right)^m = e^{\lim\limits_{m\to\infty}\left(\cos\left(\frac{x}{m}\right)-1\right)m}$ (1^∞ form)

$$= e^{\lim\limits_{m\to\infty} m\left[-2\sin^2\frac{x}{2m}\right]} = e^{\lim\limits_{m\to\infty} -2\left(\frac{\sin\frac{x}{2m}}{x/2m}\right)^2\left(\frac{x^2}{4m^2}\right)m} = e^0 = 1$$

30. (b) : Normal of parabola passes through $\left(\frac{h}{2}, 0\right)$.

Maximum number of normals can be drawn to parabola are 3 which can be possible if $\frac{h}{2} > 2 \Rightarrow h > 4$.

So maximum common normals is possible if $h \in (4, \infty)$.

31. (a)

32. (d) : We have, $f(x) = \frac{\alpha x}{x+1}, x \neq -1$

$$f(f(x)) = f\left(\frac{\alpha x}{x+1}\right) = \frac{\alpha \cdot \frac{\alpha x}{x+1}}{\frac{\alpha x}{x+1}+1} = \frac{\frac{\alpha^2 x}{x+1}}{\frac{\alpha x + x + 1}{x+1}}$$

$$\Rightarrow f(f(x)) = \frac{\alpha^2 x}{\alpha x + x + 1} = x \quad \text{(Given)}$$

This is possible if $\alpha^2 = 1$ and $x(\alpha + 1) + 1 = 1$

$\Rightarrow$ $\alpha = \pm 1$ and $x(\alpha+1) = 0$ $\Rightarrow$ $\alpha = -1$

33. (d) : $P(10^{\text{th}}$ day as Sunday$) = 1/7$.

34. (b)

35. (c) : The homogeneous linear system of equations has non-trivial solution.

$$\therefore \begin{vmatrix} 2 & 3 & 5 \\ 1 & k & 5 \\ k & -12 & -14 \end{vmatrix} = 0$$

$\Rightarrow\ 2(-14k+60)-3(-14-5k)+5(-12-k^2)=0$

$\Rightarrow\ 5k^2+13k-102=0 \Rightarrow (5k-17)(k+6)=0$

$\Rightarrow\ k=-6, \dfrac{17}{5}$

36. (d) : $n[(A\cup B)^c]=n(U)-n(A\cup B)$

$=\ 20-(n(A)+n(B)-n(A\cap B))$

$=\ 20-(12+9-4)=20-17=3$

37. (b)

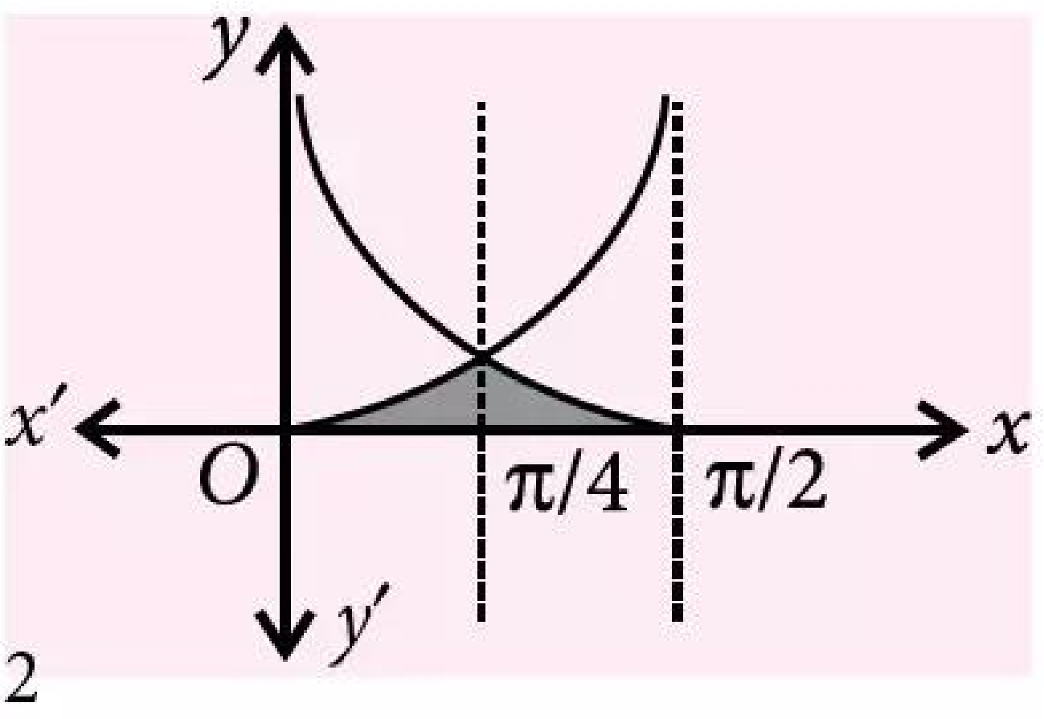
38. (c) : We have, $y=\tan x$ ...(i)

and $y=\cot x$ ...(ii)

Clearly, $\tan x$ and $\cot x$ intersect at $\pi/4$.

$\therefore$ Required area = area of shaded region

$$=\int_0^{\pi/4}\tan x\,dx+\int_{\pi/4}^{\pi/2}\cot x\,dx$$



$$=\left[-\log\cos x\right]_0^{\pi/4}+\left[\log\sin x\right]_{\pi/4}^{\pi/2}$$

$$=-\log\frac{1}{\sqrt2}-\log\frac{1}{\sqrt2}=2\log\sqrt2=\log 2\ \text{ sq. units}$$

39. (a) : Given, $|\vec a-\vec b|=|\vec a|=|\vec b|=1$

$$\Rightarrow |\vec a-\vec b|^2=|\vec a|^2+|\vec b|^2-2\vec a\cdot\vec b \Rightarrow 1=1+1-2|\vec a||\vec b|\cos\theta$$

(where θ is angle between $\vec a$ and $\vec b$)

$$\Rightarrow \cos\theta=\frac12 \Rightarrow \theta=\frac{\pi}{3}$$

40. (c)

41. (b) : We have,

$$\lim_{x\to0}\frac{8}{\sin^8 x}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)$$

$$=\lim_{x\to0}\frac{8}{\sin^8 x}\left(1-\cos\frac{x^2}{2}\right)\left(1-\cos\frac{x^2}{4}\right)$$

$$=\lim_{x\to0}\frac{8}{\sin^8 x}\times2\sin^2\frac{x^2}{4}\times2\sin^2\frac{x^2}{8}$$

$$\left[\because 1-\cos\theta=2\sin^2\frac{\theta}{2}\right]$$

$$=\lim_{x\to0}\frac{32}{\sin^8 x}\sin^2\frac{x^2}{4}\cdot\sin^2\frac{x^2}{8}$$

$$=\lim_{x\to0}\frac{32x^8}{\sin^8 x}\cdot\frac{\sin^2\frac{x^2}{4}}{\frac{x^4}{16}}\cdot\frac{\sin^2\frac{x^2}{8}}{\frac{x^4}{64}}\times\frac{1}{16\times64}$$

$$=32\times1\times1\times1\times\frac{1}{16\times64}=\frac{1}{32}$$

42. (b) **43. (a)**

44. (b) : Available letters are MMAATTHEICS

There are 11 letters in which 2 M's, 2 A's and 2 T's.

The following cases are possible :

Case I : When four letters consist of two pair of similar letters. We can choose such letters in ${}^3C_2$ ways.

$\therefore$ Number of permutations of such four letters is $\dfrac{4!}{2!2!}$

Case II : When four letters consist of one similar pair and 2 different letters. We can choose such letters in ${}^3C_1\times{}^7C_2$ ways and then number of possible permutations = $\dfrac{4!}{2!}$

Case III : When four letters consist of all different letters. We can choose such letters in ${}^8C_4$ ways and then number of permutations = 4!

$\therefore$ The required number is

$${}^3C_2\times\frac{4!}{2!2!}+{}^3C_1\times{}^7C_2\times\frac{4!}{2!}+{}^8C_4\times4!=2454.$$

45. (c) **46. (b)**

47. (d) : From the given equation, we have

$5(2\cos^2\theta-1)+(1+\cos\theta)+1=0$

$\Rightarrow\ 10\cos^2\theta+\cos\theta-3=0$

$\Rightarrow\ (5\cos\theta+3)(2\cos\theta-1)=0$

$\Rightarrow\ \cos\theta=1/2,\ \cos\theta=-3/5$

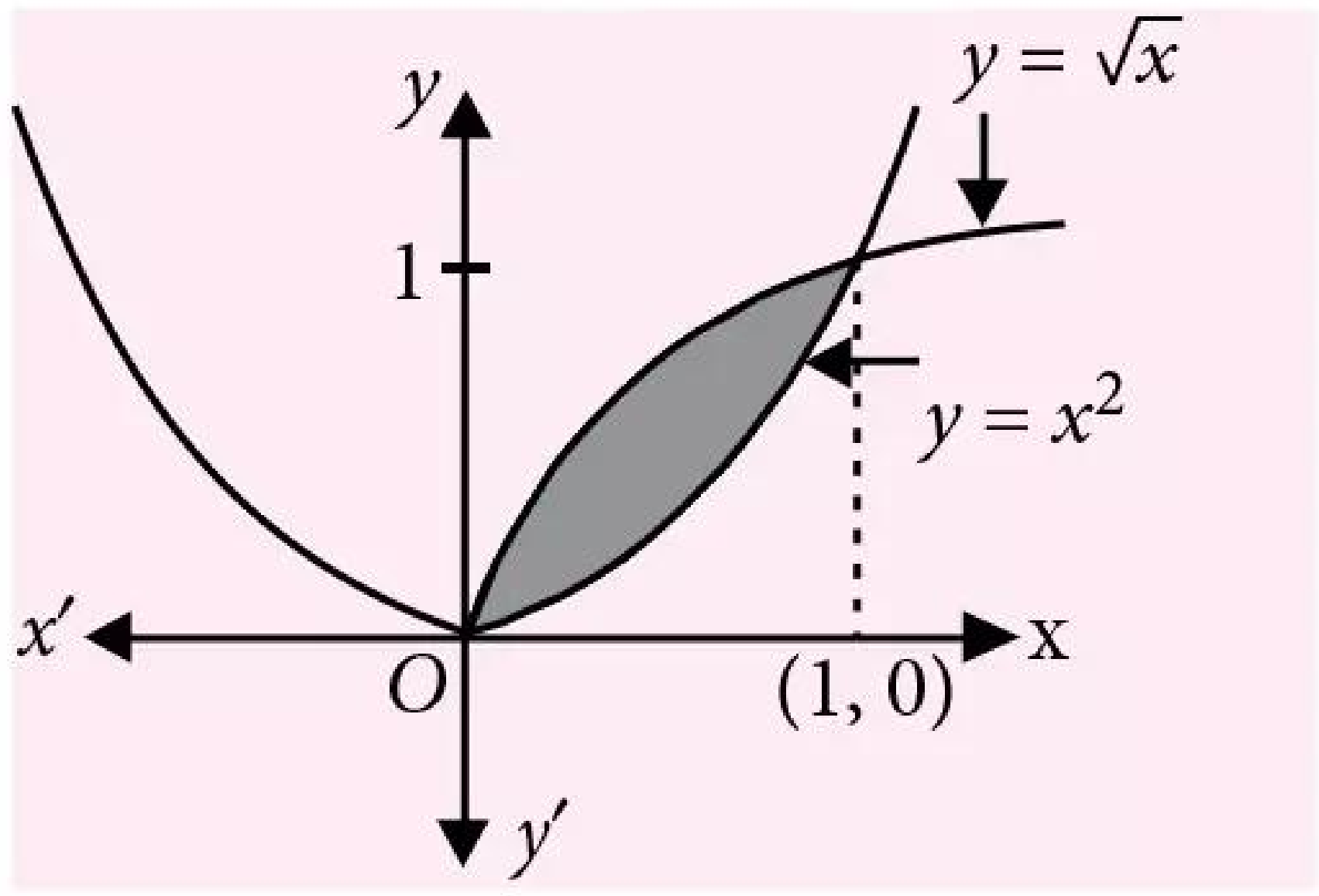
$\Rightarrow\ \theta=\pi/3,\ \pi-\cos^{-1}(3/5)$ in $-\pi<0<\pi$.

48. (b) : $x^2+y^2=4\Rightarrow 2x+2y\dfrac{dy}{dx}=0\Rightarrow y\dfrac{dy}{dx}+x=0$

49. (c) : We have, $y=x^2$ ...(i) a parabola with vertex at (0, 0)

and $y=\sqrt{x}$ ...(ii) a half portion of parabola $y^2=x$ having vertex at (0, 0).

Point of intersection of (i) and (ii) are (0, 0) and (1, 1)



$\therefore$ Required area = area of shaded region

$$=\int_0^1\left(\sqrt{x}-x^2\right)dx=\left[\frac23x^{3/2}-\frac13x^3\right]_0^1=\frac13\text{ sq. unit}$$

50. (c) : We have,

$$y=\frac{(1-x)(1+x)(1+x^2)\ldots(1+x^{2n})}{(1-x)}=\frac{1-x^{4n}}{1-x}$$

$$\Rightarrow \frac{dy}{dx}=\frac{(1-x)(-4nx^{4n-1})-(1-x^{4n})(-1)}{(1-x)^2} \Rightarrow \left[\frac{dy}{dx}\right]_{x=0}=1$$

51. (d)

52. (b) : The given equation of hyperbola is

$$\frac{16x^2}{144}-\frac{9y^2}{144}=1 \text{ or } \frac{x^2}{9}-\frac{y^2}{16}=1$$

∴ $a = 3$ and $b = 4$

Now, eccentricity $(e) = \sqrt{1+\frac{16}{9}}=\frac{5}{3}$

and directrix of the hyperbola are $x=\pm\frac{a}{e}=\pm\frac{9}{5}$

∵ It is given that the directrix of the hyperbola is $x=-\frac{9}{5}$.

∴ Required corresponding focus is $(-ae, 0) = (-5, 0)$.

53. (c)

54. (c) : Direction ratios of x-axis are (1, 0, 0) and direction ratios of the normal to the plane $2x - 3y + 6z = 11$ is (2, −3, 6).

$$\therefore \sin(\sin^{-1}\alpha)=\left|\frac{2+0+0}{\sqrt{1^2+0^2+0^2}\sqrt{2^2+(-3)^2+6^2}}\right|$$

$$\Rightarrow \alpha=\left(\frac{2}{7}\right)$$

55. (c) : Let $\tan^{-1}x = \theta \Rightarrow x = \tan\theta$

$\Rightarrow dx = \sec^2\theta\, d\theta$

Now, $x = 0 \Rightarrow \theta = 0$ and $x = 1 \Rightarrow \theta=\frac{\pi}{4}$

$$\therefore \int_0^1 \frac{x\tan^{-1}x}{(1+x^2)^{3/2}}dx=\int_0^{\pi/4}\frac{\theta\tan\theta}{\sec^3\theta}\sec^2\theta\, d\theta$$

$$=\int_0^{\pi/4}\theta\sin\theta\, d\theta=[-\theta\cos\theta]_0^{\pi/4}-\int_0^{\pi/4}(-\cos\theta)\, d\theta$$

$$=[-\theta\cos\theta]_0^{\pi/4}+[\sin\theta]_0^{\pi/4}=\frac{4-\pi}{4\sqrt{2}}$$

56. (d) : Given, $n(A) = m$ and $n(B) = n$

∴ Total number of relations from A to $B = 2^{mn}$

∴ Total number of non-empty relations from A to $B = 2^{mn} - 1$

57. (a) : $1+\frac{1}{2}\cdot n+\frac{1}{3}\cdot\frac{n(n-1)}{2!}+.....+\frac{1}{n+1}\cdot 1$

$$=\frac{1}{n+1}\left[(n+1)+\frac{(n+1)n}{2!}+\frac{(n+1)n(n-1)}{3!}+.....+1\right]$$

$$=\frac{1}{n+1}[{}^{n+1}C_1+{}^{n+1}C_2+{}^{n+1}C_3+.....+{}^{n+1}C_{n+1}]$$

$$=\frac{1}{n+1}[{}^{n+1}C_0+{}^{n+1}C_1+{}^{n+1}C_2+.....+{}^{n+1}C_{n+1}-1]$$

$[\because {}^{n+1}C_0 = 1]$

$$=\frac{1}{n+1}[2^{n+1}-1]$$

58. (a)

59. (a) : We have, $A^2 = I$

Now, $(A - I)^3 + (A + I)^3 - 7A$

$= A^3 - I^3 - 3A^2I + 3AI^2 + A^3 + I^3 + 3A^2I + 3AI^2 - 7A$

$= 2A^3 + 6AI^2 - 7A = 2A^2A + 6AI - 7A$

$= 2IA + 6A - 7A = 2A + 6A - 7A = A$ $\quad[\because A^2 = I]$

60. (a) : $\theta = \tan^{-1}(2\tan^2\theta) - \tan^{-1}((1/3)\tan\theta)$

$$\Rightarrow \tan\theta=\frac{2\tan^2\theta-(1/3)\tan\theta}{1+(2/3)\tan^3\theta}$$

$$\Rightarrow \tan\theta\left[\frac{2\tan\theta-(1/3)}{1+(2/3)\tan^3\theta}-1\right]=0,$$

which is true if $\tan\theta = 0$

or $\frac{2\tan\theta-(1/3)}{1+(2/3)\tan^3\theta}=1 \Rightarrow \tan^3\theta - 3\tan\theta + 2 = 0$

$\Rightarrow (\tan\theta - 1)^2(\tan\theta + 2) = 0$, which holds if $\tan\theta = 1$ or $\tan\theta = -2$

61. (b) **62. (b)**

63. (d) : Variance $=\frac{\Sigma x^2}{n}-\left(\frac{\Sigma x}{n}\right)^2=\frac{18000}{60}-\left(\frac{960}{60}\right)^2$

$= 300 - 256 = 44$

64. (c) : Given that $2ae=\frac{1}{2}\left(\frac{2b^2}{a}\right)$

$$\Rightarrow 2ae=\frac{b^2}{a} \Rightarrow 2e=\frac{b^2}{a^2}$$

Also, $e^2=1-\frac{b^2}{a^2} \Rightarrow e^2=1-2e \Rightarrow e=\sqrt{2}-1$

65. (d) : If $y = mx + c$ is the normal to the parabola $y^2 = 4ax$ at $(at^2, 2at)$ then $c = at^3 + 2at$ and $m = -t$

Given, focal distance is 8 units.

∴ $(at^2 - a)^2 + 4a^2t^2 = 64$

$\Rightarrow a^2[(t^2 - 1)^2 + 4t^2] = 64 \Rightarrow a^2(t^2 + 1)^2 = 64$

$\Rightarrow a(t^2 + 1) = 8 \Rightarrow t^2 + 1 = 4$ (∵ Here, $a = 2$)

$\Rightarrow t^2 = 3 \Rightarrow t = \pm\sqrt{3}$

Now, $c = at^3 + 2at$

$= 2(\sqrt{3})^3+2(2)\sqrt{3}=6\sqrt{3}+4\sqrt{3}=10\sqrt{3}$, when $t=\sqrt{3}$,

$c=2(-\sqrt{3})^3+2(2)(-\sqrt{3})=-10\sqrt{3}$, when $t=-\sqrt{3}$

∴ $|c|=10\sqrt{3}$

66. (b, c) : Summing the infinite G.P.' s, we get

$$x=\frac{1}{1-\cos^2\theta}=\frac{1}{\sin^2\theta},\ y=\frac{1}{1-\sin^2\theta}=\frac{1}{\cos^2\theta}$$

$$\therefore\ \frac{1}{x}+\frac{1}{y}=\sin^2\theta+\cos^2\theta=1\ \Rightarrow\ x+y=xy \qquad \text{...(i)}$$

Now, $z=\dfrac{1}{1-\cos^2\theta\sin^2\theta}\Rightarrow\dfrac{1}{z}=1-\cos^2\theta\sin^2\theta$

$$\Rightarrow\ \frac{1}{z}=1-\left(\frac{1}{x}\right)\left(\frac{1}{y}\right)=1-\frac{1}{xy}$$

$$\Rightarrow z=\frac{xy}{xy-1}\Rightarrow xyz=z+xy\Rightarrow xyz=z+x+y\ \text{(From (i))}$$

67. (d)

68. (a, b) : Given $\dfrac{x^2}{14-a}+\dfrac{y^2}{9-a}=1$

the equation will represent itself

(i) an ellipse if

$14-a>0$ and $9-a>0\Rightarrow a<14$ and $a<9\Rightarrow a<9$

(ii) an hyperbola if $14-a>0$ and $9-a<0$

$\Rightarrow$ $-a>-14$ and $a>9\Rightarrow a<14$ and $a>9$ $\Rightarrow 9<a<14$

or $14-a<0$ and $9-a>0$

$\Rightarrow$ $-a<-14$ and $-a>-9$

$\Rightarrow$ $a<9$ and $a>14$, which is not true.

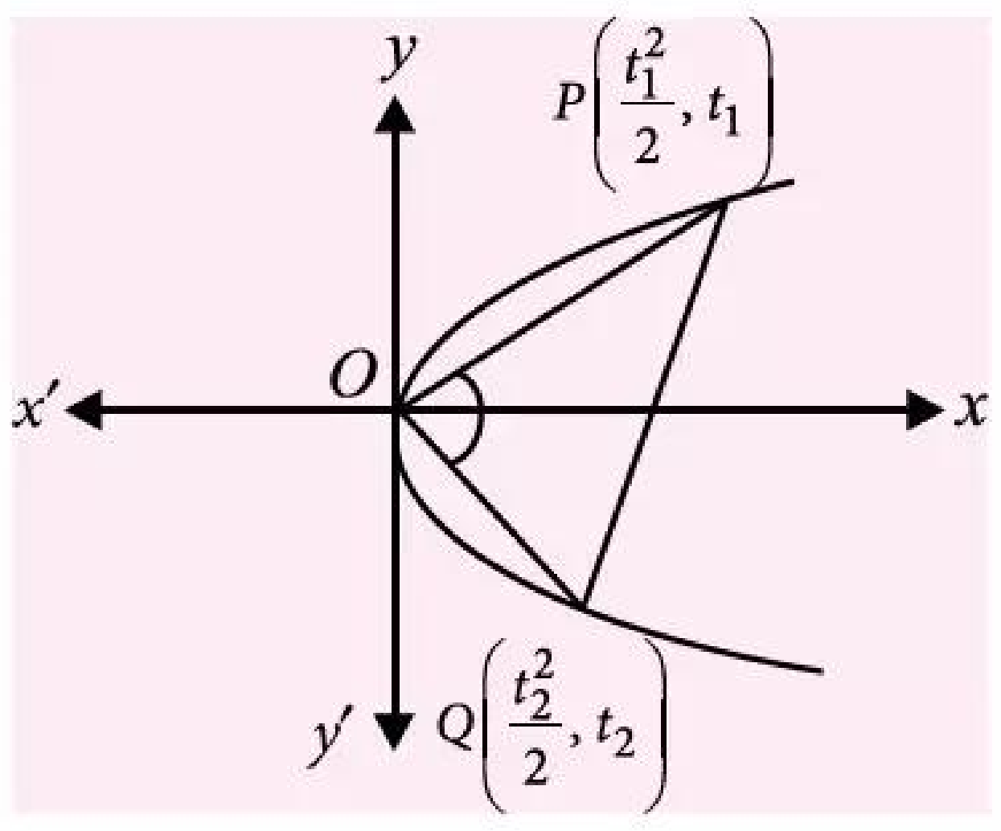
69. (a, b, c) : $A\times B=\{(1,1),(1,3),(1,5),(1,7),(1,9),(2,1),(2,3),(2,5),(2,7),(2,9),(3,1),(3,3),(3,5),(3,7),(3,9),(4,1),(4,3),(4,5),(4,7),(4,9),(5,1),(5,3),(5,5),(5,7),(5,9)\}$

Now R_4 is not a relation from A to B as $(2,4),(7,9)\in R_4$ but $(2,4),(7,9)\notin A\times B$.

70. (a, d) : Let t_1 and t_2 be parameters of P and Q respectively. Since OP and OQ are perpendicular.

$\therefore$ Product of slopes = –1

$\Rightarrow$ $t_1t_2=-4$



y
P$\left(\frac{t_1^2}{2},t_1\right)$
x′
O
x
y′
Q$\left(\frac{t_2^2}{2},t_2\right)$

Given, area of $\Delta OPQ=3\sqrt{2}$

$$\Rightarrow\ \frac{1}{4}\left|t_1^2t_2-t_1t_2^2\right|=3\sqrt{2}$$

$$\Rightarrow\ |t_1t_2||t_1-t_2|=12\sqrt{2}$$

$$\Rightarrow |t_1-t_2|=3\sqrt{2}$$

$$\Rightarrow\ \left|t_1+\frac{4}{t_1}\right|=3\sqrt{2}\quad\therefore\quad t_1+\frac{4}{t_1}=3\sqrt{2}$$

$$\Rightarrow\ t_1^2-3\sqrt{2}t_1+4=0\ \Rightarrow\ t_1=2\sqrt{2},\sqrt{2}$$

Thus $P\equiv(4,2\sqrt{2})$ or $(1,\sqrt{2})$

71. (a, b, c, d) : The curves are mirror images of each other in the two lines, $x-y=0$ and $x+y=0$. The slope of the common tangent, $m=\pm1$.

$\therefore$ The common tangents are $y=\pm x\pm\sqrt{a^2-b^2}$.

72. (a, c) : The three cube roots of p ($p<0$) (*i.e.*, solutions of $x^3-p=0$) are $p^{1/3}$, $p^{1/3}\omega$, $p^{1/3}\omega^2$.

Let $\alpha=p^{1/3}$, $\beta=p^{1/3}\omega$, $\gamma=p^{1/3}\omega^2$. Then,

$$\frac{x\alpha+y\beta+z\gamma}{x\beta+y\gamma+z\alpha}=\frac{x+y\omega+z\,\omega^2}{x\omega+y\omega^2+z}=\omega^2$$

If $\alpha=p^{1/3}$, $\gamma=p^{1/3}\omega$, $\beta=p^{1/3}\omega^2$, then

$$\frac{x\alpha+y\beta+z\gamma}{x\beta+y\gamma+z\alpha}=\frac{x+y\omega^2+z\omega}{x\omega^2+y\omega+z}=\omega\frac{(x+y\omega^2+z\omega)}{x\omega^3+y\omega^2+z\omega}=\omega.$$

Every other choice of α,β,γ will give its value ω or ω^2.

73. (a, d) : The lines $\vec{r}=\vec{a}+s\vec{b}$, $\vec{r}=\vec{c}+t\vec{d}$ are coplanar if $[\vec{c}-\vec{a}\ \vec{b}\ \vec{d}]=0$

$$\begin{vmatrix}1-2 & 4-3 & 5-4\\ 1 & 1 & -k\\ k & 2 & 1\end{vmatrix}=0\ \Rightarrow\ \begin{vmatrix}-1 & 1 & 1\\ 1 & 1 & -k\\ k & 2 & 1\end{vmatrix}=0$$

$\Rightarrow$ $-1(1+2k)-(1+k^2)+2-k=0\Rightarrow k^2+3k=0$

$\Rightarrow$ $k=0,-3$

74. (a, b) : We have, $AB=A$ and $BA=B$

Now, $AB=A$

$\Rightarrow$ $(AB)A=AA$ [Post multiplying both sides by A]

$\Rightarrow$ $A(BA)=A^2$ [By associativity of matrix multiplication]

$\Rightarrow$ $AB=A^2$ $[\because\ BA=B]$

$\Rightarrow$ $A=A^2$ $[\because\ AB=A]$

Similarly, it can be proved that $B^2=B$.

75. (c) : We have, $\dfrac{dt}{dx}-t\dfrac{g'(x)}{g(x)}=-\dfrac{t^2}{g(x)}$

$$\Rightarrow\ -\frac{1}{t^2}\frac{dt}{dx}+\frac{1}{t}\frac{g'(x)}{g(x)}=\frac{1}{g(x)}\qquad\text{...(i)}$$

Let $\dfrac{1}{t}=z\ \Rightarrow\ -\dfrac{1}{t^2}\dfrac{dt}{dx}=\dfrac{dz}{dx}$

Now by (i), we have $\dfrac{dz}{dx}+z\dfrac{g'(x)}{g(x)}=\dfrac{1}{g(x)}$

It is of type $\dfrac{dy}{dx}+Py=Q$

$$\therefore\ \text{I.F.}=e^{\int\frac{g'(x)}{g(x)}dx}=g(x)$$

$\therefore$ Solution is $z\,g(x)=x+c$

$$\Rightarrow\ \frac{1}{t}g(x)=x+c\ \Rightarrow\ t=\frac{g(x)}{x+c}$$

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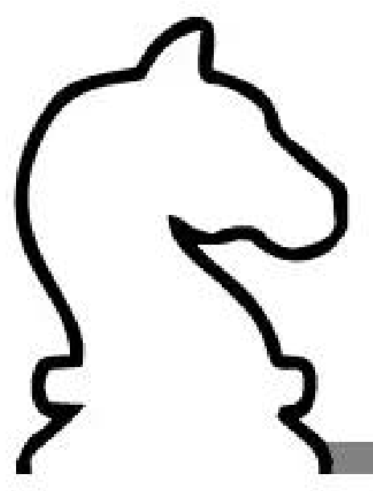
Challenging PROBLEMS For JEE

ON Sets, Relations and Functions



1. P is the set of all prime numbers. Set $Q = P \cup \{4\}$. S is the set of all prime numbers and their squares. A is the set of positive integers 'a' for which $\frac{(a-1)!}{a}$ is not an integer. Then
(a) $A = P$ (b) $A = Q$
(c) $A = S$ (d) $Q \subset A \subset S$

2. Consider two fixed 3-element disjoint subsets $A, B, \subset \{1, 2, 3, ..., 20\}$. The number of 3-element subsets of $\{1, 2, 3, ..., 20\}$ that has exactly one element in A and atleast one element in B, is
(a) 125 (b) 135 (c) 25 (d) 27

3. N is the set of first n natural numbers and two sets A, B are such that $A \subseteq B \subseteq N$, then the number of ordered pairs (A, B) is
(a) 3^n (b) $n!$
(c) $\sum_{k=0}^{n} {}^nC_k \cdot {}^nC_{n-k}$ (d) 2^n

4. If $\forall\ n \in N, P_n = \{1, 2, 3, ..., n\}, Q_1 = P_1$
$Q_2 = P_2 \Delta Q_1 = \{2\}, Q_3 = P_3 \Delta Q_2, ..., Q_{n+1} = P_{n+1} \Delta Q_n\ \forall$ $n \geq 1$, then $|Q_{2k}| =$
(a) k (b) $2k$ (c) $2k - 1$ (d) $2k + 1$

5. If $A \cap B \cap C = \phi$, $n(A \Delta B) = 100 = n(B \Delta C) = n(C \Delta A)$, then the number of elements in $A \cup B \cup C =$
(a) 150 (b) 200 (c) 250 (d) 300

6. Consider the operation $*$ that takes a pair of integers and returns an integer according to the rule $a * b = a \times (b + 1)$.
Given $a_1, a_2, ..., a_{10}$ are permutations of the set $\{1, 2, 3, ..., 10\}$. The value of a_1 which maximizes the value of $(((a_1 * a_2) * a_3) * ..... * a_9) * a_{10}$ is
(a) 1 (b) 10 (c) 5 (d) 6

7. Consider the two statements, S_1 and S_2 :
S_1 : We can divide the set $\{1, 2, 3, ..., 96\}$ into 32 subsets each containing 3 elements and the sums of the three elements in each subset are all equal.
S_2 : We can divide the set $\{1, 2, 3, ..., 99\}$ into 33 subsets each containing 3 elements, and the sums of the three elements in each subset are all equal. Then
(a) Both S_1 and S_2 are true
(b) Both S_1 and S_2 are false
(c) S_1 is true and S_2 is false
(d) S_1 is false and S_2 is true

8. Let $f: (3, 5) \to (7, 13)$ be a continuous function, then number of distinct integral values which may be equal to $4f(3) + 9f(5)$ are
(a) 77 (b) 39 (c) 80 (d) 40

9. If $f(x) = x^3 - \frac{3x^2}{7} + \frac{159x}{49} + \frac{6}{343}$ and $g(x) = f\left(f\left(x + \frac{1}{7}\right)\right) - \frac{1}{7}$, then $g(4) + g(-4) =$
(a) 0 (b) $\frac{1}{7}$ (c) $\frac{-1}{7}$ (d) 2

10. If $f(x) = \sin x + \cos ax$ is periodic, then a possible value of a is
(a) 2 (b) π (c) e (d) $\pi/2$

11. If $y = f(x)$ is symmetric about the lines $3x + 4y + 1 = 0$ and $4x - 3y = 7$, then it must be symmetric about
(a) $(0, 0)$ (b) $(1, 0)$ (c) $(0, 1)$ (d) $(1, -1)$

12. Consider $f(x) = \ln(2x - x^2) + \sin\left(\frac{\pi x}{2}\right)$ and the following four statements.
S_1 : Graph of f is symmetric about line $x = 1$
S_2 : Graph of f is symmetric about line $x = 0$
S_3 : Maximum value of $f(x)$ is 1
S_4 : Minimum value of $f(x)$ does not exist
The number of correct statements are
(a) 0 (b) 1 (c) 2 (d) 3

13. A function $f : N \to N - \{1\}$ has the property that $\log_{n+1} f(n) = \log_{f(n+2)} (n + 3)$ for any natural number n, then $f(20) =$
(a) 19 (b) 20 (c) 21 (d) 22

14. A function $f: R^+ \to R^+$ is such that
$$f\left(\frac{x+y}{2}\right) = \frac{2f(x)f(y)}{f(x) + f(y)}\ \forall\ x, y \in R^+.$$
The graph of $f(x)$ is
(a) straight line (b) part of circle
(c) part of parabola (d) part of hyperbola

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15. If $f(x)$ is an unique invertible function from $R \rightarrow R$ and $(f(x))^3 - (x^3 + 2)(f(x))^2 + 2x^3 f(x) = x^3 - f(x)$, then $f(4) + f(3) + f(2) + f(1) =$

(a) 50 (b) 100 (c) 51 (d) 101

16. Given $f(x) = e^x$; If $y = m$ and $y = n$ are two horizontal asymptotes of the function $g(x) = \dfrac{4f(x) - 7f(-x)}{f(x) + f(-x)}$, then $m + n =$

(a) 2 (b) -2 (c) 3 (d) -3

17. The number of real solutions to the equation $(\sqrt{x^4 + 4x^2 + 5} + x^2 + 2)\cdot(\sqrt{9x^2 + 1} - 3x) = 1$ is/are

(a) 0 (b) 2 (c) 4 (d) 8

SOLUTIONS

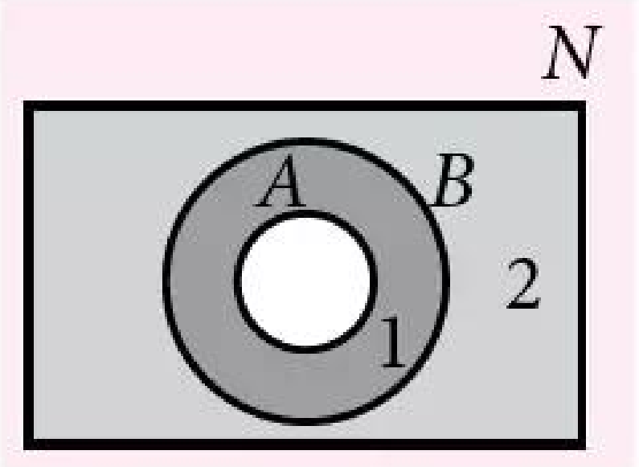
1. (b) : a does not divide $(a - 1)!$ $\Rightarrow$ $a \in P$, *i.e.*, all primes are in A.
Also $(4 - 1)! = 6$, not divisible by 4, *i.e.*, 4 is the only non-prime in A. Hence, $A = Q$.

2. (b) : Possible choices

	A,	B,	C
	1	1	1
or	1	2	0

i.e., ${}^3C_1 \cdot {}^3C_1 \cdot {}^{14}C_1 + {}^3C_1 \cdot {}^3C_2 \cdot {}^{14}C_0 = 135$

3. (a) : Any element may be in A or in shaded part-1 or in shaded part-2, *i.e.*, 3 choices for every element.
Hence, total number of ordered pairs = 3^n.



4. (a) : $Q_1 = \{1\}$, $Q_2 = \{2\}$, $Q_3 = \{1, 3\}$, $Q_4 = \{2, 4\}$.
$Q_6 = \{2, 4, 6\}$, $Q_8 = \{2, 4, 6, 8\}$.
Following the pattern, Q_{2k} contains all k even numbers upto $2k$ *i.e.*, $|Q_{2k}| = k$.

5. (a) : $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C)$

$$= \frac{1}{2}(n(A \Delta B) + n(B \Delta C) + n(C \Delta A)) = \frac{1}{2}(300) = 150$$

6. (b) : The given relation simplifies to

$a_1(a_2 + 1)(a_3 + 1) \ldots (a_{10} + 1) = \dfrac{a_1}{a_1 + 1}(11)!$, which is max. at $a_1 = 10$

7. (d) : $S_1 : 1 + 2 + 3 + \ldots\ldots + 96 = 48 \times 97$ and 32 does not divides 48×97.

$S_2 : \dfrac{1 + 2 + \ldots\ldots + 99}{33} = 150$

Possible sets are; (1, 50, 99), (3, 49, 98), ..., (33, 34, 83), (2, 66, 82), (4, 65, 81), ..., (32, 51, 67)

8. (a) : $A(3, f(3))$, $B(5, f(5))$.
Any point P on the line segment AB is $\left[\dfrac{5m + 3n}{m + n}, \dfrac{mf(5) + nf(3)}{m + n}\right]$

Replacing $m = 9$, $n = 4$. Hence, $7 < \dfrac{9f(5) + 4f(3)}{9 + 4} < 13$

9. (a) : $g(x)$ is an odd function.

$$g(x) = \left(x^3 + \frac{3x}{4}\right)^3 + \frac{3}{4}\left(x^3 + \frac{3x}{4}\right)$$

$\therefore$ $g(4) + g(-4) = 0$

10. (a) : Let T be the period of $f(x)$, then
$f(x + T) = f(x) \rightarrow f(0) = f(T)$ and $f(0) = f(-T)$
i.e., $1 = \sin T + \cos aT$ and $1 = -\sin T + \cos aT$
$\Rightarrow$ $\sin T = 0$, $\cos aT = 1 \Rightarrow T = n\pi$, $aT = 2m\pi$, $m, n \in I$

So, $a = \dfrac{2m}{n} =$ rational.

11. (d) : Given two lines are perpendicular, intersecting at $(1, -1)$.

12. (d) : S_1 : clearly, $f(1 + x) = f(1 - x)$ and domain of f is $(0, 2)$.
$S_3 : f_{\max.}$ occurs at $x = 1$.
S_4 : As $x \rightarrow 0^+$ or $x \rightarrow 2^-$, $f(x) \rightarrow -\infty$.

13. (c) : $\log_{n+1} f(n)$

$$= \frac{1}{\log_{(n+3)} f(n+2)} \rightarrow g(n) = \frac{1}{g(n+2)} = g(n+4)$$

i.e., g is periodic with period 4.

$g(1) = a$ (Let) $= \dfrac{1}{g(3)} \rightarrow f(1) = 2^a$ and $f(3) = 4^{1/a}$.

As $f(1), f(3) \in N - \{1\}$ So only possibility of $a = 1$.
So, $g(1) = g(3) = g(5) = \ldots = 1$ and $g(2) = g(4) = \ldots = 1$,
i.e., $g(n) = 1$ or $f(n) = n + 1$.

14. (d) : Let $g(x) = \dfrac{1}{f(x)}$, then the given equation becomes $g\left(\dfrac{x + y}{2}\right) = \dfrac{g(x) + g(y)}{2}$, which is satisfied by a linear function, *i.e.*, $g(x) = ax + b$.

Hence $f(x) = \dfrac{1}{ax + b}$ (Hyperbola), $a > 0$, $b \geq 0$.

15. (b) : Given expression can be factorised as
$(f(x) - 1)^2 (f(x) - x^3) = 0$
Hence, invertible $f(x) = x^3$.
$\therefore$ $f(4) + f(3) + f(2) + f(1) = 64 + 27 + 8 + 1 = 100$

16. (d) : $g(x) = \dfrac{4 - 7e^{-2x}}{1 + e^{-2x}}$, $\lim\limits_{x \to \infty} g(x) = 4 = m$

and also, $g(x) = \dfrac{4e^{2x} - 7}{e^{2x} + 1}$, $\lim\limits_{x \to -\infty} g(x) = -7 = n$

So, $m + n = 4 - 7 = -3$

17. (b) : Consider the increasing function

$$f(t) = \sqrt{t^2 + 1} + t$$

The given equation is equivalent to
$f(x^2 + 2) \cdot f(-3x) = 1$ ...(i)
Now, $f(t_1) f(t_2) = 1 \Rightarrow t_1 = -t_2$
Hence, for (i), $x^2 + 2 = -(-3x) \rightarrow x = 1, 2$

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CBSE

Held on
11th March

SOLVED PAPER 2023

CLASS **12**

General Instructions : Read the following instructions very carefully and follow them:

Time Allowed : 3 hours
Maximum Marks : 80

(a) This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
(b) Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
(c) Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
(d) Section C has 6 Short Answer (SA)-type questions of 3 marks each.
(e) Section D has 4 Long Answer (LA)-type questions of 5 marks each.
(f) Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION - A (MULTIPLE CHOICE QUESTIONS)

Each question carries 1 mark.

1. Let $A = \{3, 5\}$. Then number of reflexive relations on A is

(a) 2 (b) 4 (c) 0 (d) 8

2. $\text{Sin}\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right]$ is equal to

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

3. If for a square matrix A, $A^2 - A + I = 0$ then A^{-1} equals

(a) A (b) $A + I$ (c) $I - A$ (d) $A - I$

4. If $A=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}, B=\begin{bmatrix}x & 0\\ 1 & 1\end{bmatrix}$ and $A = B^2$, then x equals

(a) ±1 (b) –1 (c) 1 (d) 2

5. If $\begin{vmatrix}\alpha & 3 & 4\\ 1 & 2 & 1\\ 1 & 4 & 1\end{vmatrix}=0$, then the value of α is

(a) 1 (b) 2 (c) 3 (d) 4

6. The derivative x^{2x} w.r.t. x is

(a) $x^{2x} - 1$ (b) $2x^{2x}\log x$
(c) $2x^{2x}(1 + \log x)$ (d) $2x^{2x}(1 - \log x)$

7. The function $f(x) = [x]$, where $[x]$ denotes the greatest integer less than or equal to x, is continuous at

(a) $x = 1$ (b) $x = 1.5$ (c) $x = -2$ (d) $x = 4$

8. If $x = A\cos 4t + B\sin 4t$, then $\frac{d^2x}{dt^2}$ is equal to

(a) x (b) $-x$ (c) $16x$ (d) $-16x$

9. The interval in which the function $f(x) = 2x^3 + 9x^2 + 12x - 1$ is decreasing, is

(a) $(-1, \infty)$ (b) $(-2, -1)$
(c) $(-\infty, -2)$ (d) $[-1, 1]$

10. $\int \frac{\sec x}{\sec x - \tan x}dx$ equals

(a) $\sec x - \tan x + c$ (b) $\sec x + \tan x + c$
(c) $\tan x - \sec x + c$ (d) $-(\sec x + \tan x) + c$

11. $\int_{-1}^{1}\frac{|x-2|}{x-2}dx, x \neq 2$ is equal to

(a) 1 (b) –1 (c) 2 (d) –2

12. The sum of the order and the degree of the differential equation $\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^3\right)$ is

(a) 2 (b) 3 (c) 5 (d) 0

13. Two vectors $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$ are collinear if

(a) $a_1b_1 + a_2b_2 + a_3b_3 = 0$
(b) $\frac{a_1}{b_1}=\frac{a_2}{b_2}=\frac{a_3}{b_3}$
(c) $a_1 = b_1, a_2 = b_2, a_3 = b_3$
(d) $a_1 + a_2 + a_3 = b_1 + b_2 + b_3$

14. The magnitude of the vector $6\hat{i}-2\hat{j}+3\hat{k}$ is
(a) 1 (b) 5 (c) 7 (d) 12

15. If a line makes angles of 90°, 135° and 45° with the x, y and z axes respectively, then its direction cosines are

(a) $0,-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}$ (b) $-\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}$

(c) $\frac{1}{\sqrt{2}},0,-\frac{1}{\sqrt{2}}$ (d) $0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}$

16. The angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is
(a) 0° (b) 30° (c) 45° (d) 90°

17. If for any two events A and B,
$P(A)=\frac{4}{5}$ and $P(A\cap B)=\frac{7}{10}$, then $P(B/A)$ is
(a) $\frac{1}{10}$ (b) $\frac{1}{8}$ (c) $\frac{7}{8}$ (d) $\frac{17}{20}$

18. Five fair coins are tossed simultaneously. The probability of the events that atleast one head comes up is
(a) $\frac{27}{32}$ (b) $\frac{5}{32}$ (c) $\frac{31}{32}$ (d) $\frac{1}{32}$

Assertion -Reason Based Questions

In the following questions 19 and 20, a statements are Assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices:

(a) Both (A) and (R) are true and (R) is the correct explanation of (A)
(b) Both (A) and (R) are true, but (R) is not the correct explanation of the (A)
(c) (A) is true, and (R) is False.
(d) (A) is false, but (R) is true.

19. **Assertion (A) :** Two coins are tossed simultaneously. The probability of getting two heads, if it is known that at least one head comes up, is $\frac{1}{3}$.

Reason : Let E and F be two events with a random experiment, then $P(F/E)=\frac{P(E\cap F)}{P(E)}$.

20. **Assertion (A) :** $\int_{2}^{8}\frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}dx=3$.

Reason (R) : $\int_{a}^{b}f(x)dx=\int_{a}^{b}f(a+b-x)dx$.

SECTION - B

This section comprises of Very Short Answer (VSA) type questions 2 marks each.

21. Write the domain and range (principle value branch) of the following functions:
$f(x) = \tan^{-1}x$.

22. (a) If $f(x)=\begin{cases}x^2, \text{ if } x\ge 1\\ x, \text{ if } x<1\end{cases}$, then show that f is not differentiable at $x = 1$.

OR

(b) Find the values (s) of 'λ' if the function $f(x)=\begin{cases}\frac{\sin^2\lambda x}{x^2}, \text{ if } x\ne 0\\ x, \text{ if } x=0\end{cases}$ is continuous at $x = 0$.

23. Sketch the region bounded by the lines $2x + y = 8$, $y = 2$, $y = 4$ and the y-axis. Hence, obtain its area using integration.

24. If the vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}|=3, |\vec{b}|=\frac{2}{3}$ and $\vec{a}\times\vec{b}$ is a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.

OR

Find the area of a parallelogram whose adjacent sides are determined by the vectors $\vec{a}=\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=-2\hat{i}-7\hat{j}+\hat{k}$.

25. Find the vector and the cartesian equations of a line that passes through the point $A(1, 2, -1)$ and parallel to the line $5x - 25 = 14 - 7y = 35z$.

SECTION - C

This section comprises of Short Answer (SA) type questions of 3 marks each.

26. If $A=\begin{bmatrix}1 & 2 & 3\\ 3 & -2 & 1\\ 4 & 2 & 1\end{bmatrix}$, then show that $A^3 - 23A - 40I = 0$.

27. Differentiate $\sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ with respect to $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$.

OR

If $y = \tan x + \sec x$, then prove that $\frac{d^2y}{dx^2}=\frac{\cos x}{(1-\sin x)^2}$.

28. Evaluate : $\int_0^{2\pi} \frac{1}{1+e^{\sin x}}\,dx$

OR

Find : $\int \frac{x^4}{(x-1)(x^2+1)}\,dx$

29. Find the area of the following region using integration.
$\{(x, y) : y^2 \le 2x \text{ and } y \ge x - 4\}$

30. Find the coordinates of the foot of the perpendicular drawn from the point $P(0, 2, 3)$ to the line $\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$.

OR

Three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ satisfy the condition $\vec{a}+\vec{b}+\vec{c}=\vec{0}$. Evaluate the quantity $\mu=\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$, if $|\vec{a}|=3$, $|\vec{b}|=4$ and $|\vec{c}|=2$.

31. Find the distance between the lines:
$\vec{r}=(\hat{i}+2\hat{j}-4\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$
$\vec{r}=(3\hat{i}+3\hat{j}-5\hat{k})+\mu(4\hat{i}+6\hat{j}+12\hat{k})$

SECTION - D

This section comprises of Long Answer (LA) type questions of 5 marks each.

32. The median of an equilateral triangle is increasing at the rate of $2\sqrt{3}$ cm/s. Find the rate at which its side is increasing.

OR

Sum of two numbers is 5. If the sum of the cubes of this numbers is least, then find the sum of squares of these numbers.

33. Evaluate : $\int_0^{\pi/2} \sin 2x \tan^{-1}(\sin x)\,dx$

34. Solve the following linear programming problem graphically.
Maximize : $P = 70x + 40y$
subject to : $3x + 2y \le 9$, $3x + y \le 9$, $x \ge 0$, $y \ge 0$.

35. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{5}$ be the probability that he knows the answer and $\frac{2}{5}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{3}$. What is the probability that the student knows the answer, given that he answered it correctly?

OR

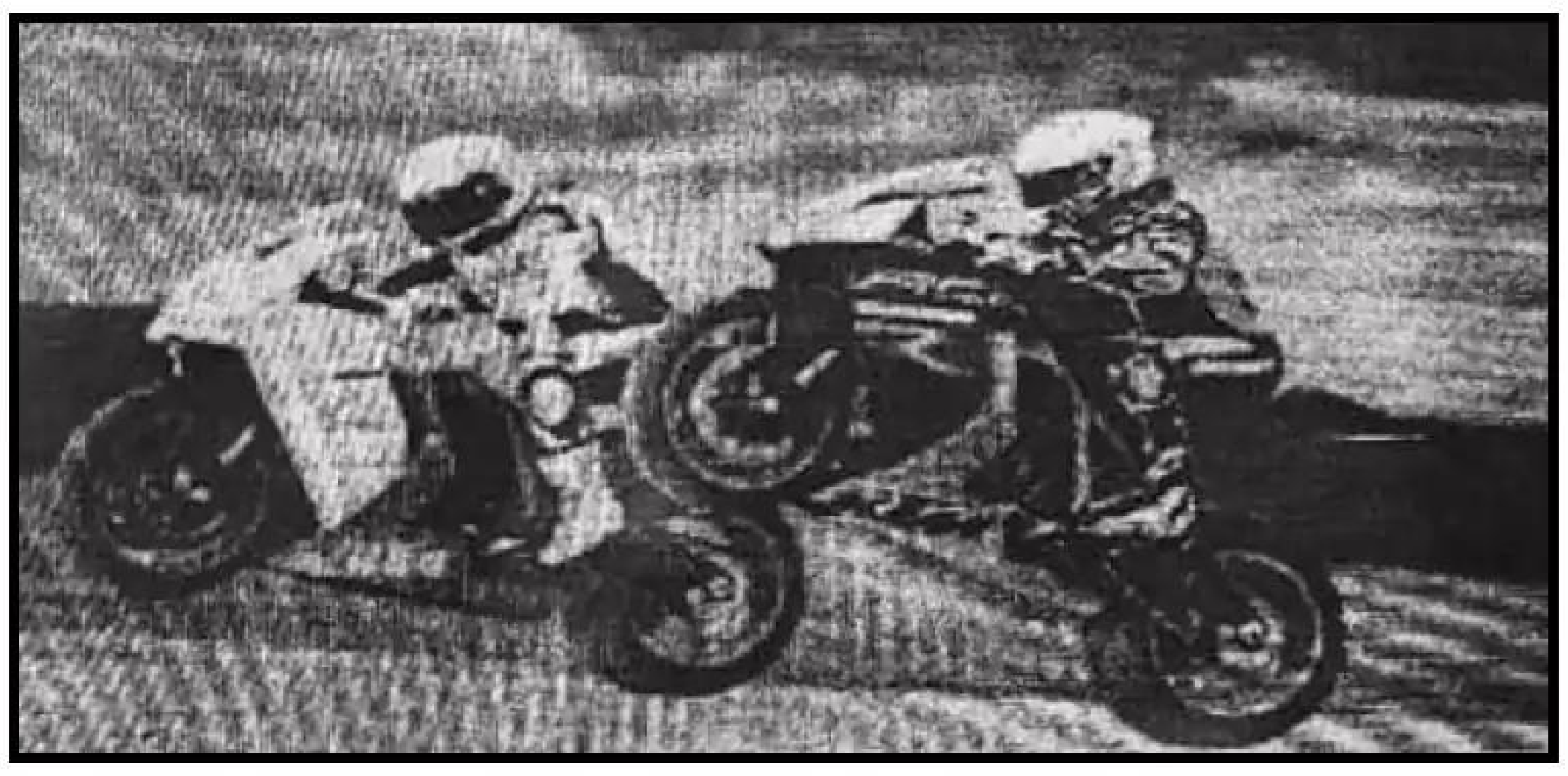
A box contains 10 tickets, 2 of which carry a prize of ₹ 8 each, 5 of which carry a prize of ₹ 4 each, and remaining 3 carry a prize of ₹ 2 each. If one ticket is drawn at random, find the mean value of the prize.

SECTION - E

This section comprises of 3 case study/passage-based questions of 4 marks each with two sub-parts. First two cases study questions have three sub-parts (I), (II), (III) of marks 1, 1, 2 respectively. The third case study questions has two sub-parts (I) and (II) of marks 2 each.

Case Study-I

36. An organization conducted bike race under two different categories - Boys and girls. There were 28 participants in all. Among all of them, finally three from category 1 and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project. Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, where B represents the set of Boys selected and G the set of Girls selected for the final race.



Based on the above information, answer the following questions.

(I) How many relations are possible from B to G?

(II) Among all the possible relations from B to G, how many functions can be formed from B to G?

(III) Let $R : B \to B$ be defined by $R = \{(x, y) : x$ and y are students of the same sex$\}$. Check if R is an equivalence relation.

OR

A function $f : B \to G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$. Check if f is bijective, justify your answer.

Case Study-II

37. Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹160. From the same shop. Vikram buys 2 pens, 1 bag and 3 instrument boxes and pays a sum of ₹190. Also Ankur buys 1 pen, 2 bags and 4 instrument boxes and pays a sum of ₹250. **Based on the above information, answer the following questions.**

(I) Convert the given above situation into a matrix equation of the form $AX = B$.

(II) Find $|A|$.

(III) Find A^{-1}.

OR

Determine $P = A^2 - 5A$.

Case Study-III

38. An equation involving derivatives of the dependent variable with respect to the independent variables is called a differential equation *A* differential equation of the form $\frac{dy}{dx} = F(x, y)$ is said to be homogeneous if $F(x, y)$ is a homogeneous function of degree zero, whereas a function $F(x, y)$ is a homogenous function of degree n if $F(\lambda x, \lambda y) = \lambda^n F(x, y)$. To solve a homogeneous differential equation of the type $\frac{dy}{dx} = F(x, y) = g\left(\frac{y}{x}\right)$, we make substitution $y = vx$ and then separate the variables. **Based on the above information, answer the following questions.**

(I) Show that $(x^2 - y^2)\,dx + 2xy\,dy = 0$ is a differential equation of the type $\frac{dy}{dx} = g\left(\frac{y}{x}\right)$.

(II) Solve the above equation to find its general solution.

SOLUTIONS

1. (b) : Total number of reflexive relations on *a* set having n number of elements $= 2^{n^2 - n}$

Here, $n = 2$

$\therefore$ Required number of reflexive relations $= 2^{2^2 - 2}$

$= 2^{4 - 2} = 2^2 = 4$

2. (a) : We have,

$$\sin\left[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)\right] = \sin\left(\frac{\pi}{3} + \sin^{-1}\sin\frac{\pi}{6}\right) = \sin\left[\frac{\pi}{3} + \frac{\pi}{6}\right]$$

$$= \sin\left(\frac{\pi}{2}\right) = 1$$

3. (c) : We have, $A^2 - A + I = O$

Pre-multiplying with A^{-1} on both sides, we get

$(A^{-1}A) \cdot A - A^{-1} \cdot A + A^{-1} \cdot I = A^{-1} \cdot O$

$\Rightarrow I \cdot A - I + A^{-1} = O \Rightarrow A^{-1} = -(A - I) = I - A$

4. (c) : We have, $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$

$$\therefore \quad B^2 = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} x^2 & 0 \\ x+1 & 1 \end{bmatrix}$$

Now, it is given that $A = B^2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} x^2 & 0 \\ x+1 & 1 \end{bmatrix}$$

On comparing, we get

$x^2 = 1$ and $x + 1 = 2 \quad \Rightarrow \quad x = \pm 1$ and $x = 1$

$\therefore \quad x = 1$

5. (d) : $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$

$\Rightarrow \alpha(2 - 4) - 3(1 - 1) + 4(4 - 2) = 0$

$\Rightarrow -2\alpha + 8 = 0 \Rightarrow 2\alpha = 8 \Rightarrow \alpha = 4$

6. (c) : Let $y = x^{2x}$

Taking log on both sides, we get

$\log y = 2x \log x$

Differentiating both sides w.r.t. x, we get

$$\frac{1}{y}\frac{dy}{dx} = 2\left\{x\frac{1}{x} + \log x\, 1\right\}$$

$$\Rightarrow \frac{dy}{dx} = 2y\{1 + \log x\} = 2x^{2x}(1 + \log x)$$

7. (b) : Let $x = 1.5$

$$\therefore \quad \text{L. H. L.} = \underset{x \to 1.5^-}{\text{Lt}} f(x) = \underset{h \to 0}{\text{Lt}} [1.5 - h] = 1$$

$$\text{and} \quad \text{R. H. L.} = \underset{x \to 1.5^+}{\text{Lt}} f(x) = \underset{h \to 0}{\text{Lt}} [1.5 + h] = 1$$

$\because$ L. H. L. = R. H. L.

$\therefore$ $f(x)$ is continous at $x = 1.5$

Also, greatest integer function is discontinous at all integeral values of x.

8. (d) : We have, $x = A\cos 4t + B\sin 4t$

Differentiating both sides w.r.t. t, we get

$$\frac{dx}{dt} = A \cdot (-\sin 4t) \cdot 4 + B\cos 4t \cdot 4 \quad \text{...(i)}$$

Again differentiating both sides of (i) w.r.t. t, we get

$$\frac{d^2x}{dt^2} = -4A(\cos 4t) \cdot 4 + 4B(-\sin 4t) \cdot 4$$

$= -16A\cos 4t - 16B\sin 4t = -16(A\cos 4t + B\sin 4t) = -16x$

9. (b) : We have, $f(x) = 2x^3 + 9x^2 + 12x - 1$

$\Rightarrow f'(x) = 6x^2 + 18x + 12$

For decreasing, $f'(x) < 0$

$\therefore \quad 6x^2 + 18x + 12 < 0 \Rightarrow x^2 + 3x + 2 < 0$

$\Rightarrow \quad (x + 1)(x + 2) < 0 \Rightarrow -2 < x < -1$

So, $f(x)$ is decreasing, if $x \in (-2, -1)$

10. (b) : Let $I=\int \frac{\sec x}{\sec x-\tan x}dx$

$=\int \frac{\sec x(\sec x+\tan x)}{(\sec x-\tan x)(\sec x+\tan x)}dx$

$=\int\left(\frac{\sec^2 x+\sec x\tan x}{\sec^2 x-\tan^2 x}\right)dx$

$=\int \sec^2 x\,dx+\int \sec x\tan x\,dx \quad [\because \sec^2 x-\tan^2 x=1]$

$= \tan x + \sec x + c$

11. (d) : Let $I=\int_{-1}^{1}\frac{|x-2|}{x-2}dx$

$=\int_{-1}^{1}\frac{-(x-2)}{x-2}dx=\int_{-1}^{1}-1\cdot dx=[-x]_{-1}^{1}\ -[1-(-1)] = -2$

12. (b) : [There is error in question. The given differential equation must be as $\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^3\right)=0$.]

The given differential equation is,

$$\frac{d}{dx}\left(\left(\frac{dy}{dx}\right)^3\right)=0 \quad\Rightarrow\quad 3\left(\frac{dy}{dx}\right)^2\left(\frac{d^2y}{dx^2}\right)=0$$

∴ Order = 2 and degree = 1

So, required sum = 2 + 1 = 3

13. (b)

14. (c) : Given vector is $6\hat{i}-2\hat{j}+3$

∴ Its magnitude $=\sqrt{6^2+(-2)^2+3^2}$

$=\sqrt{36+4+9}=\sqrt{49}=7$ units

15. (a) : Direction cosines are $\langle \cos 90°, \cos 135°, \cos 45°\rangle$

$=\left\langle 0, \cos(90°+45°), \frac{1}{\sqrt{2}}\right\rangle=\left\langle 0, -\sin 45°, \frac{1}{\sqrt{2}}\right\rangle$

$=\left\langle 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right\rangle$

16. (d) : The given equation of lines can be rewritten as

$\frac{x-0}{1/2}=\frac{y-0}{1/3}=\frac{z-0}{-1}$ and $\frac{x-0}{1/6}=\frac{y-0}{-1}=\frac{z-0}{-1/4}$

∴ $a_1=\frac{1}{2},\ b_1=\frac{1}{3}, c_1=-1$

and $a_2=\frac{1}{6},\ b_2=-1,\ c_2=\frac{-1}{4}$

Now, $\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

$$=\frac{\frac{1}{2}\cdot\frac{1}{6}+\frac{1}{3}\cdot(-1)+(-1)\cdot\left(\frac{-1}{4}\right)}{\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{3}\right)^2+(-1)^2}\sqrt{\left(\frac{1}{6}\right)^2+(-1)^2+\left(\frac{-1}{4}\right)^2}}=0$$

⇒ $\cos\theta = 0 \Rightarrow \theta = 90°$

17. (c) : We know that, $P(B/A)=\frac{P(B\cap A)}{P(A)}=\frac{7/10}{4/5}=\frac{7}{8}$

18. (c) : Since each coin turns up on either a head or tail.

∴ Total possible outcomes = 2^5 = 32

Let A be the event that all tails comes up.

∴ $n(A)$ = 1 {i.e., (T, T, T, T, T)}

So, required probability $=1-P(A)=1-\frac{1}{32}=\frac{31}{32}$

19. (a) : Sample space = {HH, HT, TH, TT}

Let A be the event of coming up two heads

∴ $A=\{HH\} \Rightarrow P(A)=\frac{1}{4}$

and B be the even of coming π atleast one head

∴ B = {HH, HT, TH} ⇒ $P(B)=\frac{3}{4}$

Also, $A\cap B=\{HH\}\Rightarrow P(A\cap B)=\frac{1}{4}$

So, required probability $=P(A/B)=\frac{P(A\cap B)}{P(B)}$

$=\frac{\frac{1}{4}}{\frac{3}{4}}=\frac{1}{3}$

So, assertion is true.

Also, reason is true and it is the correct explanation of assertion.

20. (a) : Let $I=\int_2^8\frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}dx$ ... (i)

$=\int_2^8\frac{\sqrt{10-(10-x)}}{\sqrt{10-x}+\sqrt{10-(10-x)}}dx$

$\left(\because \int_a^b f(x)dx={}_{a}^{b}f(a+b-x)dx\right)$

$=\int_2^8\frac{\sqrt{x}}{\sqrt{10-x}+\sqrt{x}}dx$ ... (ii)

Adding (i) and (ii), we get

$$2I=\int_2^8\frac{\sqrt{10-x}+\sqrt{x}}{\sqrt{x}+\sqrt{10-x}}dx=\int_2^8 1\,dx=[x]_2^8$$

$\Rightarrow\ I=\frac{1}{2}(8-2)=\frac{6}{2}=3$

Hence, both assertion and reason are true and reason is the correct explanation of assertion.

21. Domain of $\tan^{-1}x = (-\infty, \infty)$ and range of $\tan^{-1}x$ is $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$.

22. We have, $f(x) = \begin{cases} x^2, & \text{if } x \geq 1 \\ x, & \text{if } x < 1 \end{cases}$

$$\text{L.H.D.} = \underset{x \to 1^-}{\text{Lt}} \frac{f(x) - f(1)}{x - 1} = \underset{x \to 1}{\text{Lt}} \frac{x^2 - 1}{x - 1}$$

$$= \underset{x \to 1}{\text{Lt}} \frac{(x-1)(x+1)}{x - 1} = \underset{x \to 1}{\text{Lt}} (x+1) = 2$$

$$\text{R.H.D.} = \underset{x \to 1^+}{\text{Lt}} \frac{f(x) - f(1)}{x - 1} = \underset{x \to 1}{\text{Lt}} \frac{x - 1}{x - 1} = 1$$

$\because$ L.H.D. ≠ R.H.D.

$\therefore$ $f(x)$ is not differentiable at $x = 1$

OR

We have, $f(x) = \begin{cases} \frac{\sin^2 \lambda x}{x^2}, & x \neq 0 \\ 1 & x = 0 \end{cases}$ is continuous at $x = 0$

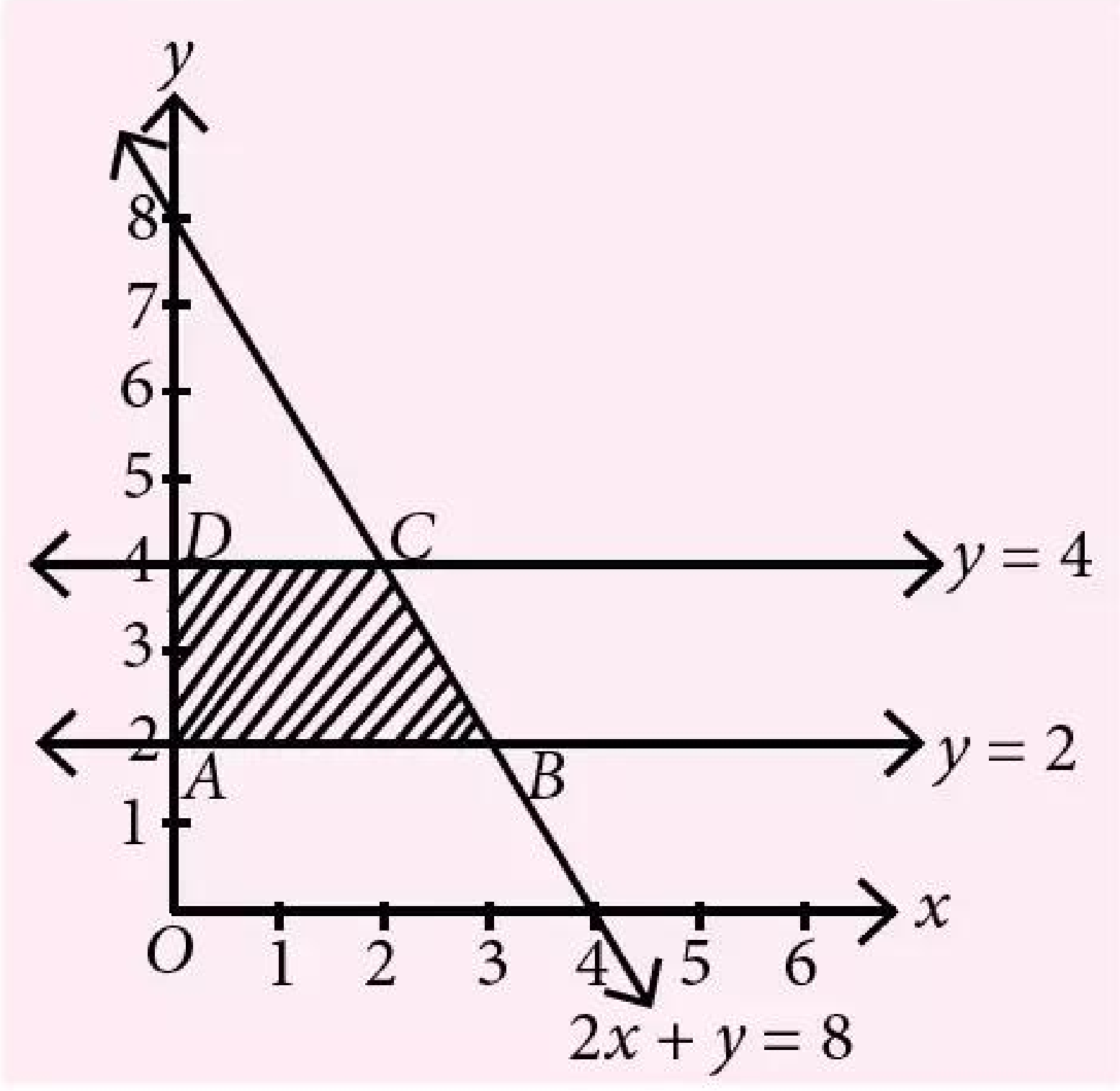
$\therefore$ L.H.L. = R.H.L.

$$\Rightarrow \underset{x \to 0}{\text{Lt}} \frac{\sin^2 \lambda x}{x^2} = 1 \Rightarrow \underset{x \to 0}{\text{Lt}} \frac{\sin^2 \lambda x}{\lambda^2 x^2} \cdot \lambda^2 = 1$$

$$\Rightarrow \lambda^2 \underset{\lambda x \to 0}{\text{Lt}} \left(\frac{\sin \lambda x}{\lambda x}\right)^2 = 1$$

$$\Rightarrow \lambda^2 \cdot 1 = 1 \Rightarrow \lambda^2 = 1 \Rightarrow \lambda = \pm 1$$

23. From the graph, *ABCD* in the required region.



$$\text{Now, Area} = \int_2^4 \left(\frac{8-y}{2}\right) dy = \frac{1}{2}\int_2^4 (8-y)\,dy$$

$$= \frac{1}{2}\left[8y - \frac{y^2}{2}\right]_2^4 = \frac{1}{2}\left[\left(32 - \frac{16}{2}\right) - \left(16 - \frac{4}{2}\right)\right]$$

$$= \frac{1}{2} \times 10 = 5 \text{ sq.units}$$

24. We know that, $|\vec{a} \times \vec{b}| = |\vec{a}|\,|\vec{b}| \sin\theta$

$$\Rightarrow \quad 1 = 3 \times \frac{2}{3} \sin\theta \qquad (\because \vec{a} \times \vec{b} \text{ is a unit vector})$$

$$\Rightarrow \sin\theta = \frac{1}{2} = \sin 30° \quad \Rightarrow \theta = 30°$$

So, angle between $\vec{a}$ and $\vec{b}$ is 30°

OR

$$\text{Area of paralleogram } = \vec{a} \times \vec{b} = \left\| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 3 \\ 2 & -7 & 1 \end{matrix} \right\|$$

$$= |(-1+21)\hat{i} - (1-6)\hat{j} + (-7+2)\hat{k}| = |20\hat{i} + 5\hat{j} - 5\hat{k}|$$

$$= \sqrt{(20)^2 + (5)^2 + (-5)^2}$$

$$= \sqrt{400 + 25 + 25} = \sqrt{450} = 15\sqrt{2} \text{ sq.units}$$

25. Let the equation of line passing through $A(1, 2, -1)$ be $\frac{x-1}{a} = \frac{y-2}{b} = \frac{z+1}{c}$ ...(i)

Now, given equation of line is,

$5x - 25 = 14 - 7y = 35z$

$$\Rightarrow \frac{x-5}{1/5} = \frac{y-2}{-1/7} = \frac{z-0}{1/35}$$

$$\Rightarrow \frac{x-25}{7} = \frac{y-2}{-5} = \frac{z-0}{1} \qquad \text{...(ii)}$$

Since (i) and (ii) are parallel lines.

$$\therefore \frac{a}{7} = \frac{b}{-5} = \frac{c}{1}$$

From (i), we get

$\frac{x-1}{7} = \frac{y-2}{-5} = \frac{z+1}{1}$ is the cartesian equation of line

Also, the equation of line is, $(\hat{i} + 2\hat{j} - \hat{k}) + \lambda(7\hat{i} - 5\hat{j} + \hat{k})$

26. *Refer to solution 30, Pg. 35-36, MTG CBSE Champion, Class 12.*

27. Let $u = \sec^{-1}\left(\frac{1}{\sqrt{1-x^2}}\right)$ ...(i)

and $v = \sin^{-1}(2x\sqrt{1-x^2})$ ...(ii)

Put $x = \sin t$ in (i) and (ii), we get

$$u = \sec^{-1}\left(\frac{1}{\sqrt{1-\sin^2 t}}\right) = \sec^{-1}\left(\frac{1}{\cos t}\right) = \sec^{-1}(\sec t) = t$$

$$\text{and } v = \sin^{-1}(2\sin t\sqrt{1-\sin^2 t}) = \sin^{-1}(\sin 2t) = 2t$$

$$\therefore \frac{du}{dt} = 1 \text{ and } \frac{dv}{dt} = 2 \quad \text{So, } \frac{du}{dv} = \frac{du}{dt} \times \frac{dt}{dv} = \frac{1}{2}$$

OR

Refer to solution 77, Pg. 92-93, MTG CBSE Champion, Class 12.

28. Let $I = \int_0^{2\pi} \frac{1}{1 + e^{\sin x}} dx$ ...(i)

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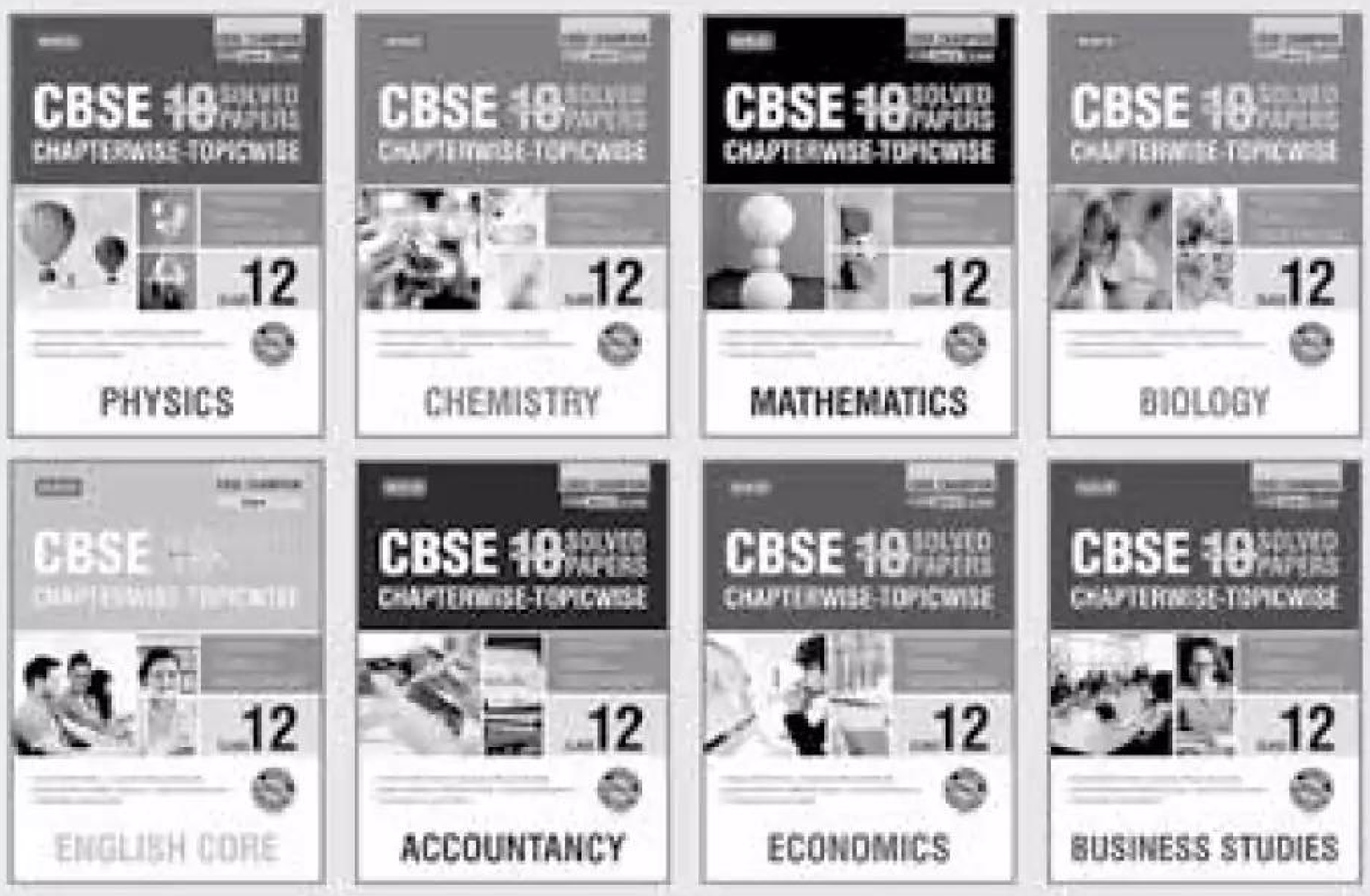
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$$\Rightarrow \quad I=\int_0^{2\pi}\frac{1}{1+e^{\sin(2\pi-x)}}dx \quad \left[\because \int_0^a f(x)dx=\int_0^a f(a-x)dx\right]$$

$$=\int_0^{2\pi}\frac{1}{1+e^{-\sin x}}dx=\int_0^{2\pi}\frac{e^{\sin x}}{e^{\sin x}+1}dx \qquad \text{...(ii)}$$

Adding (i) and (ii), we get

$$2I=\int_0^{2\pi}\left(\frac{1+e^{\sin x}}{1+e^{\sin x}}\right)dx=\int_0^{2\pi}1\cdot dx$$

$$\Rightarrow \quad I=\frac{1}{2}[x]_0^{2\pi}=\frac{1}{2}\times 2\pi=\pi$$

OR

$$\text{Let } I=\int\frac{x^4}{(x-1)(x^2+1)}dx=\int\frac{(x^4-1)+1}{(x-1)(x^2+1)}dx$$

$$=\int\frac{(x^2+1)(x-1)(x+1)+1}{(x-1)(x^2+1)}dx$$

$$=\int\left(x+1+\frac{1}{(x-1)(x^2+1)}\right)dx \qquad \text{...(i)}$$

$$\text{Let } \frac{1}{(x-1)(x^2+1)}=\frac{Ax+B}{x^2+1}+\frac{C}{x-1}$$

$\Rightarrow \quad 1 = (Ax + B)(x – 1) + C(x^2 + 1)$

Comparing the coefficients of x, x^2 and constant terms, we get

$A + C = 0, –A + B = 0, –B + C = 1$

Solving above equations, we get

$$A=\frac{-1}{2},\ B=\frac{-1}{2}, C=\frac{1}{2}$$

$$\therefore \quad \frac{1}{(x-1)(x^2+1)}=\frac{-(x+1)}{2(x^2+1)}+\frac{1}{2(x-1)}$$

So, from (i)

$$I=\int\left[(x+1)-\frac{1}{2}\frac{x}{x^2+1}-\frac{1}{2(x^2+1)}+\frac{1}{2(x-1)}\right]dx$$

$$=\frac{x^2}{2}+x-\frac{1}{4}\log|x^2+1|-\frac{1}{2}\tan^{-1}(x)+\frac{1}{2}\log|x-1|+C$$

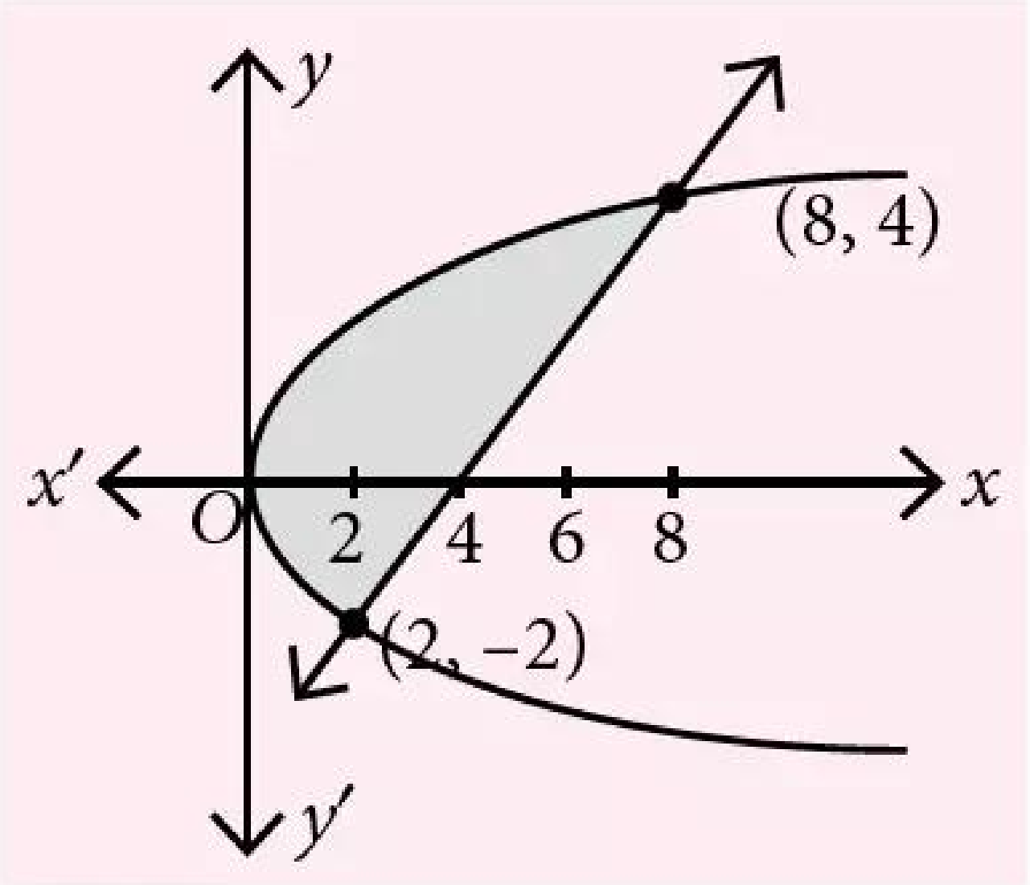
29. We have, $y^2 \le 2x$ ... (i)

and $y \ge x – 4$ ... (ii)

Here, the shaded area represents the required area.

$\therefore$ Required area

$$=\int_{-2}^{4}(y+4)\,dy-\int_{-2}^{4}\frac{y^2}{2}dy$$



$$=\left[\frac{y^2}{2}+4y\right]_{-2}^{4}-\frac{1}{2}\left[\frac{y^3}{3}\right]_{-2}^{4}$$

$$=\left[\left(\frac{16}{2}+16\right)-\left(\frac{4}{2}-8\right)\right]-\frac{1}{2}\left[\frac{64}{3}-\left(\frac{-8}{3}\right)\right]$$

= 30 – 12 = 18 sq. units

30. Let M be the foot of the perpendicular drawn from point $P(0, 2, 3)$ to the line

$$\frac{x+3}{5}=\frac{y-1}{2}=\frac{z+4}{3}=\lambda\,(\text{say}) \qquad \text{... (i)}$$

$\therefore$ Any point on line (i) is $(5\lambda – 3, 2\lambda + 1, 3\lambda – 4)$

So, coordinates of $M \equiv (5\lambda – 3, 2\lambda + 1, 3\lambda – 4)$ ... (ii)

Now, direction ratios of

PM are $\langle 5\lambda – 3 – 0, 2\lambda + 1 – 2, 3\lambda – 4 – 3 \rangle$

i.e., $\langle 5\lambda – 3, 2\lambda – 1, 3\lambda – 7 \rangle$

Since, PM is perpendicular to line (i)

$\therefore \quad 5(5\lambda – 3) + 2(2\lambda – 1) + 3(3\lambda – 7) = 0$

$\Rightarrow \quad 25\lambda – 15 + 4\lambda – 2 + 9\lambda – 21 = 0$

$\Rightarrow \quad 38\lambda – 38 = 0 \Rightarrow \lambda = 1$

So, coordinates of M are $(2, 3, –1)$

OR

We have, $\vec{a}+\vec{b}+\vec{c}=0$

$\Rightarrow \quad \vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=0 \Rightarrow |\vec{a}|^2+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}=0$ ... (i)

Similarly, $\vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b}+\vec{b}\cdot\vec{c}=0$

$\Rightarrow \quad \vec{a}\cdot\vec{b}+|\vec{b}|^2+\vec{b}\cdot\vec{c}=0$ ... (ii)

And, $\vec{c}\cdot\vec{a}+\vec{c}\cdot\vec{b}+\vec{c}\cdot\vec{c}=0$

$\Rightarrow \quad \vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}+|\vec{c}|^2=0$ ... (iii)

Adding (i), (ii) and (iii), we get

$$|\vec{a}|^2+\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}+\vec{a}\cdot\vec{b}+|\vec{b}|^2+\vec{b}\cdot\vec{c}+\vec{a}\cdot\vec{c}+\vec{b}\cdot\vec{c}+|\vec{c}|^2=0$$

$\Rightarrow \quad |\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{a}\cdot\vec{c})=0$

$\Rightarrow \quad (3)^2 + (4)^2 + (2)^2 + 2\mu = 0$

$$\Rightarrow \quad \mu=\frac{-(9+16+4)}{2}=\frac{-29}{2}$$

31. Given lines are

$l_1 : \vec{r}=\hat{i}+2\hat{j}-4\hat{k}+\lambda(2\hat{i}+3\hat{j}+6\hat{k});$

$l_2 : \vec{r}=3\hat{i}+3\hat{j}-5\hat{k}+\mu(4\hat{i}+6\hat{j}+12\hat{k})$

$\therefore$ We have $\vec{a}_1=\hat{i}+2\hat{j}-4\hat{k}, \vec{b}_1=2\hat{i}+3\hat{j}+6\hat{k}$

and $\vec{a}_2=3\hat{i}+3\hat{j}-5\hat{k}, \vec{b}_2=4\hat{i}+6\hat{j}+12\hat{k}$

So, $\vec{a}_2-\vec{a}_1=2\hat{i}+\hat{j}-\hat{k}$

Also, $\vec{b}_2=4\hat{i}+6\hat{j}+12\hat{k}=2\vec{b}_1 \Rightarrow \vec{b}_1 \,||\, \vec{b}_2$

Hence l_1 and l_2 are parallel lines.

Shortest distance between two parallel lines is,

$$d=\left|\frac{\vec{b}\times(\vec{a}_2-\vec{a}_1)}{|\vec{b}|}\right| \Rightarrow d=\left|\frac{(2\hat{i}+3\hat{j}+6\hat{k})\times(2\hat{i}+\hat{j}-\hat{k})}{\sqrt{2^2+3^2+6^2}}\right|$$

$$\Rightarrow d=\left|\frac{-9\hat{i}+14\hat{j}-4\hat{k}}{7}\right|$$

$$\Rightarrow d=\frac{\sqrt{(-9)^2+14^2+(-4)^2}}{7}=\frac{\sqrt{293}}{7}\text{ units.}$$

32. Let the side of an equilateral triangle be $2x$ cm, then its median = $\sqrt{3}x$

Let $M=\sqrt{3}x \Rightarrow \frac{dM}{dt}=\sqrt{3}\frac{dx}{dt}$

$$\Rightarrow 2\sqrt{3}=\sqrt{3}\frac{dx}{dt} \Rightarrow \frac{dx}{dt}=\frac{2\sqrt{3}}{\sqrt{3}}=2\text{ cm/sec}$$

Now, side of triangle = $2x$

$\Rightarrow S = 2x$

$$\Rightarrow \frac{ds}{dt}=2\frac{dx}{dt}=2\times 2\text{ cm/sec}=4\text{ cm/sec}$$

So, side is increasing at the rate of 4 cm/sec.

OR

Refer to solution 39, Pg. 115, MTG CBSE Champion, Class 12.

33. Let $I=\int_0^{\pi/2}\sin 2x\tan^{-1}(\sin x)\,dx$

$$=\int_0^{\pi/2}2\sin x\cos x\cdot\tan^{-1}(\sin x)\,dx$$

Put $\sin x = t \Rightarrow \cos x\,dx = dt$

When $x = 0$, $t = 0$ and $x = \pi/2$, $t = 1$

$$\therefore\ I=\int_0^1 2t\cdot\tan^{-1}t\,dt=2\int_0^1 t\cdot\tan^{-1}t\,dt$$

$$=2\left\{\left[\tan^{-1}t\cdot\frac{t^2}{2}\right]_0^1-\int_0^1\frac{1}{1+t^2}\times\frac{t^2}{2}\,dt\right\}$$

(Integration by parts)

$$=2\left\{\left[\frac{\pi}{4}\cdot\frac{1}{2}-0\right]-\frac{1}{2}\int_0^1\left(\frac{t^2+1}{t^2+1}-\frac{1}{t^2+1}\right)dt\right\}$$

$$=2\left\{\frac{\pi}{8}-\frac{1}{2}\left(\int_0^1 dt-\int_0^1\frac{dt}{t^2+1}\right)\right\}=2\left\{\frac{\pi}{8}-\frac{1}{2}\left([t]_0^1-\left[\tan^{-1}t\right]_0^1\right)\right\}$$

$$=2\left\{\frac{\pi}{8}-\frac{1}{2}\left(1-0-\frac{\pi}{4}+0\right)\right\}=2\left\{\frac{\pi}{8}-\frac{1}{2}+\frac{\pi}{8}\right\}=\frac{\pi}{2}-1$$

34. We have, maximize $P = 70x + 40y$

Subject to : $3x + 2y \le 9$; $3x + y \le 9$; $x \ge 0, y \ge 0$

Convert all inequations into equation, we get

$3x + 2y = 9$ ... (i)

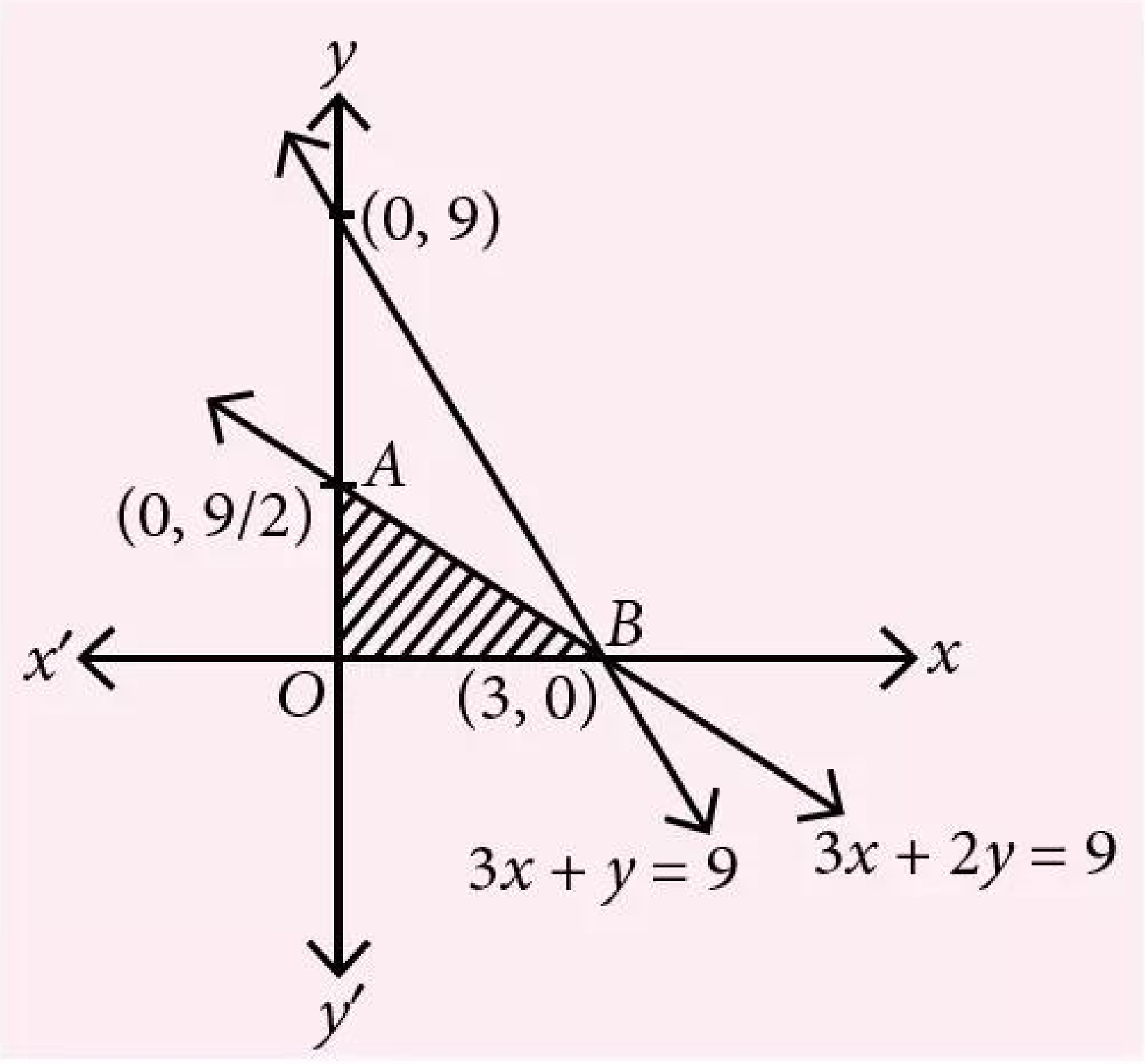
$3x + y = 9$ ... (ii)

$x = 0$ and $y = 0$

Solving (i) and (ii), we get $x = 3, y = 0$

So, point of of equation intersection (i) and (ii) are (3, 0).

The given shaded region is the feasible region.



The corner points of the feasible region are $O(0, 0)$, $A(0, 9/2)$ and $B(3, 0)$.

Corner points	Value of $p = 70x + 40y$
$O(0, 0)$	$70 \times 0 + 40 \times 0 = 0$
$A(0, 9/2)$	$70 \times 0 + 40 \times \frac{9}{2} = 180$
$B(3, 0)$	$70 \times 3 + 40 \times 0 = 210$ (maximum)

So, P is maximum at point $B(3, 0)$.

35. Let E_1, E_2 and A be the events defined as follows :

E_1 : The student knows the answer

E_2 : The student guesses the answer

A : The student answers correctly

We have, $P(E_1)=\frac{3}{5}, P(E_2)=\frac{2}{5}$

Also, $P(A/E_2)=\frac{1}{3}$ and $P(A/E_1) = 1$

$\therefore$ Required probability

$$=P(E_1/A)=\frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)\cdot P(A/E_2)}$$

$$=\frac{\frac{3}{5}\cdot 1}{\frac{3}{5}\cdot 1+\frac{2}{5}\cdot\frac{1}{3}}=\frac{3\times 3}{3\times 3+2}=\frac{9}{11}.$$

OR

Refer to solution 77, Pg. 296, MTG CBSE Champion, Class 12.

36. (I) Here $n(B) = 3$ and $n(G) = 2$

$\therefore$ Number of relation from B to $G = 2^{3\times 2} = 2^6$

(II) Number of functions formed from B to $G = 2^3 = 8$

MONTHLY TEST DRIVE CLASS XII ANSWER KEY

1.	(b)	**2.**	(d)	**3.**	(a)	**4.**	(a)	**5.**	(b)
6.	(d)	**7.**	(c)	**8.**	(a,c)	**9.**	(a,c)	**10.**	(a,d)
11.	(b,d)	**12.**	(a,b)	**13.**	(a,c,d)	**14.**	(b)	**15.**	(c)
16.	(c)	**17.**	(7)	**18.**	(3)	**19.**	(5)	**20.**	(1)

(III) We have, $R = \{(x, y) = x$ and y are students of the same sex$\}$

Set B represents the set of Boys.

$\therefore$ R is reflexive as $(x, x) \in R$.

R is symmetric as $(x, y) \in R \Rightarrow (y, x) \in R$.

Since, $(x, y) \in R$ and $(y, z) \in R \Rightarrow (y, z) \in R$

Hence, R is an equivalence relations.

OR

We have $f : B \to G$ be defined by $f = \{(b_1, g_1), (b_2, g_2), (b_3, g_1)\}$

Since, elements b_1 and b_3 have the same image, therefore, the functions is not one-one but it is many one functions.

Since, to every element in G has its pre-image in B, so the functions is onto.

Hence, the function is surjective.

37. (I) Let the cost of each pen, each bag and each instrument box be x, y and z respectively.

Acording to question, we have

$5x + 3y + z = 160$ ....(i)

$2x + y + 3z = 190$ ...(ii)

$x + 2y + 4z = 250$ ...(iii)

$$\Rightarrow \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 190 \\ 250 \end{bmatrix}$$

i.e., $AX = B$

(II) Now, $|A| = \begin{vmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{vmatrix}$

$= 5(4 - 6) - 3(8 - 3) + 1(4 - 1) = -10 - 15 + 3 = -22$

(III) $\therefore$ $A^{-1} = \dfrac{\text{adj } A}{|A|} = \dfrac{-1}{22}\begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$

$$= \begin{bmatrix} 1/11 & 5/11 & -4/11 \\ 5/22 & -19/22 & 13/22 \\ -3/22 & 7/22 & 1/22 \end{bmatrix}$$

OR

$$A^2 = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 32 & 20 & 18 \\ 15 & 13 & 17 \\ 13 & 13 & 23 \end{bmatrix}$$

$P = A^2 - 5A$

$$= \begin{bmatrix} 32 & 20 & 18 \\ 15 & 13 & 17 \\ 13 & 13 & 23 \end{bmatrix} - \begin{bmatrix} 25 & 15 & 5 \\ 10 & 5 & 15 \\ 5 & 10 & 20 \end{bmatrix} = \begin{bmatrix} 7 & 5 & 13 \\ 5 & 8 & 2 \\ 8 & 3 & 3 \end{bmatrix}$$

38. (I) We have, $(x^2 - y^2)\,dx + 2xy\,dy = 0$

$$\Rightarrow \frac{dy}{dx} = -\frac{(x^2 - y^2)}{2xy} = \frac{y^2 - x^2}{2xy}$$

Now, putting $\dfrac{dy}{dx} = F(x, y)$ and find $F(\lambda x, \lambda y)$,

$$\Rightarrow F(x, y) = \frac{y^2 - x^2}{2xy}$$

$$\therefore F(\lambda x, \lambda y) = \frac{\lambda^2 y^2 - \lambda^2 x^2}{2\lambda^2 xy} = \frac{y^2 - x^2}{2xy} = F(x, y)$$

So, $F(x, y)$ is a homogeneous function and the given differential equation is of the type $g\left(\dfrac{y}{x}\right)$.

(II) We have, $\dfrac{dy}{dx} = \dfrac{y^2 - x^2}{2xy}$

Put $y = vx \Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$

$$\therefore v + x\frac{dv}{dx} = \frac{v^2x^2 - x^2}{2x \cdot vx} = \frac{v^2 - 1}{2v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v^2 - 1}{2v} - v = \frac{v^2 - 1 - 2v^2}{2v}$$

$$\Rightarrow x\frac{dv}{dx} = -\left(\frac{v^2 + 1}{2v}\right) \Rightarrow \int \frac{2v}{v^2 + 1}\,dv = -\int \frac{dx}{x}$$

$$\Rightarrow \int \frac{2v}{v^2 + 1}\,dv + \int \frac{dx}{x} = \log C$$

$$\Rightarrow \log|v^2 + 1| + \log x = \log C$$

$$\Rightarrow \log\left|\left(\frac{y^2 + x^2}{x^2}\right) \times x\right| = \log C$$

$$\Rightarrow \frac{y^2 + x^2}{x} = C \Rightarrow x^2 + y^2 = Cx$$

is the required general solution.

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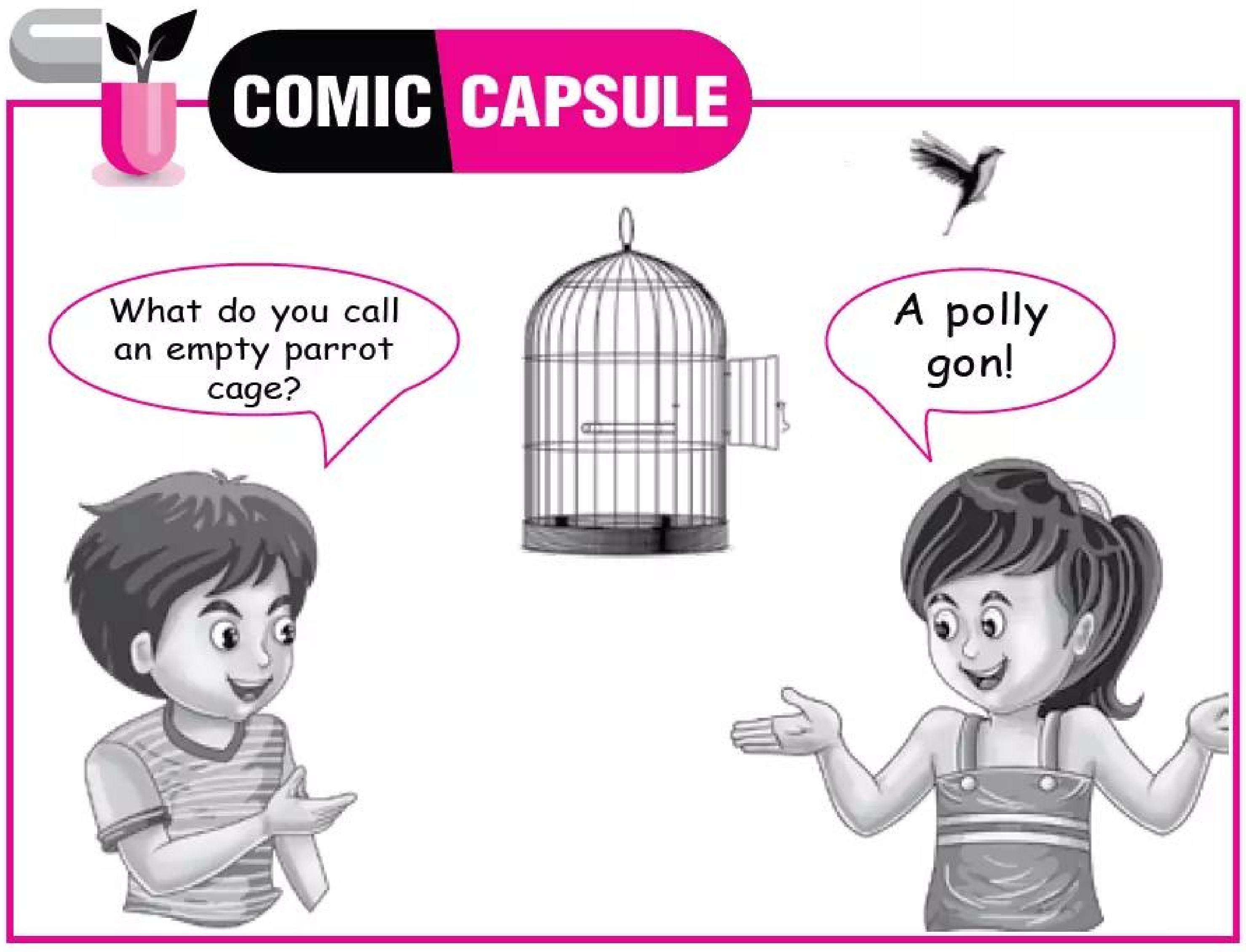
COMIC CAPSULE


What do you call an empty parrot cage?
A polly gon!

GK CORNER

Enhance Your General Knowledge with Current Updates!

SCIENCE AND TECHNOLOGY

Stratospheric Observatory for Infrared Astronomy (SOFIA)

- It is a joint program between NASA and the German Aerospace Centre DLR. In this mission a 2.7 m infrared telescope fly in an aeroplane. It was designed to observe cosmic objects in far-infrared wavelengths.
- SOFIA discovered water molecules (H_2O) on the Sun facing side of Moon.

DNA vaccine against dengue

- Researchers at India's National Centre for Biological Sciences, in collaboration with nine other institutions in India, Africa, and the US, have developed India's first and only **DNA vaccine** candidate for dengue fever.
- Earlier, world's first DNA vaccine—ZyCoV-D was approved in 2021 for emergency use against COVID-19.

VAIBHAV Fellowship

- The VAIBHAV Fellowship is launched by ministry of Science and Technology.
- The fellowship offer NRI researchers to work in higher educational institutions or research institution in the country for a minimum period of one month and maximum period of two months per year.

CHANDRAYAAN-3

- Indian Space Research Organisation (ISRO) successfully conducts key rocket engine test for Chandrayaan-3.
- Chandrayaan-3 (to be launched in later 2023) is a follow-on mission to Chandrayaan-2 to demonstrate end-to-end capability in safe landing and roving on lunar surface. Chandrayaan-2 was launched in 2019, while Chandrayaan-1 was launched in 2008.

MISSIONS TO MOON
Artemis:
United States
Chandrayaan-3:
India
Chang'e Project:
China
Luna-25:
Russia

Hubble Space Telescope (HST)

- NASA's HST has captured, a Jelly fish Galaxy known as J0201 in the Constellation Cetus.
- Hubble Space Telescope is a large, space-based observatory launched in low Earth orbit in 1990 by NASA and ESA (European Space Agency). It has a crystal-clear view of the universe. Scientists have used Hubble to observe some of the most distant stars and galaxies yet seen, as well as the planets in our solar system.

CE-20 Cryogenic Engine

- Indian Space Research Organisation (ISRO) has successfully completed the flight acceptance hot test for the CE-20 cryogenic engine, which will power the LVM3 launch vehicle's Cryogenic Upper Stage for the upcoming Chandrayaan-3 mission.

North East India's first-ever Compressed Biogas Plant

- North East India will now have its first-ever 'Compressed Biogas Plant'. The Chief Minister of Assam inaugurated the plant on February 25, 2023 at Damora Pathar village in Sonapur, Assam.

BharOS Software

- Recently, an IIT Madras-incubated company has developed the BharOS. It is an indigenous mobile Operating System (OS), like Android or iOS. It is focused on privacy and security.

Megha-Tropiques-1 (MT1)

- Recently, the Indian Space Research Organisation (ISRO) has successfully carried out the controlled Re-entry experiment for the decommissioned Megha-Tropiques-1 (MT-1) Satellite. It is an Indo-French Earth Observation Satellite, which was launched in October 2011 for carrying out tropical weather and climate studies.

Planet's New Layer, The Innermost Inner Core

- The researchers, in a new study, have confirmed the existence of a fifth new layer. The four known layers of the Earth include the crust, mantle, outer liquid and inner solid core. The fifth layer "the innermost inner core" lies at the Earth's centre, within the inner core. The fifth layer is made of iron and nickel, the same materials that comprise the rest of the inner core.

Test Yourself!

1. The VAIBHAV Fellowship, launched by ministry of Science and Technology. What is the aim of this scheme?
   (a) To bring Indian-origin researchers to higher educational institutions in the country for a maximum period of two months per year.
   (b) To provide adequate training in market-relevant skills to over 40 crore youth by 2024.
   (c) Development and innovation of products and services and increasing the employment rate in India.
   (d) Facilitating the industry to attain and sustain a pre-eminent global standing in the manufacture and export of clothing.

2. Recently 'BharOS' was in news. Select the correct option regarding it.
   (a) Software that simulates human-like conversations
   (b) Diesel-electric attack submarine
   (c) Mobile operating system designed by IIT Madras.
   (d) Hypersonic cruise missile of India

3. Consider the following statements regarding the Prithvi-II missile:
   I. It is a surface-to-air tactical missile with a strike range of 1500 km.
   II. It is capable of carrying 500 kg to 1000 kg of warheads, both conventional as well as nuclear.
   Which of the statements given above is/are correct?
   (a) I only (b) II only
   (c) Both I and II (d) None of the above

4. Who has launched the Space X Crew-6 mission?
   (a) ISRO (b) NASA
   (c) JAXA (d) ESA

5. India launched the "Initiative on critical and Emerging Technology" with which country?
   (a) Australia (b) France
   (c) USA (d) UK

6. Which layer is responsible for Earth's magnetic field?
   (a) Mantle (b) Crust
   (c) Core (d) Liquid

7. Indian Space Research Organisation (ISRO) conducted CE-20 Cryogenic Engine test successfully recently. What is cryogenic technology?
   (a) Rocket engine that uses solid propellants
   (b) Use of rocket propellants at extremely low temperatures
   (c) Propulsion in which the thrust is provided by the product of a chemical reaction, usually burning (or oxidizing) a fuel
   (d) None of these

8. Hubble Space Telescope was launched in 1990 jointly by
   (a) ISRO and NASA (b) JAXA and NASA
   (c) ESA and ISRO (d) NASA and ESA

9. Which North East state get India's first-ever Compressed Biogas Plant?
   (a) Sikkim (b) Manipur
   (c) Nagaland (d) Assam

10. Which institutes develop India's first prospective DNA vaccine against dengue?
    (a) National Centre for Biological Sciences
    (b) Indian Institute of Technology–Madras
    (c) National Institute of science & Technology
    (d) Indian National Science Academy

Answer Key

1.	(a)	**2.**	(c)	**3.**	(b)	**4.**	(b)	**5.**	(c)
6.	(c)	**7.**	(b)	**8.**	(d)	**9.**	(d)	**10.**	(a)

◆ ◆

beat the TIME TRAP

Duration : 60 minutes

SECTION-I

Single Option Correct Type

1. $ABCD$ is a rhombus, its diagonals AC and BD intersect at the point R where $BD = 2AC$. Its points D and R represent the complex numbers $1 + i$ and $2 - i$ respectively, then the complex number represented by A is
(a) $(3, -1/2)$ or $(1, -1/2)$
(b) $(3, -1/2)$ or $(1, -3/2)$
(c) $(-1/2, -3/2)$ or $(-3/2, -1/2)$
(d) None of these

2. If a and b are roots of $x^2 - P(x + 1) - c = 0$, then the equation whose roots are $(1 + a)(1 + b)$ and $\frac{a^2+2a+1}{a^2+2a+c}+\frac{b^2+2b+1}{b^2+2b+c}$ is
(a) $x^2 + (2 - c)x + (1 + c) = 0$
(b) $x^2 - (2 - c)x + (1 + c) = 0$
(c) $x^2 + (c - 2)x + 1 - c = 0$
(d) None of these

3. The range of $f(x) = {}^{15-x}C_{2x-1} + {}^{20-3x}C_{4x-5}$ is
(a) $[1300, 4224]$ (b) $\{650, 1122\}$
(c) $\{0, 4224\}$ (d) None of these

4. If $\lim_{x\to 0}\frac{1-\cos^n(1-\cos x)}{\tan^m x}=1$, then value of $(m + n)$ equals
(a) 4 (b) 6 (c) 8 (d) 12

5. The angle between the pair of tangents drawn from the point $(2, 4)$ to the circle $x^2 + y^2 = 4$ is
(a) $\tan^{-1}(3/8)$ (b) $\tan^{-1}(4/3)$
(c) 90° (d) None of these

6. If m and M are the least and greatest values of $(\cos^{-1}x)^2 + (\sin^{-1}x)^2$, then $\frac{M}{m} =$
(a) 10 (b) 5 (c) 4 (d) 2

7. Let k and K be the minimum and the maximum values of the function $f(x)=\frac{(1+x)^{0.6}}{1+x^{0.6}}$ in $[0, 1]$ respectively, then the ordered pair (k, K) is equal to
(a) $(1, 2^{0.6})$ (b) $(2^{-0.4}, 2^{0.6})$
(c) $(2^{-0.6}, 1)$ (d) $(2^{-0.4}, 1)$

8. Two adjacent sides of a parallelogram $ABCD$ are given by $\overrightarrow{AB}=2\hat{i}+10\hat{j}+11\hat{k}$ and $\overrightarrow{AD}=-\hat{i}+2\hat{j}+2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then $\cos\alpha =$
(a) $\frac{8}{9}$ (b) $\frac{\sqrt{17}}{9}$ (c) $\frac{1}{9}$ (d) $\frac{4\sqrt{5}}{9}$

9. If the line, $\frac{x-3}{1}=\frac{y+2}{-1}=\frac{z+\lambda}{-2}$ lies in the plane, $2x - 4y + 3z = 2$, then the shortest distance between this line and the line, $\frac{x-1}{12}=\frac{y}{9}=\frac{z}{4}$ is
(a) 0 (b) 3 (c) 1 (d) 2

10. If $f(x) = \int_0^x \frac{dt}{f(t)}$ and $f(1) = \sqrt{2}$, $f(x) \geq 0$, then $\int_0^1 f(x)dx =$
(a) $\frac{2}{3}$ (b) $\frac{\sqrt{2}}{3}$ (c) $\frac{2\sqrt{2}}{3}$ (d) $\frac{1}{3}$

SECTION-II

Numerical Answer Type

11. Normals AO, AA_1, AA_2 are drawn to parabola $y^2 = 8x$ from the point $A(h, 0)$. If triangle OA_1A_2 is equilateral, then the value of h is ________ .

12. If $\sum_{i=1}^{9}(x_i-5)=9$ and $\sum_{i=1}^{9}(x_i-5)^2=45$, then the standard deviation of the 9 items $x_1, x_2, ..., x_9$ is _____ .

13. Let $f(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$,

find k if $f(\theta).f(\phi)=kf(\theta+\phi)$.

14. If

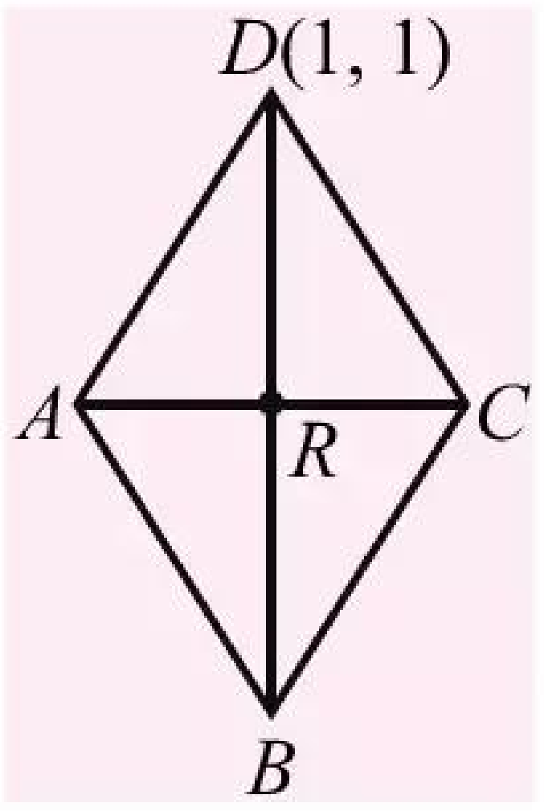
$$\int\frac{(3x^2+2x)\,dx}{x^6+2x^5+x^4+2x^3+2x^2+5}=\frac{1}{A}\tan^{-1}\left(\frac{x^B+x^2+C}{D}\right)+K,$$

then the value of $A+B+C+D$ is_____.

15. A peon has eight master keys to open several new rooms. Only one master key will open any given room. If 40% of these rooms are usually left unlocked, then the probability that the peon gets into specific room if he selects three master keys at random before leaving the office is $\frac{5}{p}$ where p is_____.

SOLUTIONS

1. (b) : Say the affix of point is $A(z)$ or $A(x, y)$



Given, $BD = 2AC \Rightarrow RD = 2AR$

$D(z_1) = 1 + i \therefore D \cong (1, 1)$

$R(z_2) = 2 - i \therefore R \cong (2, -1)$

Also, $DR \perp AR$

$(2AR)^2 = (RD)^2$

$\Rightarrow 4[(x-2)^2 + (y+1)^2]$

$= (1-2)^2 + (1+1)^2$

$\Rightarrow 4[(x-2)^2 + (y+1)^2] = 5$ ...(i)

Again, $m_{AR}\, m_{RD} = -1 \Rightarrow \left(\frac{y+1}{x-2}\right)\left(\frac{1+1}{1-2}\right)=-1$

$\Rightarrow 2(y+1) = x-2$ ...(ii)

By (i) and (ii), we get

$4[4(y+1)^2 + (y+1)^2] = 5$

$\Rightarrow 4\cdot 5\,(y+1)^2 = 5$

$\Rightarrow (y+1)^2 = \frac{1}{4} \Rightarrow y+1 = \pm\frac{1}{2}$

$\Rightarrow y = \pm\frac{1}{2} - 1 \Rightarrow y = -1/2, -3/2$

$\therefore x = 2(y+2)$

By using $y = -1/2, -3/2$, we get $x = 3, 1$

$\therefore A(z) = 3 - \frac{1}{2}i$ or $1 - \frac{3}{2}i$

2. (c) : Given equation is $x^2 - Px - (P+c) = 0$.

As a, b are its roots.

$\therefore a + b = P$ and $ab = -(P+c)$

Now, $(1+a)(1+b) = ab + a + b + 1 = 1 - c$

and $\frac{a^2+2a+1}{a^2+2a+c}+\frac{b^2+2b+1}{b^2+2b+c}$

$$=\frac{(a+1)^2}{(a+1)^2-(1-c)}+\frac{(b+1)^2}{(b+1)^2-(1-c)}$$

$$=\frac{(a+1)^2}{(a+1)^2-(1+a)(1+b)}+\frac{(b+1)^2}{(b+1)^2-(1+a)(1+b)}$$

$$=\frac{a+1}{a-b}+\frac{b+1}{b-a}=1$$

Now, required equation is

x^2 – (sum of roots) x + product of roots = 0

$\Rightarrow x^2 - (1 - c + 1)\,x + (1 - c) = 0$

$\Rightarrow x^2 + (c-2)\,x + (1-c) = 0$

3. (b)

4. (d) : $\lim_{x\to 0}\frac{1-\cos^n(1-\cos x)}{\tan^m x}=1$

$$\Rightarrow \lim_{x\to 0}\left(\frac{1-\cos^n(1-\cos x)}{(1-\cos x)^2}\right)\left(\frac{1-\cos x}{x^2}\right)^2\cdot\left(\frac{x^4}{\tan^m x}\right)=1$$

$$\Rightarrow \lim_{x\to 0}\frac{1-\cos^n(1-\cos x)}{(1-\cos x)^2}\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right)^2\lim_{x\to 0}\left(\frac{x^4}{\tan^m x}\right)=1$$

$$\Rightarrow \lim_{x\to 0}\frac{1-\cos^n(1-\cos x)}{(1-\cos x)^2}\lim_{x\to 0}\left(\frac{2\sin^2\frac{x}{2}}{4\frac{x^2}{4}}\right)^2\lim_{x\to 0}\left(\frac{x^4}{\tan^m x}\right)=1$$

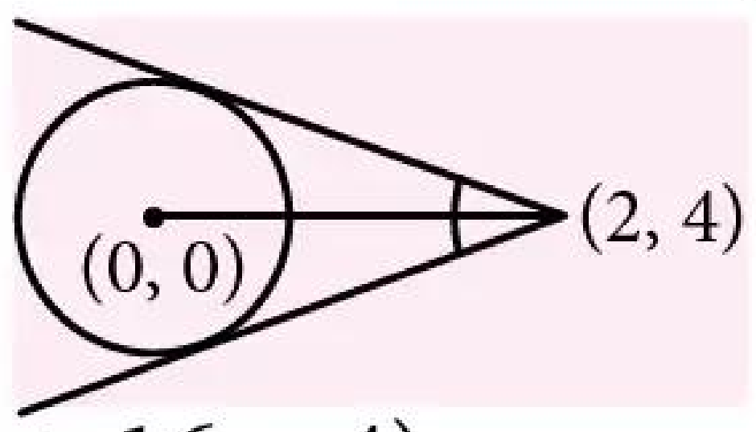
$$\Rightarrow \lim_{x\to 0}\frac{-n\cos^{n-1}(1-\cos x)(-\sin(1-\cos x))(\sin x)}{2(1-\cos x)(\sin x)}$$

$$\cdot\frac{1}{4}\cdot\lim_{x\to 0}\frac{x^4}{\tan^m x}=1$$

$$\Rightarrow \lim_{x\to 0}\frac{n}{2}\cos^{n-1}(1-\cos x)\frac{\sin(1-\cos x)}{(1-\cos x)}\cdot\frac{1}{4}\cdot\lim_{x\to 0}\frac{x^4}{\tan^m x}=1$$

Let $m = 4 \Rightarrow \frac{n}{2}(1)\frac{1}{4}=1 \Rightarrow n = 8 \therefore m + n = 12$

5. (b) : Equation of pair of tangents to a circle is $T^2 = SS_1$



where $T = xx_1 + yy_1 - 4$

$S_1 = {x_1}^2 + {y_1}^2 - 4$

$S = x^2 + y^2 - 4$

$\therefore (2x + 4y - 4)^2 = (x^2 + y^2 - 4)(4 + 16 - 4)$

$\Rightarrow 4(x + 2y - 2)^2 = 16(x^2 + y^2 - 4)$

$\Rightarrow x^2 + 4y^2 + 4xy - 8y - 4x + 4 = 4x^2 + 4y^2 - 16$

$\Rightarrow 3x^2 - 4xy + 8y + 4x - 20 = 0$

$= ax^2 + 2hxy + by^2 + 2gx + 2fy + c$

$\therefore \tan\theta = \frac{2\sqrt{h^2-ab}}{a+b}=\frac{2\times 2}{3} \Rightarrow \theta = \tan^{-1}(4/3)$

6. (a) : $(\cos^{-1} x)^2 + (\sin^{-1} x)^2$

$= (\cos^{-1} x + \sin^{-1} x)^2 - 2 \sin^{-1} x \cos^{-1} x$

$$=\frac{\pi^2}{4}-2\sin^{-1}x\left(\frac{\pi}{2}-\sin^{-1}x\right)$$

$$=2\left[(\sin^{-1}x)^2-\frac{\pi}{2}\sin^{-1}x+\frac{\pi^2}{8}\right]$$

$$=2\left[\left(\sin^{-1}x-\frac{\pi}{4}\right)^2+\frac{\pi^2}{16}\right]=f(x)$$

$$m=f\left(\frac{1}{\sqrt{2}}\right)=\frac{\pi^2}{8}$$

$$M=f(-1)=2\left[\left(-\frac{\pi}{2}-\frac{\pi}{4}\right)^2+\frac{\pi^2}{16}\right]=\frac{5\pi^2}{4}$$

$\therefore \quad \dfrac{M}{m} = 10$

7. (d) : Let $f(x)=\dfrac{(1+x)^{3/5}}{1+x^{3/5}}$

When $f'(x) = 0 \quad \Rightarrow \quad x = 1$

$f(x)$ is strictly decreasing in $(0, 1)$.

Also, $f(0)=1$, and $f(1)=\dfrac{2^{0.6}}{2}=2^{-0.4}$

$\therefore \quad f(x) \in (2^{-0.4}, 1)$

8. (b)

9. (a) : Point $(3, -2, -\lambda)$ lies on plane $2x - 4y + 3z - 2 = 0$

$\therefore \quad 6 + 8 - 3\lambda - 2 = 0 \Rightarrow \quad 3\lambda = 12 \Rightarrow \lambda = 4$

Now, $\dfrac{x-3}{1}=\dfrac{y+2}{-1}=\dfrac{z+4}{-2}=k_1$ (say) ...(i)

$\dfrac{x-1}{12}=\dfrac{y}{9}=\dfrac{z}{4}=k_2$ (say) ...(ii)

Point on first line is $(k_1 + 3, -k_1 - 2, -2k_1 - 4)$

Point on second line is $(12k_2 + 1, 9k_2, 4k_2)$

$\therefore \quad k_1 + 3 = 12k_2 + 1; -k_1 - 2 = 9k_2; -2k_1 - 4 = 4k_2$

On solving these equations, we get

$k_2 = 0$ and $k_1 = -2$

$\therefore$ Point $(1, 0, 0)$ lies on both lines.

So, given lines intersect each other.

$\therefore$ Shortest distance = 0

10. (c) : $f(x) = \displaystyle\int_0^x \frac{dt}{f(t)} \Rightarrow f'(x) = \frac{1}{f(x)}$,

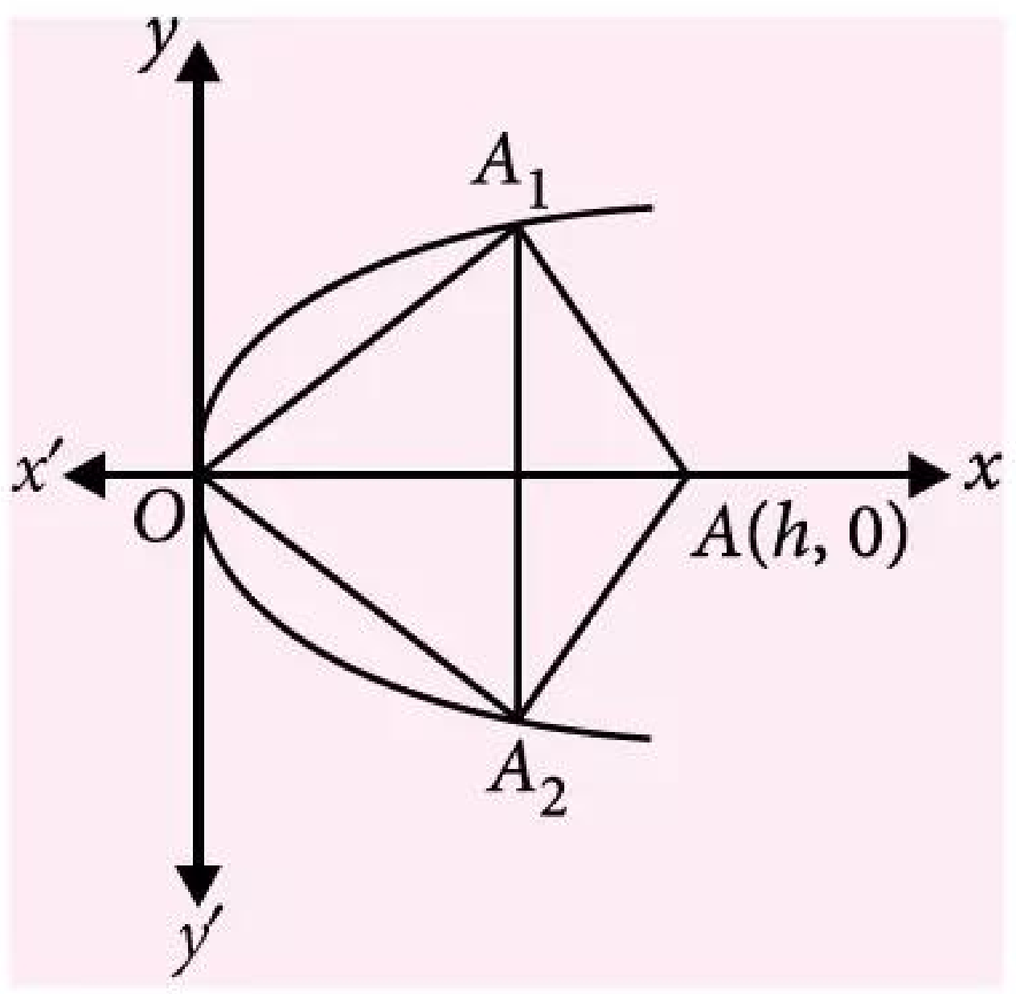
$ff' = 1, fdf = dx.$

Integrating, $f^2 = 2x + c, f(1) = \sqrt{2} \Rightarrow c = 0$

$\therefore \quad f(x)=\sqrt{2x} \Rightarrow \displaystyle\int_0^1 f(x)dx=\frac{2\sqrt{2}}{3}$

11. (28) : Let $A_1 \equiv (2t_1^2, 4t_1)$, $A_2 \equiv (2t_1^2, -4t_1)$

Clearly, $\angle A_1OA = \pi/6$



$$\Rightarrow \frac{2}{t_1}=\frac{1}{\sqrt{3}} \Rightarrow t_1=2\sqrt{3}$$

Equation of normal at A_1 is

$y = -t_1x + 4t_1 + 2t_1^3$

$\Rightarrow h = 4 + 2t_1^2$

$= 4 + 2 \times 4 \times 3 = 28$

12. (2) : $\Sigma x_i - 45 = 9 \Rightarrow \quad \Sigma x_i = 54$

Now, $\Sigma x_i^2 - 10 \times 54 + 25 \times 9 = 45 \Rightarrow \Sigma x_i^2 = 360$

$$\therefore \quad \sigma=\sqrt{\frac{360}{9}-\left(\frac{54}{9}\right)^2}=2$$

13. (1) :

$$f(\theta)\cdot f(\phi)=\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\phi & -\sin\phi & 0\\ \sin\phi & \cos\phi & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos(\theta+\phi) & -\sin(\theta+\phi) & 0\\ \sin(\theta+\phi) & \cos(\theta+\phi) & 0\\ 0 & 0 & 1\end{bmatrix}$$

So, $k = 1$.

14. (8) : Let $I=\displaystyle\int\frac{3x^2+2x}{(x^3+x^2)^2+2(x^3+x^2)+5}dx$

Put $x^3 + x^2 = t \quad \Rightarrow \quad (3x^2 + 2x)dx = dt$

So, $I=\dfrac{1}{2}\tan^{-1}\left(\dfrac{x^3+x^2+1}{2}\right)+K$

$\therefore \quad A + B + C + D = 8$

15. (8) : Let A_1 and A_2 denote the event that a specific room is unlocked and locked respectively.

$\therefore \quad P(A_1) = 0.4, P(A_2) = 0.6$

If A denotes the event that the peon gets into a specific room, then

$$P(A/A_1) = 1,\ P(A/A_2) = \frac{^7C_2}{^8C_3}=\frac{3}{8}$$

$\therefore \quad P(A) = P(A_1)\,P(A/A_1) + P(A_2)\,P(A/A_2)$

$$=(0.4)(1)+(0.6)\left(\frac{3}{8}\right)=\frac{4}{10}+\frac{18}{80}=\frac{5}{8}$$

$\therefore \quad p$ is 8.

* * *

UNSCRAMBLED WORDS

MARCH 2023

1-e- RECIPROCAL
2-j- PERCENTILE
3-h- INVARIANT
4-c- OBLIQUE
5-a- REGRESSION
6-g- INTERVAL
7-i- MULTIVARIABLE
8-b- COVARIANCE
9-d- QUOTIENT
10-f-SUPERSET

QUANTITATIVE APTITUDE

For Various Competitive Exams

1. Two friends P and Q started a business investing in the ratio of 5 : 6. R joined them after six months investing an amount equal to that of Q's. At the end of the year, 20% profit was earned which was equal to ₹ 98000. What was the amount invested by R?

(a) ₹ 105000 (b) ₹ 175000
(c) ₹ 210000 (d) Data inadequate

2. 24 mango trees, 40 banana trees and 72 trees of oranges are planted in row so that each row consisting the same number of trees of one variety only. The minimum number of rows in which the above trees may be planted is

(a) 12 (b) 17 (c) 4 (d) 60

3. Mahavir purchase 1000 articles at the rate ₹ 5 each and sold 850 articles at the rate ₹ 7.00 each and rest articles at the rate ₹ 3.50 each. Find the average profit per article sold.

(a) ₹ 1.50 (b) ₹ 2.47 (c) ₹ 1.47 (d) ₹ 1.75

4. 10 years before, the ratio of ages of A and B was 13 : 17. 17 years from now, the ratio of their ages will be 10 : 11. The present age of B is

(a) 23 years (b) 27 years (c) 40 years (d) 44 years

5. A person spends 30% of monthly salary on rent, 25% on food, 20% on children education, 12% on electricity and deposits remaining ₹ 1040 in the bank, what is the monthly salary of the person?

(a) ₹ 10600 (b) ₹ 9600 (c) ₹ 9000 (d) ₹ 8000

6. If the goods be purchased for ₹ 672 and $\left(\frac{1}{4}\right)^{\text{th}}$ be sold at 20% loss, at what gain% should sell the remaining part so as to make an overall profit of 15% on the total transaction?

(a) $\frac{80}{3}\%$ (b) $\frac{70}{3}\%$ (c) 17.5% (d) $\frac{50}{3}\%$

7. A man buys two horses for ₹ 1280. He sells one so as to gain 27% and other so as to loss 21%. What is cost of the horses if there is no profit and no loss in the whole transaction?

(a) ₹ 680, ₹ 600 (b) ₹ 700, ₹ 580
(c) ₹ 560, ₹ 720 (d) ₹ 630, ₹ 650

8. Ram and Shyam working separately can do the work in 10 days and 12 days respectively. If they working together for 1 day alternatively, Ram start the work, in how many days will the work be completed?

(a) $10\frac{2}{3}$ days (b) $10\frac{5}{6}$ days
(c) $5\frac{5}{6}$ days (d) None of these

9. Two pipes A and B can separately fill a tank in 12 minutes and 15 minutes respectively. But 4 minutes after the starts, pipe A is turned off. How much time will it take to fill the tank?

(a) 5 minutes (b) 4 minutes
(c) 8 minutes (d) 6 minutes

10. A plane P take off, from Hyderabad airport at 4 a.m. and reaches Delhi Palam airport at 6 a.m. Another plane Q takes off from Delhi Palam airport at 5 a.m. and reaches Hyderabad airport at 6:20 a.m. At what time do the two planes cross each other?

(a) 6:24 a.m. (b) 5:24 a.m.
(c) 4:24 a.m. (d) None of these

11. The simple interest on a sum of money will be ₹ 152 after 5 years. In next 5 years principal becomes 7 times, what will be the total interest at the end of 10 years?

(a) ₹ 1672 (b) ₹ 1520 (c) ₹ 304 (d) ₹ 1216

12. A sum invested for 3 years compounded at 5%, 10% and 20% respectively. In these three years sum amounts to ₹ 16632, then find the sum.

(a) ₹ 12000 (b) ₹ 11000 (c) ₹ 15000 (d) ₹ 9000

13. Find the value of $\log_5 \sqrt{5\sqrt{5\sqrt{5........\infty}}}$.

(a) 0 (b) 1 (c) 5 (d) infinite

14. Four equal sized maximum circular plates are cut off from a square sheet of iron of area 784 cm^2. The circumference of each plate is

(a) 44 cm (b) 22 cm (c) 176 cm (d) 88 cm

15. The diagonals of three faces of a cuboid are x, y, z respectively, what is the volume of the cuboid?

(a) $\dfrac{\sqrt{(x^2+y^2)(y^2+z^2)(z^2+x^2)}}{2\sqrt{2}}$

(b) $\dfrac{x^2y^2z^2}{2\sqrt{2}}$

(c) $\dfrac{\sqrt{(x^2+y^2-z^2)(y^2+z^2-x^2)(x^2+z^2-y^2)}}{2\sqrt{2}}$

(d) None of these

16. In a tea party there are 16 guests and arrange themselves along two sides of a long table with 8 chairs on each side. Three guests are close friends and wish to sit on one particular side and from the remaining guests 2 on the other side. In how many ways they can be arrange themselves?

(a) ${}^{16}P_3 \times {}^{16}P_2 \times 11!$ (b) $2! \times 16!$

(c) ${}^{8}P_3 \times {}^{8}P_2 \times 11!$ (d) $2! \times 8! \times 8!$

17. The odd against of an event is 3 : 2 and odds in favour of another independent event is 3 : 5. What is the probability that at least one of the event must occur?

(a) $\frac{9}{64}$ (b) $\frac{25}{64}$ (c) $\frac{1}{2}$ (d) $\frac{5}{8}$

18. If θ be a positive acute angle satisfying $\cos^2\theta + \cos^4\theta = 1$, then the value of $\tan^4\theta + \tan^2\theta$ equals

(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\frac{3}{2}$

19. If $t = \dfrac{2\sin\theta}{1-\cos\theta+\sin\theta}$, then $\dfrac{1+\cos\theta+\sin\theta}{1+\sin\theta}$ equals

(a) $1+t$ (b) t (c) $\frac{1}{t}$ (d) $1-t$

20. If $x = \dfrac{2ab}{a+b}\,(a \neq b)$, then $\dfrac{x+a}{x-a} + \dfrac{x+b}{x-b}$ equals

(a) 8 (b) 6 (c) 2 (d) 4

SOLUTIONS

1. (c) : Let the initial investment be ₹ x.
Then, 20% of x = 98000

$$\Leftrightarrow x = \left(\frac{98000 \times 100}{20}\right) = 490000.$$

Let the capitals of P, Q and R be ₹ $5x$, ₹ $6x$ and ₹ $6x$ respectively. Then,

$(5x \times 12) + (6x \times 12) + (6x \times 6) = 490000 \times 12$

$\Leftrightarrow 168x = 490000 \times 12$

$$\Leftrightarrow x = \left(\frac{490000 \times 12}{168}\right) = ₹\,35000.$$

∴ R's investment = $6x = 6 \times 35000$ = ₹ 210000.

2. (b) : For minimum number of rows, the number of trees in each row must be maximum.

∴ Number of trees in each row
= H.C.F of (24, 40, 72) = 8

∴ Number of rows = $\dfrac{24+40+72}{8} = 17$

3. (c) : C.P. of 1000 articles = ₹ 5 × 1000 = ₹ 5000
S.P. of 850 articles = ₹ 7 × 850 = ₹ 5950
S.P. of 150 articles = ₹ 3.50 × 150 = ₹ 525
∴ Total S.P. of 1000 articles = ₹ 6475
∴ Profit = ₹(6475 – 5000) = ₹ 1475

∴ Profit per article $= \dfrac{1475}{1000} = ₹\,1.47$

4. (b) : In the problem $x = 13k$, $y = 17k$
(as 10 years ago the ratio of ages was 13 : 17)
∴ Present age of $A = 13k + 10$
and present age of $B = 17k + 10$

∴ According to problem, we have $\dfrac{13k+10+17}{17k+10+17} = \dfrac{10}{11}$

(as after 17 years the ratio of their ages is 10 : 11)

$$\Rightarrow \frac{13k+27}{17k+27} = \frac{10}{11} \Rightarrow 27k = 27 \Rightarrow k = 1$$

∴ Present age of $B = 17k + 10 = 17(1) + 10 = 27$ years

SAMURAI SUDOKU

ANSWER · MARCH 2023

9	6	5	8	7	3	1	4	2				9	6	8	4	5	1	3	2	7
1	3	7	6	4	2	8	9	5				3	5	2	8	6	7	4	9	1
8	2	4	5	1	9	6	7	3				7	1	4	2	3	9	6	5	8
6	5	8	7	3	1	4	2	9				6	2	5	7	1	4	9	8	3
4	7	2	9	5	6	3	1	8				1	8	9	3	2	6	7	4	5
3	1	9	4	2	8	7	5	6				4	3	7	5	9	8	2	1	6
2	9	1	3	6	7	5	8	4	6	1	9	2	7	3	9	8	5	1	6	4
7	4	6	2	8	5	9	3	1	8	2	7	5	4	6	1	7	2	8	3	9
5	8	3	1	9	4	2	6	7	3	5	4	8	9	1	6	4	3	5	7	2
						4	5	8	9	3	2	1	6	7						
						6	1	2	7	4	8	3	5	9						
						7	9	3	1	6	5	4	2	8						
9	3	2	8	4	6	1	7	5	4	8	6	9	3	2	4	5	6	8	7	1
7	5	4	3	9	1	8	2	6	5	9	3	7	1	4	9	8	2	5	6	3
6	8	1	7	2	5	3	4	9	2	7	1	6	8	5	3	1	7	2	9	4
8	4	9	5	7	2	6	3	1				2	4	6	1	7	9	3	5	8
3	1	7	6	8	9	2	5	4				3	5	1	6	4	8	7	2	9
2	6	5	4	1	3	9	8	7				8	7	9	5	2	3	4	1	6
1	2	8	9	5	4	7	6	3				5	9	8	2	6	4	1	3	7
5	9	6	2	3	7	4	1	8				1	6	7	8	3	5	9	4	2
4	7	3	1	6	8	5	9	2				4	2	3	7	9	1	6	8	5

5. (d) : Let the monthly income be ₹ x.
According to problem, we have
[100 – (30 + 25 + 20 + 12)]% of x = 1040
$\Rightarrow$ [100 – (87)]% of x = 1040
$\Rightarrow$ $13 \times x = 1040 \times 100$ $\Rightarrow$ $x = 8000$

6. (a) : Cost price of goods = ₹ 672

Cost price of $\frac{1}{4}$ of goods = ₹ $\frac{672}{4}$ = ₹168

and cost price of $\frac{3}{4}$ of goods = ₹ (672 – 168) = ₹ 504

Now, S.P. of $\frac{1}{4}$ of goods (sold at 20% loss)

= ₹ $\frac{168 \times 80}{100}$ = ₹ $\frac{672}{5}$

and total selling price = ₹ $\frac{672 \times 115}{100}$ = ₹ $\frac{3864}{5}$

∴ S.P. of $\frac{3}{4}$ of goods = ₹ $\left(\frac{3864}{5} - \frac{672}{5}\right)$ = ₹ $\frac{3192}{5}$

But cost price of $\frac{3}{4}$ of goods is ₹ 504

∴ Profit on $\frac{3}{4}$ of goods = ₹ $\left(\frac{3192}{5} - 504\right)$ = ₹ $\frac{672}{5}$

∴ Required profit % on $\frac{3}{4}$ of goods

$$= \frac{\text{Profit on 3/4 of goods}}{\text{C.P. of 3/4 of goods}} \times 100 = \frac{672}{5 \times 504} \times 100 = \frac{80}{3}\%$$

7. (c) : Here, there is no profit no loss
∴ Loss on one horse = gain on the other
$\Rightarrow$ 21% of the C.P. of one horse = 27% of the C.P. of other
$\Rightarrow$ Ratio of their cost prices = 21 : 27 or 27 : 21
$\Rightarrow$ Ratio of their cost prices = 7 : 9

∴ C.P. of one horse = $\frac{7}{16} \times 1280$ = ₹ 560

C.P. of other horse = $\frac{9}{16} \times 1280$ = ₹ 720

∴ Cost of horses (₹ 560, ₹ 720) or (₹ 720, ₹ 560)

8. (b) : Ram's and Shyam's 2 days work = $\frac{1}{10} + \frac{1}{12} = \frac{11}{60}$
(As they work alternatively)
As both work alternatively, so both works 5 days and rest work done by Ram.
∴ Total 10 days work
= 5 × two days work of Ram and Shyam = $5 \times \frac{11}{60} = \frac{11}{12}$

∴ Remaining work = $1 - \frac{11}{12} = \frac{1}{12}$

Now $\frac{1}{10}$ of the work done by Ram in 1 day

1 of the work done by Ram in 10 days

$\frac{1}{12}$ of the work done by Ram in $10 \times \frac{1}{12} = \frac{5}{6}$ days

∴ The work will finished in $10 + \frac{5}{6} = 10\frac{5}{6}$ days

9. (d) : Work done by pipe A in 1 minute = $\frac{1}{12}$

Work done by pipe B in 1 minute = $\frac{1}{15}$

Work done by (A + B) in 1 minute = = $\frac{1}{12} + \frac{1}{15}$

Work done by (A + B) in 4 minutes = $4\left(\frac{1}{12} + \frac{1}{15}\right) = \frac{3}{5}$

So unfilled part of the tank = $1 - \frac{3}{5} = \frac{2}{5}$, which is to be filled by pipe B

Now to fill $\frac{1}{15}$ part of tank by pipe B, time taken = 1 minute
∴ To fill 1 part of tank by pipe B, time taken
= 15 minutes

Time taken to fill $\frac{2}{5}$ part of tank by pipe B

$= \frac{15 \times 2}{5} = 6$ minutes

10. (b) : Let the distance between Hyderabad and Palam be x km.
And let the planes meet y hours after 5 a.m.
Clearly plane P covers x km in 2 hours

And plane Q covers x km in 1 h 20 mins, *i.e.*, $\frac{4}{3}$ h

∴ Speed of plane $P = \frac{x}{2}$ km/h

Speed of plane $Q = \frac{3x}{4}$ km/h

Now distance covered by P in (y + 1) hrs
+ Distance covered by Q in y hrs = x km

∴ $\frac{x(y+1)}{2} + \frac{3x}{4} \times y = x$

$\Rightarrow$ $2(y + 1) + 3y = 4$ $\Rightarrow$ $5y = 2$

$\Rightarrow$ $y = \frac{2}{5}$ hr $= \frac{2}{5} \times 60$ mins = 24 mins

∴ Planes meet each other at 5 a.m. + 24 mins = 5:24 a.m.

11. (d) : Simple interest $= \frac{P \times r \times 5}{100} = 152$

∴ $5Pr = 152 \times 100$
Again, for next 5 years principal becomes 7 times at the same rate.

$\therefore$ Simple interest $=\frac{7P\times r\times 5}{100}=\frac{5Pr\times 7}{100}$

$= 7 \times 152 = \text{₹}\ 1064$

$\therefore$ Total interest after 10 years = ₹ (152 + 1064)

= ₹ 1216

12. (a) : Let the sum invested be ₹ P_0, which amounts to ₹ 16632 at the rate 5%, 10%, 20% respectively.

$$\therefore\ P_0\left(1+\frac{5}{100}\right)\left(1+\frac{10}{100}\right)\left(1+\frac{20}{100}\right)=16632$$

$$\Rightarrow\ P_0\left(\frac{21}{20}\times\frac{11}{10}\times\frac{6}{5}\right)=16632$$

$$\Rightarrow\ P_0=\frac{16632\times 20\times 10\times 5}{21\times 11\times 6}=\text{₹}12000$$

13. (b) : $\log_5\left(\sqrt{5\sqrt{5\sqrt{5\ldots\ \infty}}}\right)$

$$=\log_5 5^{\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots\infty\right)}=\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots\ \infty=\frac{1/2}{1-\frac{1}{2}}=1$$

14. (a) : Area of iron sheet = 784 cm^2

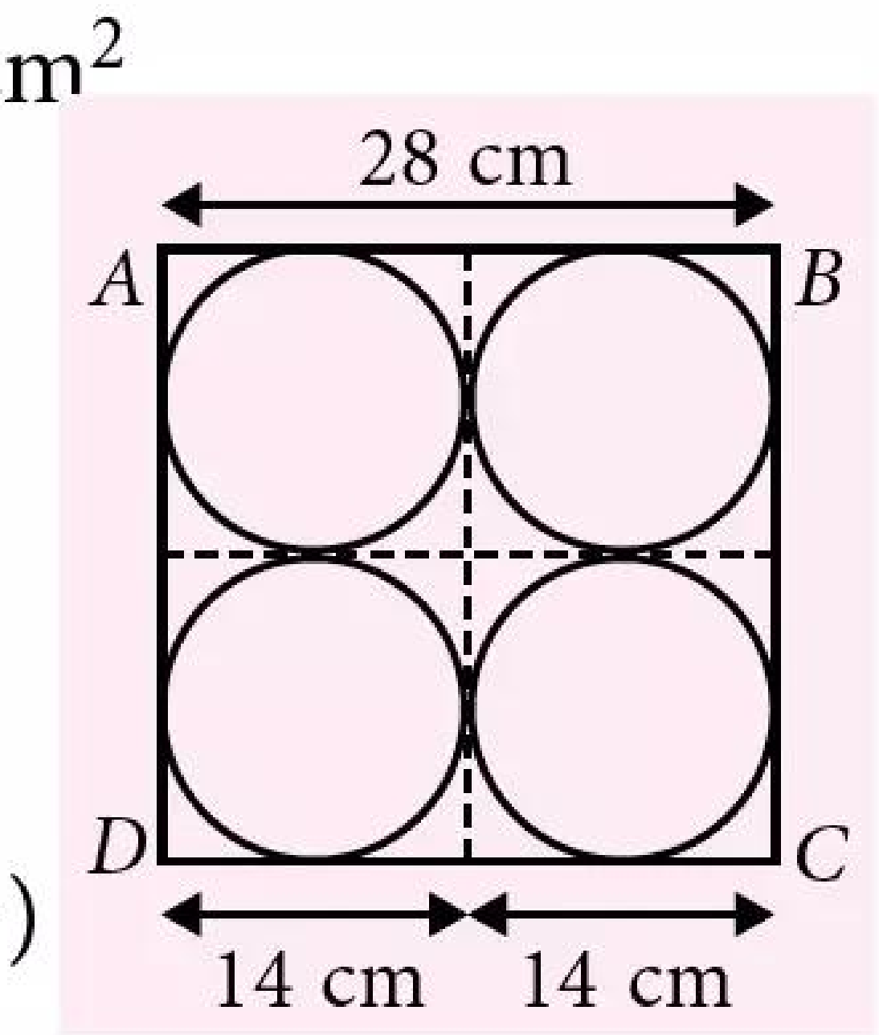
$\therefore$ Side of iron sheet

$=\sqrt{784}\text{ cm}=28\text{ cm}$

Diameter of largest circle

$=\frac{28}{2}\text{cm}=14\text{ cm}$

$\Rightarrow$ Radius $=\frac{14}{2}\text{cm}=7\text{cm}\ (=r\text{ say})$



$\therefore$ Circumference of each plate = $2\pi r$

$$=2\times\frac{22}{7}\times 7=44\text{ cm}$$

15. (c) : Let l, m, n be the sides (edges) of the cuboids whose diagonals are x, y and z respectively.

$\therefore\ l^2 + m^2 = x^2$ ...(1)

$m^2 + n^2 = y^2$ ...(2)

$n^2 + l^2 = z^2$ ...(3)

On adding (1), (2) and (3), we have

$2(l^2 + m^2 + n^2) = x^2 + y^2 + z^2$

or $l^2 + m^2 + n^2 = \frac{1}{2}(x^2+y^2+z^2)$ ...(4)

Now from (1) and (4), we have

$$x^2+n^2=\frac{1}{2}(x^2+y^2+z^2)$$

$$\Rightarrow\ n^2=\frac{1}{2}(y^2+z^2-x^2)\Rightarrow n=\sqrt{\frac{y^2+z^2-x^2}{2}}$$

Similarly $l=\sqrt{\frac{(x^2+z^2-y^2)}{2}}$ and $m=\sqrt{\frac{x^2+y^2-z^2}{2}}$

Now, volume of cube = lmn

$$=\sqrt{\frac{x^2+z^2-y^2}{2}}\times\sqrt{\frac{x^2+y^2-z^2}{2}}\times\sqrt{\frac{y^2+z^2-x^2}{2}}$$

$$=\frac{\sqrt{(x^2+y^2-z^2)(y^2+z^2-x^2)(z^2+x^2-y^2)}}{2\sqrt{2}}$$

16. (c) : Total guests = 16

Let the two sides be A and B. Assume that 3 guests wish to sit on side A.

Three guests who wish to sit on side A can accommodated on 8 chairs by ${}^8P_3$ ways and two guests who wish to sit on side B can be arranged on 8 chairs in ${}^8P_2$ ways. By doing this 11 guests are remains to arrange themselves on 11 chairs in 11! ways.

$\therefore$ Required ways in which 16 guests can arrange themselves = ${}^8P_3 \times {}^8P_2 \times 11!$

17. (d) : P(atleast one must occurs)

= (1 – none of them will occurs)

$$=(1-P(\bar{A}_1)\times P(\bar{A}_2))=\left(1-\frac{3}{5}\times\frac{5}{8}\right)=\frac{5}{8}$$

18. (c) : Given, $\cos^4\theta + \cos^2\theta = 1$

$\Rightarrow\ \cos^4\theta = 1 - \cos^2\theta = \sin^2\theta\ \Rightarrow\ \frac{\sin^2\theta}{\cos^4\theta}=1$

$\Rightarrow\ \tan^2\theta\sec^2\theta = 1\ \Rightarrow\ \tan^2\theta\,(1 + \tan^2\theta) = 1$

$\Rightarrow\ \tan^2\theta + \tan^4\theta = 1$

19. (b) : $\frac{1+\cos\theta+\sin\theta}{1+\sin\theta}=\frac{(1+\sin\theta)^2-\cos^2\theta}{(1+\sin\theta)(1-\cos\theta+\sin\theta)}$

$$=\frac{(1-\cos^2\theta)+\sin^2\theta+2\sin\theta}{(1+\sin\theta)(1-\cos\theta+\sin\theta)}$$

$$\frac{2\sin\theta(\sin\theta+1)}{(1+\sin\theta)(1-\cos\theta+\sin\theta)}=\frac{2\sin\theta}{1-\cos\theta+\sin\theta}=t$$

20. (c) : Given $x=\frac{2ab}{a+b}\ \Rightarrow\ \frac{x}{a}=\frac{2b}{a+b}$ and $\frac{x}{b}=\frac{2a}{a+b}$

Applying componendo and dividendo in both terms, we get

$$\frac{x+a}{x-a}=\frac{2b+a+b}{2b-a-b}\text{ and }\frac{x+b}{x-b}=\frac{2a+a+b}{2a-a-b}$$

$$\Rightarrow\ \frac{x+a}{x-a}=\frac{3b+a}{b-a}\text{ and }\frac{x+b}{x-b}=\frac{3a+b}{a-b}\qquad\text{...(i)}$$

Now, $\frac{x+a}{x-a}+\frac{x+b}{x-b}=\frac{3b+a}{b-a}+\frac{3a+b}{a-b}$ (Using (i))

$$\Rightarrow\ \frac{x+a}{x-a}+\frac{x+b}{x-b}=\frac{3a+b-3b-a}{a-b}=\frac{2(a-b)}{a-b}=2$$

◆◆

Brain Teaser

LOGICAL REASONING

for Various Competitive Exams

Direction Q.(1 and 2) : What will come in place of '?' in the given series?

1. 12, 22, 69, 272, 1365, ?
(a) 8196 (b) 8184 (c) 8195 (d) 6830

2. AZ, CX, EV, GT, ? , KP, ?
(a) IR and NM (b) RI and MN
(c) RI and NM (d) IR and MN

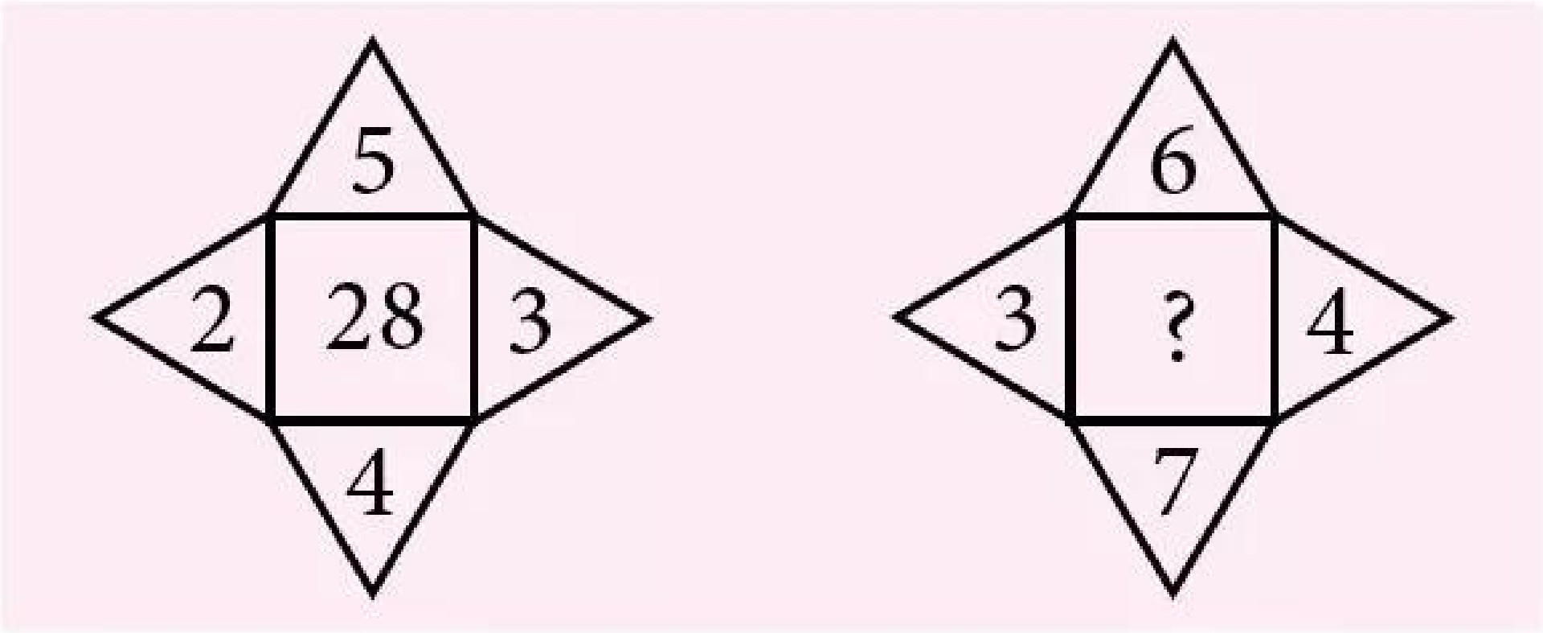
Direction Q. (3 and 4): The two words/numbers on the right of (::) are related in some way as the words/numbers on the left of (::). Identify what will come in place of blank?

3. Cells : Cytology :: Insects : ....
(a) Ornithology (b) Entomology
(c) Physiology (d) Morphology

4. 5 : 75 :: 10 : ....
(a) 100 (b) 200 (c) 300 (d) 400

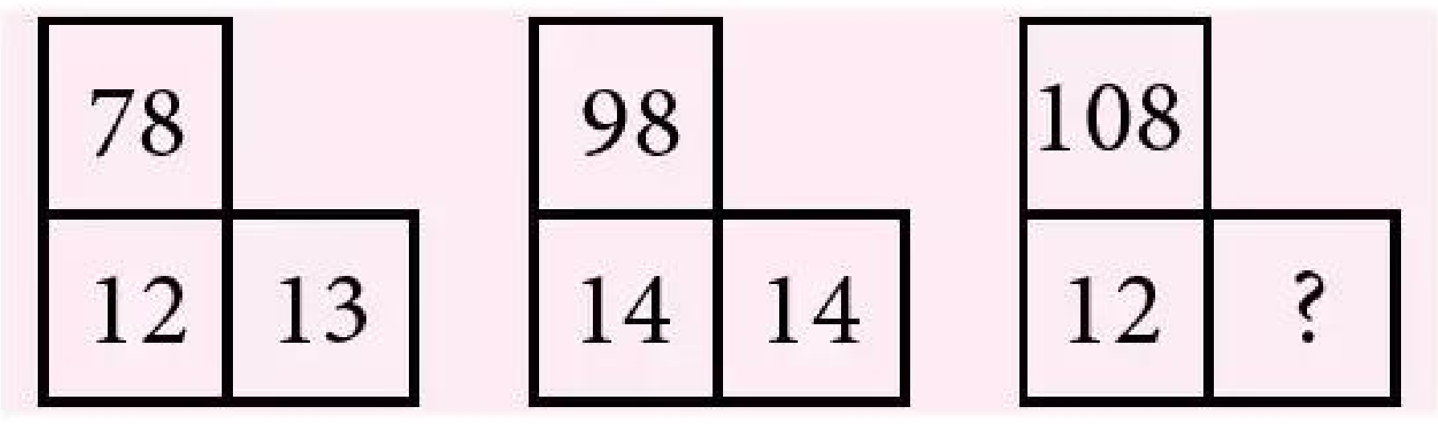
Directions Q. (5 and 6) : In each of the following figures, find the number which replaces the sign of ?

5.



(a) 66 (b) 48 (c) 30 (d) 60

6.



(a) 6 (b) 18 (c) 14 (d) 16

7. In a code language, 'DESCRIBE' is coded as 'FCJSDTFE', then 'CONSIDER' will be coded as
(a) SFEJTOPD (b) SEFJTOPD
(c) QFEJTOPD (d) QEFJTOPD

8. Pointing to a photograph, Arun said, "She is the mother of my brother's son's wife's daughter". How is the lady related to Arun?
(a) Cousin (b) Daughter-in-law
(c) Uncle (d) Brother-in-law

9. Pointing to a man in the photograph, a woman says, "He is the son of my brother's grandfather's only son". How is that man related to the woman?
(a) Brother (b) Uncle
(c) Father (d) Brother-in-law

10. Ram and Sham start to walk in opposite directions. Ram covers 6 km and Sham 8 km. Then Ram turns right and walks 8 km and Sham turns left and walks 6 km. How far each is from the starting point?
(a) 11 km (b) 8 km (c) 9 km (d) 10 km

11. The post office is to the east of the school while my house is to the south of the school. The market is to the north of the post office. If the distance of the market from the post office is equal to the distance of my house from the school, in which direction is the market with respect to my school?
(a) North (b) East
(c) North-east (d) South-west

Direction Q. (12 and 13) In these questions relationships between different elements is shown in the statements. These statements are followed by four conclusions numbered (I), (II), (III) and (IV). Study the statements and conclusions carefully and select the correct answer.

12. Statements: $Y > R \geq S = T < M \leq N < J$

Conclusions:

I. $T < Y$ II. $N > S$ III. $J < S$ IV. $S \geq Y$
(a) Only conclusions (I), II and (III) are true.
(b) Only conclusions (I) and (II) are true.
(c) Only conclusions (II) and (III) are true.
(d) Only conclusions (I), (III) and (IV) are true.

13. Statements: $S > H \geq T \leq C$; $Y > T \geq N$

Conclusions:

I. $H > Y$ II. $S > C$ III. $C = Y$ IV. $C \geq N$
(a) Only conclusions (I) and (IV) are true.
(b) Only conclusion (III) is true.
(c) Only conclusions (II) and (IV) are true.
(d) Only conclusion (IV) is true.

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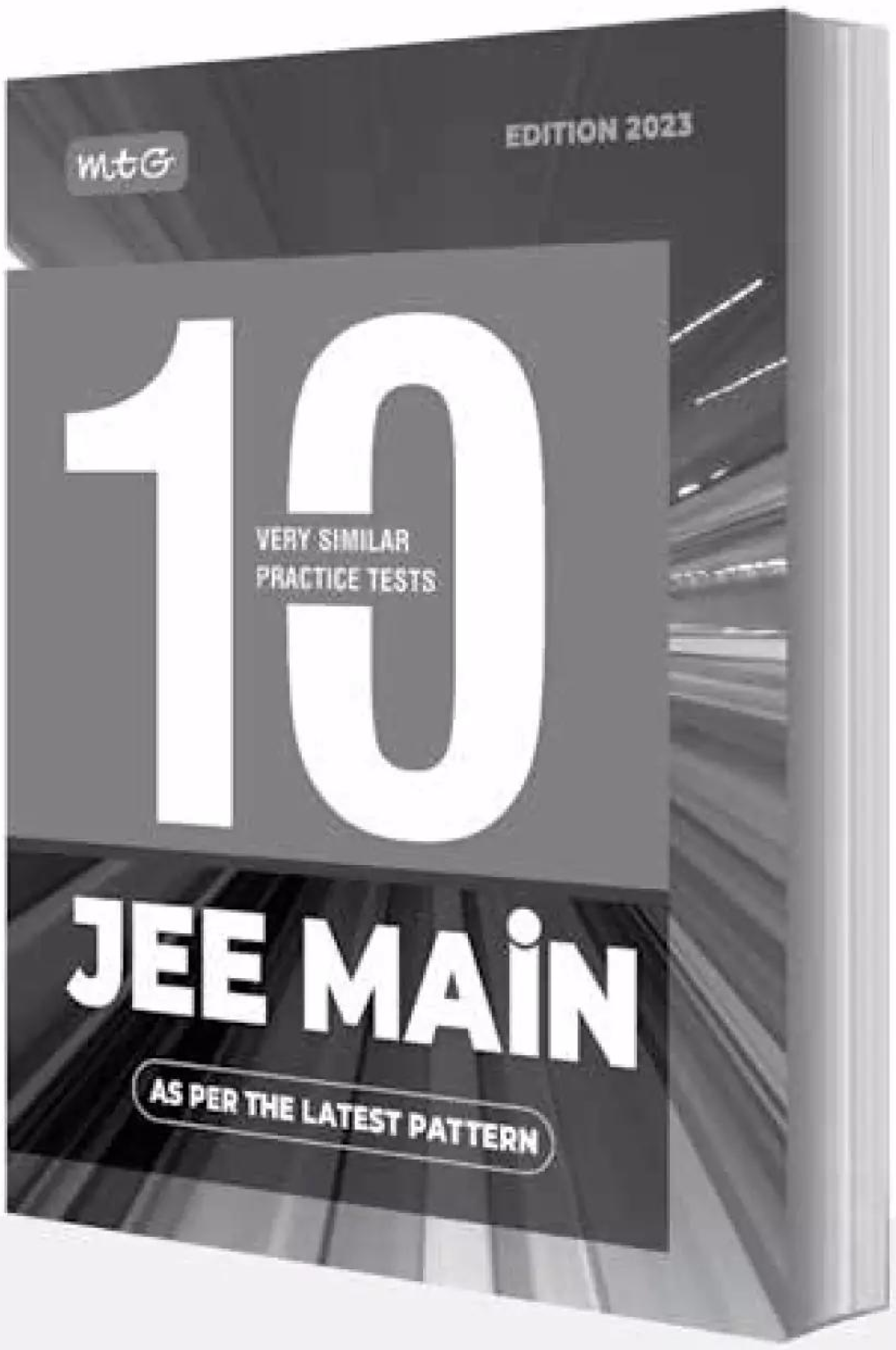
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10
VERY SIMILAR PRACTICE TESTS
JEE MAIN
AS PER THE LATEST PATTERN

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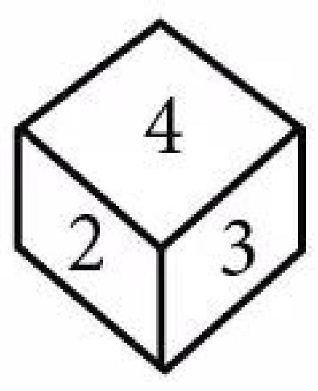
Direction Q. (14 and 15): A cuboid of size 5 cm × 3 cm × 3 cm is painted such that the faces which are square are painted with red colour. While out of remaining faces 2 adjacent are painted yellow while 2 adjacent are painted pink. Now the cube is cut into small cuboid of size 1 cm.

14. How many cubes will have 2 faces painted red?
(a) 0 (b) 8 (c) 12 (d) 16

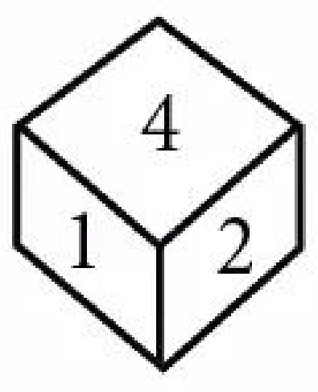
15. How many cubes will have 2 faces yellow?
(a) 5 (b) 8 (c) 16 (d) 20

Directions Q. (16 and 17): Different views of a cube are given below. Study each view and answer the questions given below.

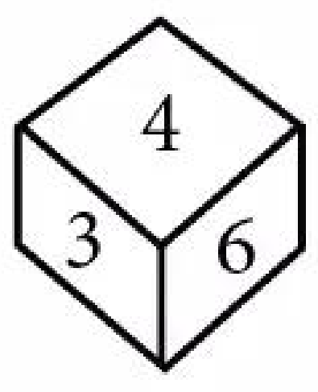
16. What numbers occur at the bottom face in the three positions of the same die?



(i)



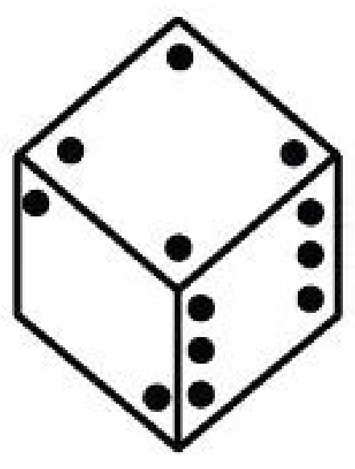
(ii)

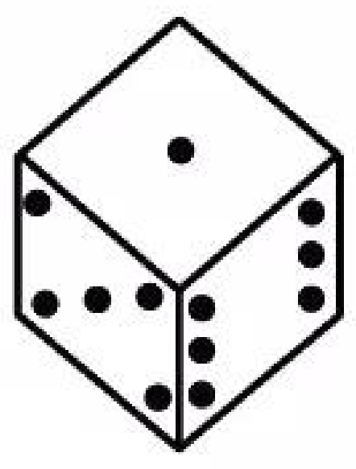


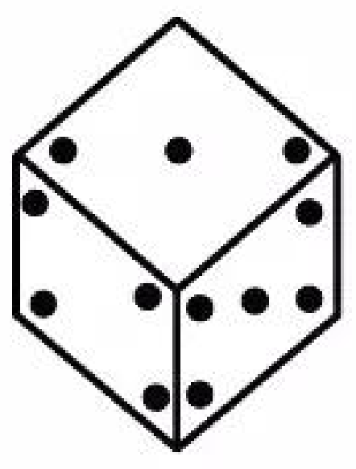
(iii)

(a) 6, 6, 2 (b) 5, 6, 1 (c) 5, 5, 5 (d) 6, 5, 2

17. How many dots lie opposite 2 dots?

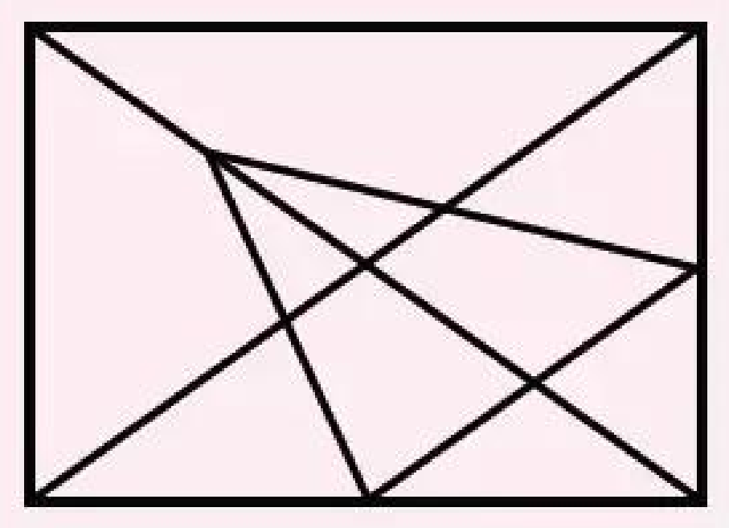

(i)


(ii)


(iii)

(a) 1 (b) 3 (c) 5 (d) 6

18. Find the number of triangles in the given figure.



(a) 15 (b) 17 (c) 18 (d) 21

Directions Q. (19 and 20): Study the following information carefully and answer the questions given below.

I. Sudha weighs more than Bharat and Abhishek.
II. Rahul weighs less than only Karan.
III. Parul weighs as much as Sudha but less than Dana.
IV. Abhishek does not weigh the minimum.

19. Who amongst the following is the third heaviest?
(a) Karan (b) Rahul (c) Bharat (d) Dana

20. Which of the following is true?
(a) Only four people are heavier than Rahul
(b) Bharat weighs more than Parul
(c) No one weighs less than Bharat
(d) Only two people are heavier than Karan

SOLUTIONS

1. (b) : $12 \xrightarrow{\times 2 - 2} 22 \xrightarrow{\times 3 + 3} 69 \xrightarrow{\times 4 - 4} 272 \xrightarrow{\times 5 + 5} 1365 \xrightarrow{\times 6 - 6} \boxed{8184}$

2. (d) : AZ, CX, EV, GT, $\boxed{}$, KP, $\boxed{}$

$A \xrightarrow{+2} C \xrightarrow{+2} E \xrightarrow{+2} G \xrightarrow{+2} \boxed{I} \xrightarrow{+2} K \xrightarrow{+2} \boxed{M}$

Similarly, $Z \xrightarrow{-2} X \xrightarrow{-2} V \xrightarrow{-2} T \xrightarrow{-2} \boxed{R} \xrightarrow{-2} P \xrightarrow{-2} \boxed{N}$

Hence, IR and MN.

3. (b) : The study of cells is called cytology. Similarly the study of insects is called entomology.

4. (c) : The relationship is $x : 3x^2$.

5. (d) : Figure 1 : $(5^2 + 4^2) - (3^2 + 2^2) = 28$
Figure 2 : $? = (6^2 + 7^2) - (3^2 + 4^2) = 60$.

SAMURAI SUDOKU

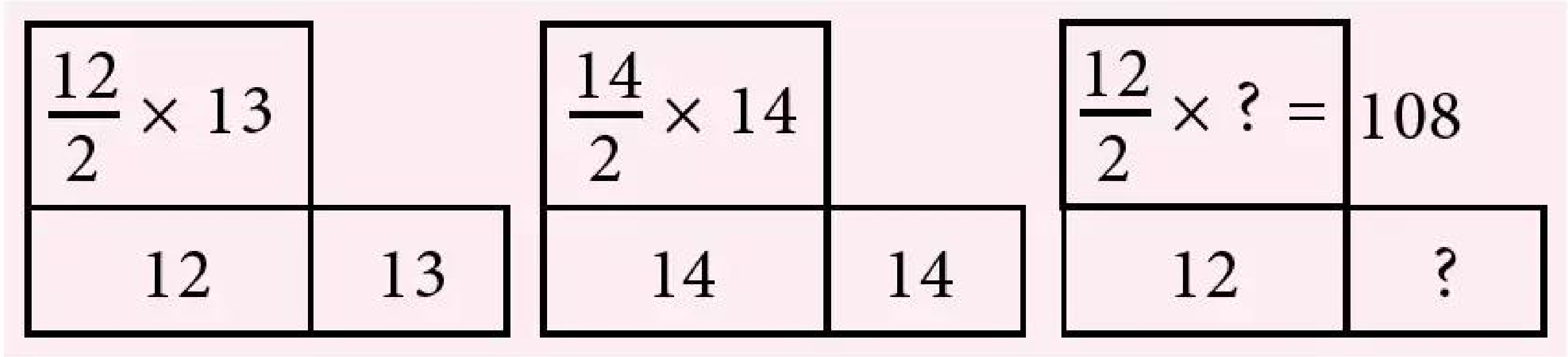
Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique answer.

2					3	7	8					5		8			2	4		
		8	9				6	3				4	7		1			9		
3			8	2		9							2	1		7		5		
8	9			6				2					1		2		9			5
6				3	2			8				2			5	4			1	
	3	2	1		8									4	6		7		2	
	8	1	2			3	7				2		4			2		1	9	
	6		7	8				4		6		1				5				7
		4	3			8	1			9				2			1			4
							9	8	7			5								
								2	4			9	1							
						1		7		5			2							
		6			8	2	4		8		5		9		2	8	7			4
8					1	9		5	2	1		8			5	9		6		
5		1	6				8				9	2			4	6			9	
1	4		7					6				4	6			7	2			
		5			6	1		7						9	6		4		8	
	6	8	1	5								1		5					7	6
3			8			6		4					4					9		2
	1	9		3				8						7			5	3		
	8			6			1							2			6	7		

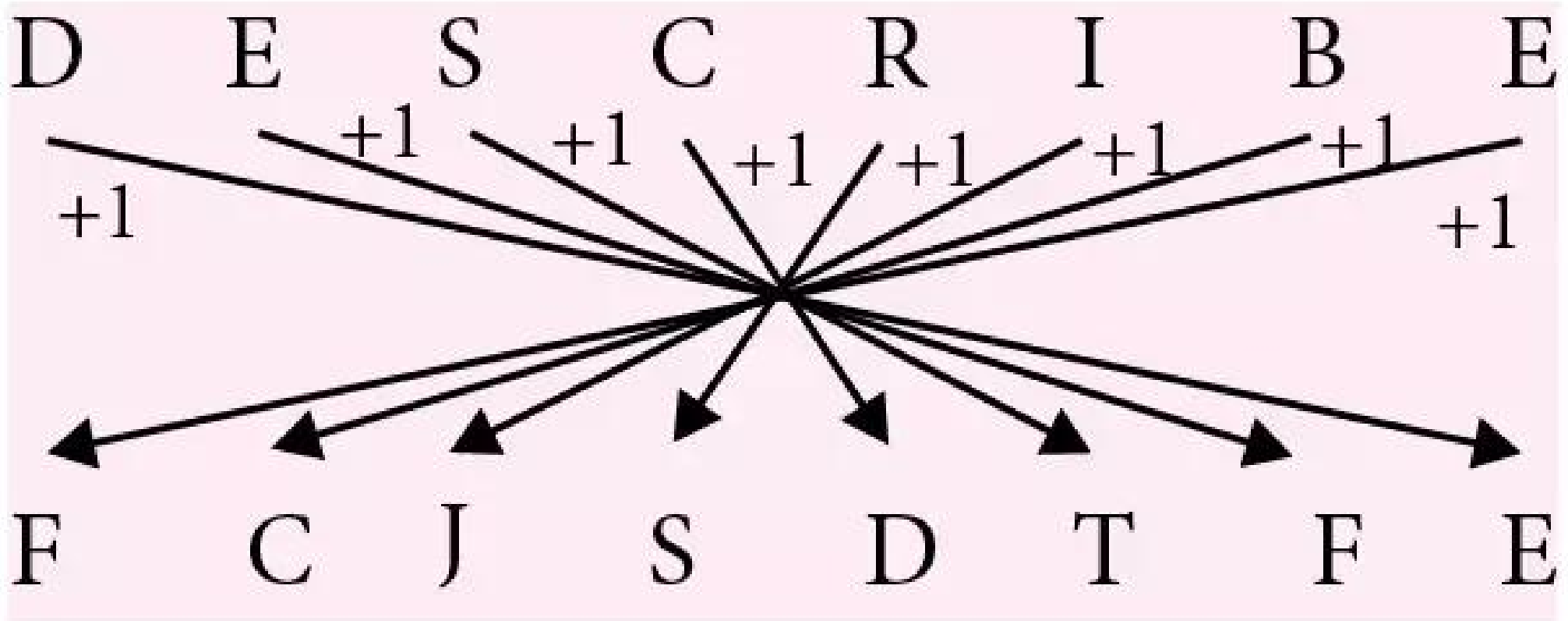
Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

6. (b) :

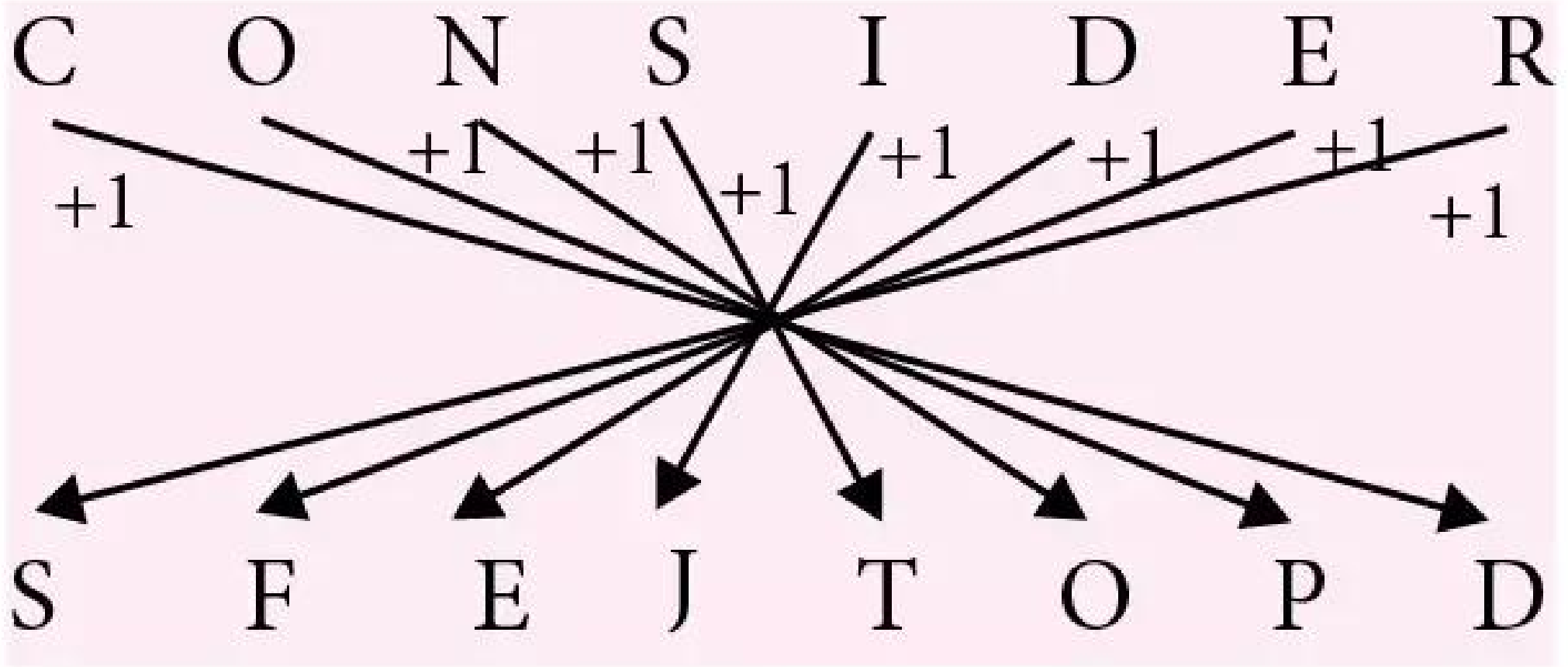

$\frac{12}{2}\times 13$
12
13
$\frac{14}{2}\times 14$
14
14
$\frac{12}{2}\times ? = 108$
12
?

$\therefore$? = 18 .

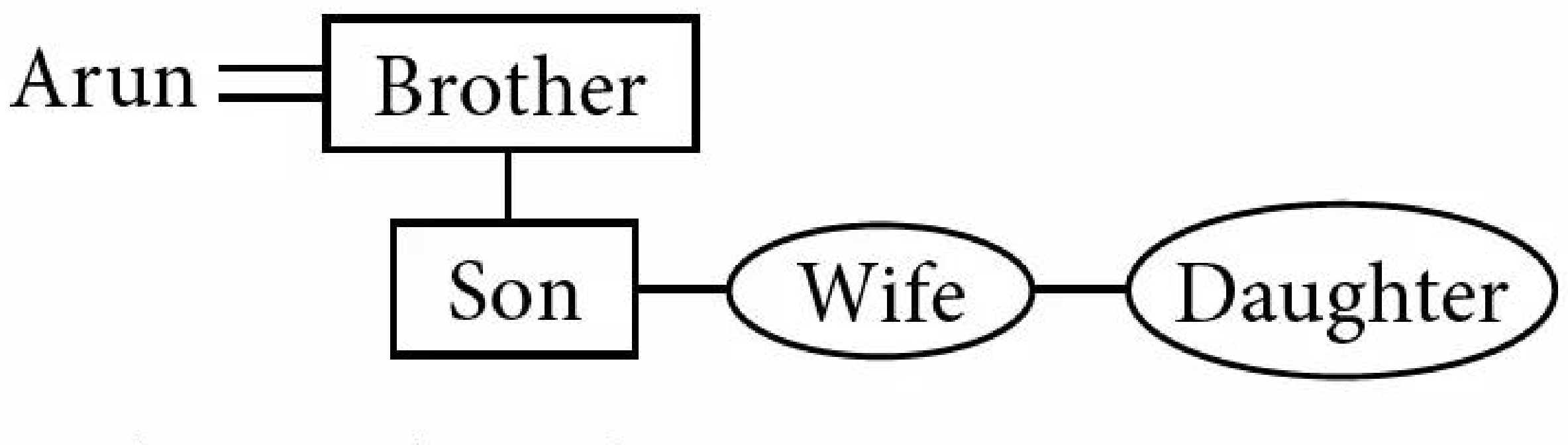
7. (a) :


D E S C R I B E
+1 +1 +1 +1 +1 +1 +1 +1
F C J S D T F E

Similarly,

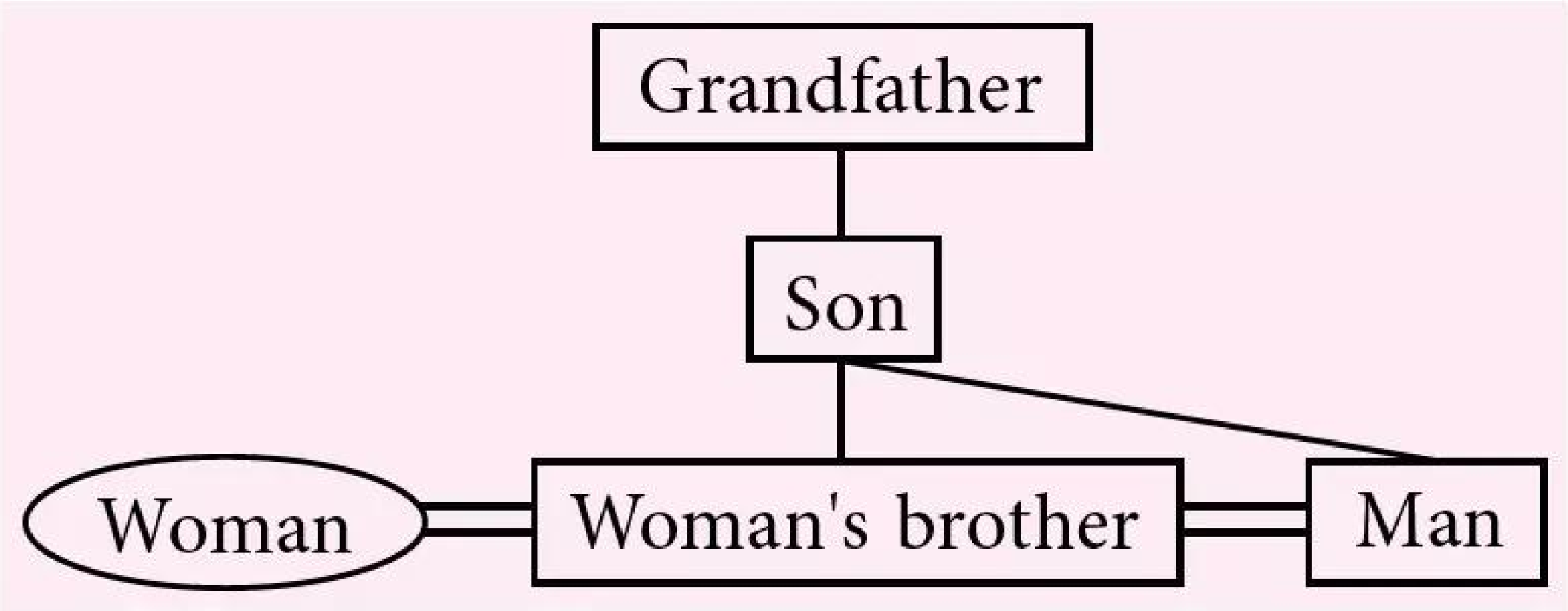

C O N S I D E R
+1 +1 +1 +1 +1 +1 +1 +1
S F E J T O P D

8. (b) :


Arun
Brother
Son
Wife
Daughter

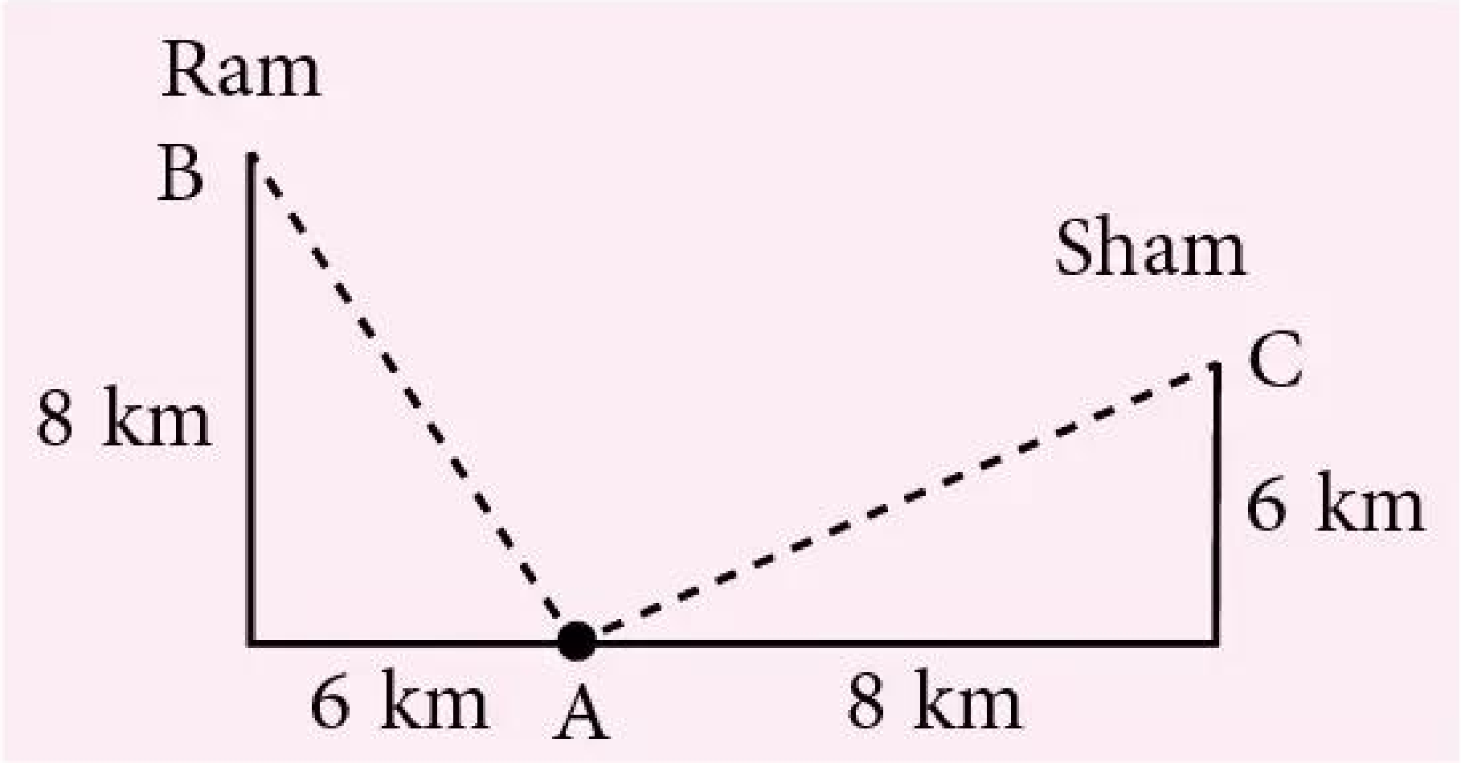
Lady is daughter-in-law of Arun.

9. (a) :


Grandfather
Son
Woman
Woman's brother
Man

The man is brother of that woman.

10. (d) :

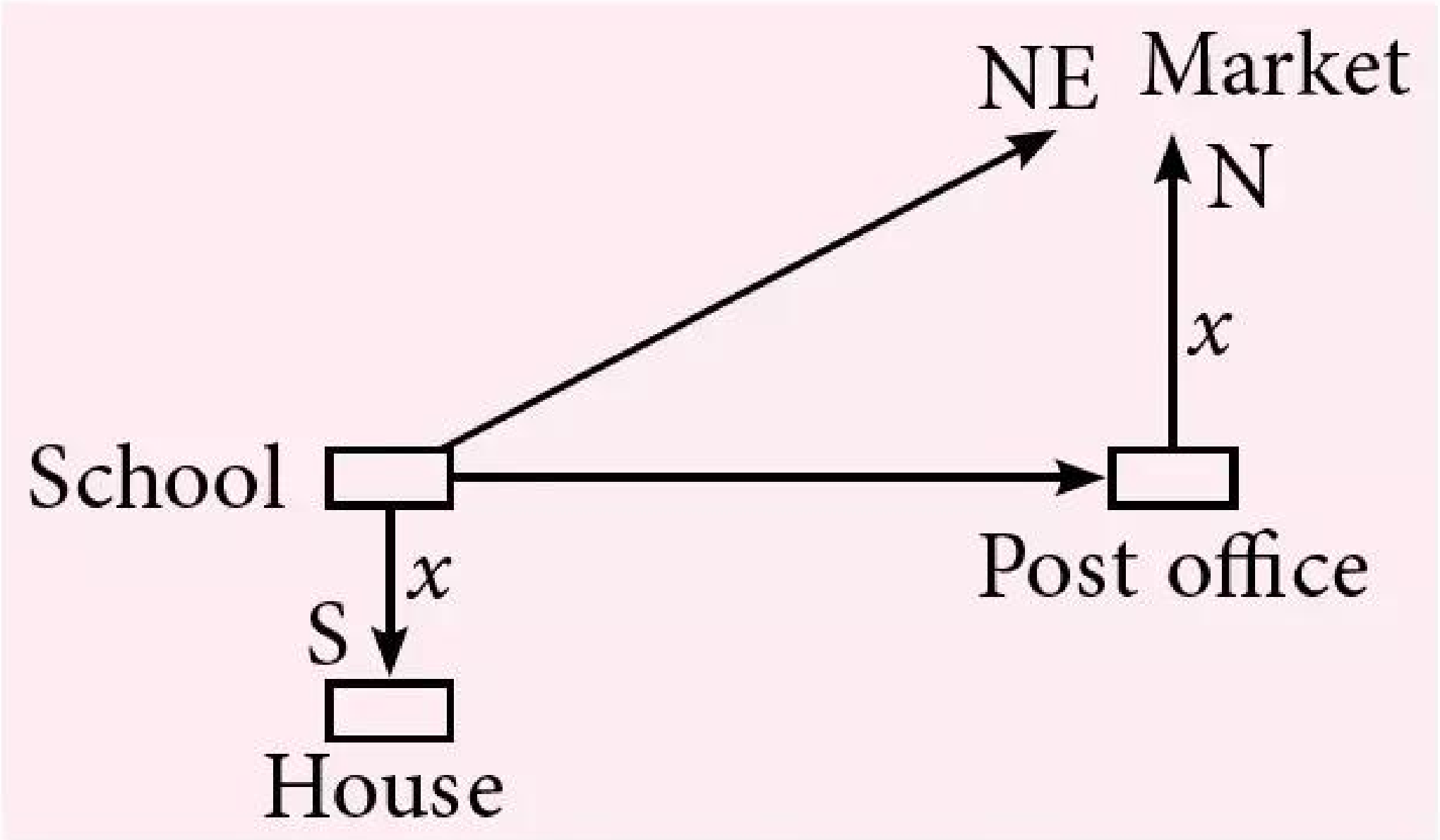

Ram
B
8 km
6 km
A
8 km
Sham
C
6 km

$AB = \sqrt{6^2 + 8^2} = 10$ km

$AC = \sqrt{8^2 + 6^2} = 10$ km.

11. (c) : Distance of house from school = x

Distance of market from post office


NE
Market
N
x
School
Post office
x
S
House

So, direction of market with respect to school = North-east

12. (b) : Statements: $Y > R \geq S = T < M \leq N < J$

$Y > R \geq S = T$

$S = T < M \leq N$

$S = T < M \leq N < J$

$S \leq R < Y$

Conclusions:

I. $T < Y$: True II. $N > S$: True

III. $J < S$: Not true IV. $S \geq Y$: Not true

Hence, only conclusion (I) and (II) are true.

13. (d) : Statements: $S > H \geq T \leq C$; $Y > T \geq N$

$S > H \geq T < Y$

$Y > T \leq C$

$N \leq T \leq C$

Conclusions:

I. $H > Y$: Not true II. $S > C$: Not true

III. $C = Y$: Not true IV. $C \geq N$: True

Hence, only conclusion (IV) is true.

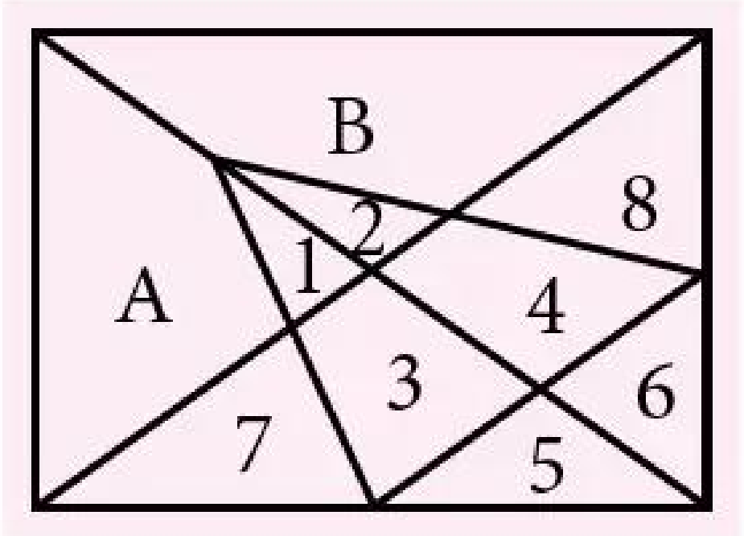
14. (a) : Red is on opposite faces.

15. (a) : All the small cubes on the only edge of the large cube where two yellow faces meet.

16. (c) : From figure (i), (ii) and (iii) 1, 2, 3 and 6 appears on sides of dice and 4 appears on top. So number at bottom on each dice is 5, i.e., 5, 5, 5.

17. (c) : From figure (i) and (ii), 5 lies opposite 2. Hence, only cubes I and III can be formed.

18. (d) :


B
8
A
1
2
4
3
6
7
5

The given figure has 21 triangles as below :
1, 2, 5, 6, 7, 8, 12, 13, 24, 56, 1A, 2B, 135, 246, 357, 468, 12AB, 1357A, 2468B, 345678, 1234.

(19 and 20)

Sudha > Bharat, Abhishek

Karan > Rahul

Dana > Parul = Sudha

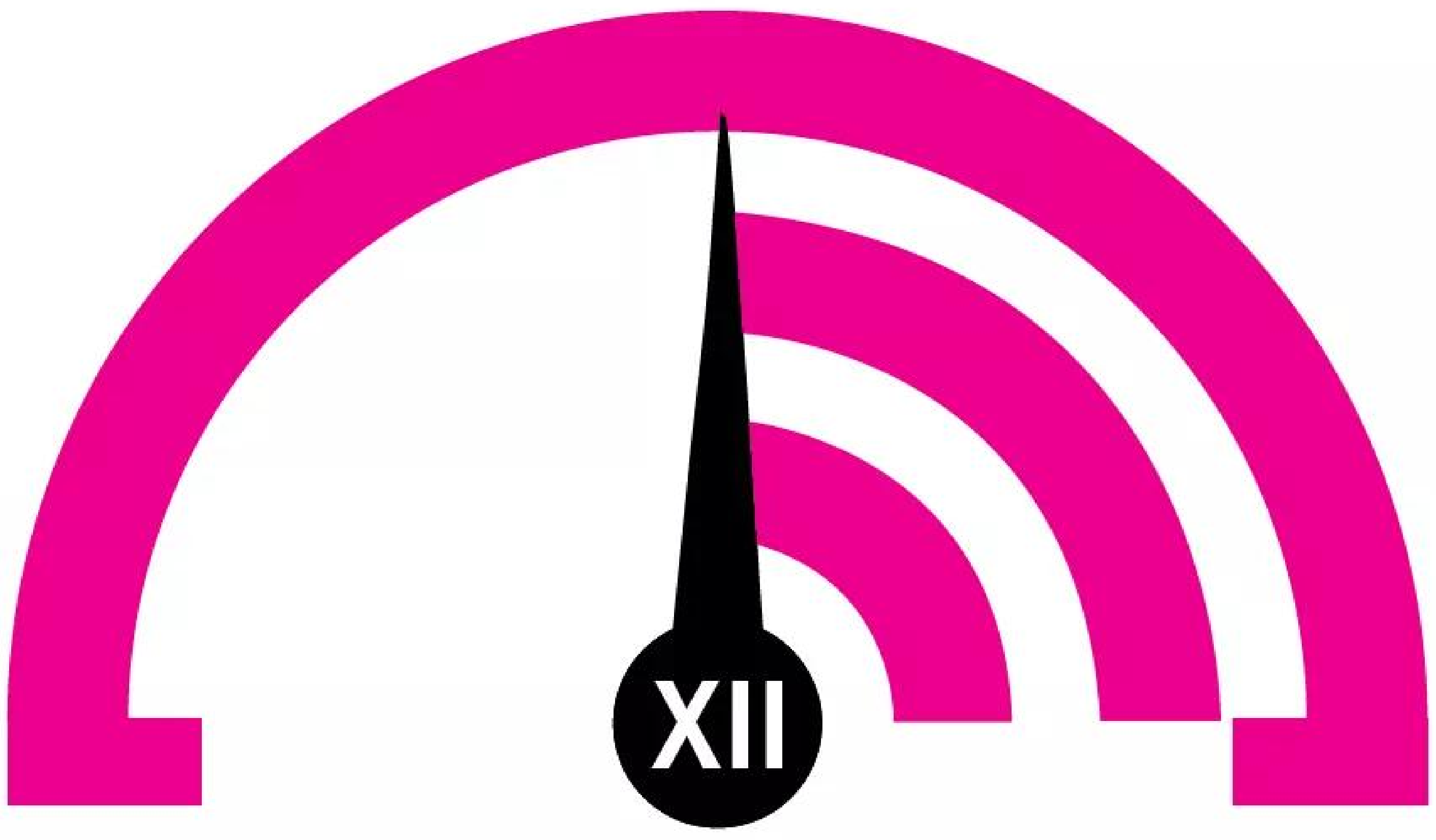
Karan > Rahul > Dana > Parul = Sudha > Abhishek > Bharat

19. (d) : Dana is the third heaviest.

20. (c) : Bharat weighs minimum.

◆ ◆

MONTHLY TEST DRIVE



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 9 : Vector Algebra, Three Dimensional Geometry

Time Taken : 60 Min.

Only One Option Correct Type

1. The direction ratios of a normal to the plane through (1, 0, 0), (0, 1, 0), which makes an angle of $\pi/4$ with the plane $x + y = 3$ are

(a) $<1, \sqrt{2}, 1>$ (b) $<1, 1, \sqrt{2}>$

(c) $<1, 1, 2>$ (d) $<\sqrt{2}, 1, 1>$

2. If $\vec{b}$ and $\vec{c}$ are unit vectors, then for any arbitrary vector $\vec{a}$, $(((\vec{a}\times\vec{b})+(\vec{a}\times\vec{c}))\times(\vec{b}\times\vec{c}))\cdot(\vec{b}-\vec{c})$ is always equal to

(a) $|\vec{a}|$ (b) $\frac{1}{2}|\vec{a}|$ (c) $\frac{1}{3}|\vec{a}|$ (d) zero

3. A line makes angles α, β, γ with the coordinate axes. If $\alpha + \beta = \pi/2$, then $(\cos\alpha + \cos\beta + \cos\gamma)^2$ is equal to

(a) $1 + \sin 2\alpha$ (b) $1 + \cos 2\alpha$

(c) $1 - \sin 2\alpha$ (d) none of these

4. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ be three non-zero vectors such that $|\vec{c}| = 1$, angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$ and $\vec{c}$ is perpendicular to $\vec{a}$ and $\vec{b}$, then

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}^2 = \lambda(a_1^2 + a_2^2 + a_3^2)\,(b_1^2 + b_2^2 + b_3^2),$$

where λ is equal to

(a) 1/2 (b) 1/4 (c) 1 (d) 2

5. $\vec{a}$ and $\vec{b}$ are two non-collinear unit vectors. If $\vec{\mu}_1 = \vec{a} - (\vec{a}\cdot\vec{b})\vec{b}$ and $\vec{\mu}_2 = \vec{a}\times\vec{b}$, then $|\vec{\mu}_2|$ is always equal to

(a) $|\vec{\mu}_1| + |\vec{\mu}_1\cdot\vec{a}|$ (b) $|\vec{\mu}_1| + |\vec{\mu}_1\cdot\vec{b}|$

(c) $|\vec{\mu}_1\cdot(\vec{a}+\vec{b})|$ (d) $|\vec{\mu}_1\cdot(\vec{a}-\vec{b})|$

6. The length of projection of the line segment joining the points (1, 0, –1) and (–1, 2, 2) on the plane $x + 3y - 5z = 6$ is equal to

(a) 2 (b) $\sqrt{\frac{271}{53}}$ (c) $\sqrt{\frac{472}{31}}$ (d) $\sqrt{\frac{474}{35}}$

One or More Than One Option(s) Correct Type

7. Let $A_1, A_2, \ldots, A_n$ $(n > 2)$ be the vertices of a regular polygon of n sides with its centre at the origin. Let $\vec{a}_k$ be the position vector of the point A_k, $k = 1, 2, \ldots, n$. If $\left|\sum_{k=1}^{n-1}(\vec{a}_k\times\vec{a}_{k+1})\right| = \left|\sum_{k=1}^{n-1}(\vec{a}_k\cdot\vec{a}_{k+1})\right|$, then the minimum value of n is

(a) 1 (b) 2 (c) 8 (d) 9

8. Let P_1 denote the equation of the plane to which the vector $(\hat{j} + \hat{i})$ is normal and which contains the line L whose equation is $\vec{r} = \hat{i} + \hat{j} + \hat{k} + \lambda(\hat{i} - \hat{j} - \hat{k})$. Let P_2 denote the equation of the plane containing the line L and a point with position vectors $\hat{j}$. Which of the following holds goods?

(a) Equation of P_1 is $x + y = 2$

(b) Equation of P_2 is $\vec{r}\cdot\left(\hat{i} - 2\hat{j} + \hat{k}\right) = 2$.

(c) The acute angle between P_1 and P_2 is $\cot^{-1}\left(\sqrt{3}\right)$.

(d) The angle between the plane P_2 and the line L is $\tan^{-1}\sqrt{3}$.

9. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - \hat{j}$, then the vectors $(\vec{a}\cdot\hat{i})\hat{i} + (\vec{a}\cdot\hat{j})\hat{j} + (\vec{a}\cdot\hat{k})\hat{k}$, $(\vec{b}\cdot\hat{i})\hat{i} + (\vec{b}\cdot\hat{j})\hat{j} + (\vec{b}\cdot\hat{k})\hat{k}$, and $\hat{i} + \hat{j} - 2\hat{k}$

(a) are mutually perpendicular

(b) are coplanar

(c) form a parallelopiped of volume 6 units

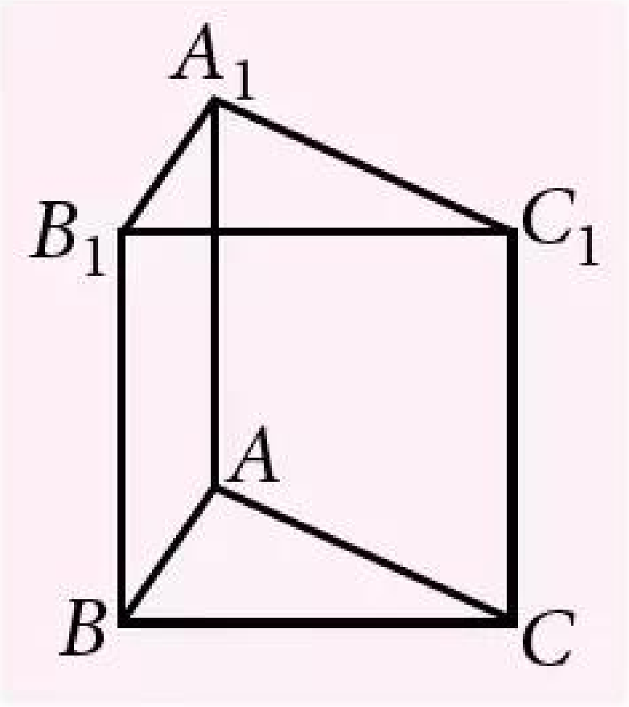
(d) form a parallelopiped of volume 3 units

10. The projection of the line $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-3}{4}$ on the plane $x+2y+z=9$ is the line of intersection of which of the following planes?

(a) $x+2y+z-9=0$ (b) $9x+2y+3z-4=0$

(c) $x-2y+z+3=0$ (d) $9x-2y-5z+4=0$

11. The volume of a right triangular prism $ABCA_1B_1C_1$ is equal to 3. Find the co-ordinates of the vertex A_1, if the co-ordinates of the base vertices of the prism are $A(1, 0, 1)$, $B(2, 0, 0)$ and $C(0, 1, 0)$.



(a) (–2, 0, 2) (b) (0, –2, 0)

(c) (0, 2, 0) (d) (2, 2, 2)

12. The equations of two lines through the origin which intersect the line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ at an angle 60° is

(a) $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1}$ (b) $\frac{x}{1}=\frac{y}{-1}=\frac{z}{2}$

(c) $\frac{x}{2}=\frac{y}{1}=\frac{z}{-1}$ (d) $\frac{x}{2}=\frac{y}{-1}=\frac{z}{1}$

13. The line of intersection of the planes $\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})=0$ and $\vec{r}\cdot(3\hat{i}+2\hat{j}+\hat{k})=0$ is

(a) equally inclined to $\hat{i}$ and $\hat{k}$

(b) equally inclined to $\hat{i}$ and $\hat{j}$

(c) inclined at an angle $\cos^{-1}\sqrt{\frac{2}{3}}$ with $\hat{j}$

(d) inclined at an angle $\cos^{-1}\left(\frac{-1}{\sqrt{6}}\right)$ with $\hat{i}$

Comprehension Type

Paragraph for Q. No. 14 and 15

A ray of light comes along the line $L=0$ and strikes the plane mirror kept along the plane $P=0$ at B. $A(2, 1, 6)$ is a point on the line $L=0$ whose image about $P=0$ is A'. It is given that $L=0$ is $\frac{x-2}{3}=\frac{y-1}{4}=\frac{z-6}{5}$ and $P=0$ is $x+y-2z=3$.

14. The coordinates of A' are

(a) (6, 5, 2) (b) (6, 5, –2)

(c) (6, –5, 2) (d) none of these

15. The coordinates of B are

(a) (5, 10, 6) (b) (10, 15, 11)

(c) (–10, –15, –14) (d) none of these

Matrix Match Type

16. Match the following columns:

Column-I		Column-II	
P.	If $\lvert\vec{a}\rvert=\lvert\vec{b}\rvert=\lvert\vec{c}\rvert$ such that angle between each pair of vectors is $\pi/3$ and $\lvert\vec{a}+\vec{b}+\vec{c}\rvert=\sqrt{6}$, then $2\lvert\vec{a}\rvert=$	1.	3
Q.	If $\vec{a}$ is perpendicular to $\vec{b}+\vec{c}$, $\vec{b}$ is perpendicular to $\vec{c}+\vec{a}$, $\vec{c}$ is perpendicular to $\vec{a}+\vec{b}$, $\lvert\vec{a}\rvert=2$, $\lvert\vec{b}\rvert=3$ and $\lvert\vec{c}\rvert=6$, then $\lvert\vec{a}+\vec{b}+\vec{c}\rvert-2=$	2.	2
R.	If $\vec{a}=2\hat{i}+3\hat{j}-\hat{k}$, $\vec{b}=-\hat{i}+2\hat{j}-4\hat{k}$, $\vec{c}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{d}=3\hat{i}+2\hat{j}+\hat{k}$, then $\frac{1}{7}(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d})=$	3.	4
S.	If $\lvert\vec{a}\rvert=\lvert\vec{b}\rvert=\lvert\vec{c}\rvert=2$ and $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=2$, then $[\vec{a}\ \vec{b}\ \vec{c}]\cos 45°$ is equal to	4.	5

	P	Q	R	S		P	Q	R	S
(a)	2	3	4	1	(b)	3	1	2	4
(c)	2	4	1	3	(d)	3	1	4	2

Numerical Answer Type

17. The distance of the point $\hat{i}+2\hat{j}+3\hat{k}$ from the plane $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=5$ measured parallel to the vectors $2\hat{i}+3\hat{j}-6\hat{k}$ must be ______.

18. The distance of the point (3, 0, 5) from the line $x-2y+2z-4=0=x+3z-11$ is ______.

19. If the lines $x-y+z=-1=3x-y+az+1$ and $x+4y-z-1=0=x+2y+bz+1$ are perpendicular, then $3ab+a-b+c=0$, where c is ______.

20. Let $\vec{p}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{q}=\hat{i}+2\hat{j}-\hat{k}$ and $\vec{r}=\hat{i}+\hat{j}-2\hat{k}$. If $\vec{V}=\vec{p}+\lambda\vec{q}$ and projection of $\vec{V}$ on $\vec{r}$ is $\frac{4}{\sqrt{6}}$, then value of λ is equal to ______.

◆◆

Keys are published in this issue. Search now! ☺

ELF CHECK

No. of questions attempted ……

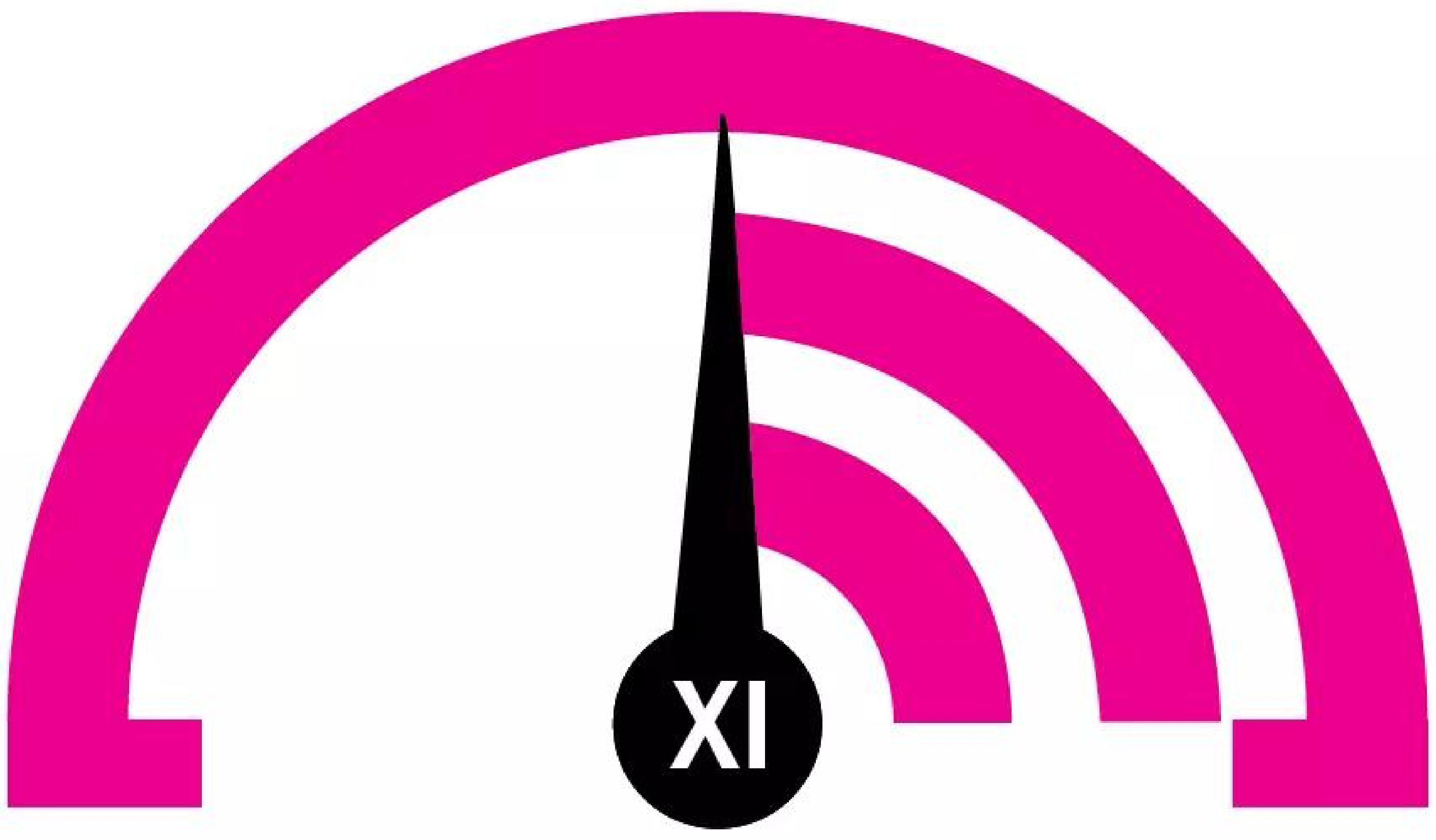
No. of questions correct ……

Marks scored in percentage ……

Check your score! If your score is

> 90%	**EXCELLENT WORK !**	You are well prepared to take the challenge of final exam.
90-75%	**GOOD WORK !**	You can score good in the final exam.
74-60%	**SATISFACTORY !**	You need to score more next time.
< 60%	**NOT SATISFACTORY!**	Revise thoroughly and strengthen your concepts.

MONTHLY TEST DRIVE



This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 9 : Conic Sections

Time Taken : 60 Min.

Only One Option Correct Type

1. The set of points on the axis of the parabola $y^2 = 4x + 8$ from which the 3 normals to the parabola are all real and different is

(a) $\{(k, 0) \mid k \le -2\}$ (b) $\{(k, 0) \mid k > -2\}$
(c) $\{(0, k) \mid k > -2\}$ (d) None of these

2. PA an PB are the tangents drawn to $y^2 = 4x$ from point P. These tangents meet the y-axis at the points A_1 and B_1 respectively. If the area of triangle PA_1B_1 is 2 sq. units, then locus of P is

(a) $(y^2 + 2x)\,x^2 = 8$ (b) $(y^2 + 4x)\,x^2 = 16$
(c) $(y^2 - 4x)\,x^2 = 8$ (d) $(y^2 - 4x)\,x^2 = 16$

3. The equation of the image of the circle $(x - 3)^2 + (y - 2)^2 = 1$ by the mirror $x + y = 19$ is :

(a) $(x - 14)^2 + (y - 13)^2 = 1$
(b) $(x - 15)^2 + (y - 14)^2 = 1$
(c) $(x - 16)^2 + (y - 15)^2 = 1$
(d) $(x - 17)^2 + (y - 16)^2 = 1$

4. PA and PB are tangents drawn to $y^2 = 4ax$ from arbitrary point P. If the angle between tangents is $\frac{\pi}{4}$, then locus of point P is

(a) $y^2 = x^2 + a^2 + 6ax$ (b) $y^2 = x^2 - a^2 + 6ax$
(c) $y^2 = x^2 + a^2 - 6ax$ (d) $y^2 = x^2 - a^2 - 6ax$

5. The locus of the centre of the circles passing through (1, 1) and cutting $x^2 + y^2 = 4$ orthogonally is

(a) $x + y = 3$ (b) $x + 2y = 3$
(c) $2x + y = 3$ (d) $2x - y = 3$

6. Area of the triangle formed by the lines $x - y = 0$, $x + y = 0$ and any tangent to the hyperbola $x^2 - y^2 = a^2$ is

(a) $|a|$ (b) $\frac{1}{2}|a|$
(c) a^2 (d) $\frac{1}{2}a^2$

One or More Than One Option(s) Correct Type

7. The points (2, 3), (0, 2), (4, 5) and (0, t) are concyclic, if the value of t is

(a) 2 (b) 1
(c) 17 (d) 19

8. The centre of the circle(s) passing through the points (0, 0), (1, 0) and touching the circle $x^2 + y^2 = 9$, is (are)

(a) $\left(\frac{3}{2}, \frac{1}{2}\right)$ (b) $\left(\frac{1}{2}, \frac{3}{2}\right)$
(c) $\left(\frac{1}{2}, 2^{1/2}\right)$ (d) $\left(\frac{1}{2}, -2^{1/2}\right)$

EXAM ALERT 2023

Exam	Date
J & K CET	4th April
JEE Main Session 2	6th, 8th, 10th, 11th & 12th April
WB JEE	30th April
MHT CET PCM	9th to 13th May
KCET BM	20th May
KCET PC	21st May
CUET	21st to 31st May
KCET Language Test	22nd May
BITSAT Session I	22nd to 26th May
JEE Advanced	4th June
BITSAT Session II	18th to 22nd June

9. The equations of the common tangents to the parabola $y = x^2$ and $y = -(x-2)^2$ is/are

(a) $y = 4(x-1)$ (b) $y = 0$

(c) $y = -4(x-1)$ (d) $y = -30x - 50$

10. The equation of the directrix of the parabola with vertex at the origin and having the axis along the x-axis and a common tangent of slope of 2 with the circle $x^2 + y^2 = 5$ is/are

(a) $x = 10$ (b) $x = 20$ (c) $x = -10$ (d) $x = -20$

11. $\dfrac{x^2}{r^2 - r - 6} + \dfrac{y^2}{r^2 - 6r + 5} = 1$ will represents the ellipse, if r lies in the interval

(a) $(-\infty, -2)$ (b) $(3, \infty)$

(c) $(5, \infty)$ (d) $(1, \infty)$

12. The distance of a point on the ellipse $\dfrac{x^2}{6} + \dfrac{y^2}{2} = 1$ from the centre is 2 unit. Then the eccentric angle of the point is

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{3\pi}{4}$ (c) $\dfrac{5\pi}{6}$ (d) $\dfrac{\pi}{6}$

13. Circles are drawn on chords of the rectangular hyperbola $xy = 4$ parallel to the line $y = x$ as diameters. All such circles pass through two fixed points whose co-ordinates are

(a) $(2, 2)$ (b) $(2, -2)$ (c) $(-2, 2)$ (d) $(-2, -2)$

Comprehension Type

Paragraph for Q. No. 14 and 15

Let P be a variable point on the ellipse $x^2 + 2y^2 = 2$ and S, S_1 be the foci.

14. The locus of the centroid of triangle PSS_1 is a conic of eccentricity $e =$

(a) $\dfrac{1}{2}$ (b) $\sqrt{\dfrac{3}{2}}$

(c) $\dfrac{1}{\sqrt{2}}$ (d) $\sqrt{2} - 1$

15. The locus of the incentre I of triangle PSS_1 is a conic of eccentricity $e =$

(a) $\sqrt{2} - 1$ (b) $\sqrt{\sqrt{2} - 1}$

(c) $\sqrt{2\sqrt{2} - 2}$ (d) $\dfrac{1}{\sqrt{2}}$

Matrix Match Type

16. Let $A(t_1)$ and $B(t_2)$ be points on the parabola $y^2 = 4ax$ with vertex O. Then find the value of $t_1t_2 + 2$ if

	Column-I		Column-II
(P)	The tangents at A and B intersect on the directrix	(1)	1
(Q)	The normal at A and B intersect on the parabola	(2)	–2
(R)	$\angle AOB = \dfrac{\pi}{2}$	(3)	2
(S)	AB is a focal chord	(4)	4

	P	Q	R	S
(a)	1	4	2	1
(b)	2	2	4	3
(c)	3	4	1	2
(d)	1	2	3	4

Numerical Answer Type

17. A circle of centre P touches externally both the circles $x^2 + y^2 - 4x + 3 = 0$ and $x^2 + y^2 - 6y + 5 = 0$. The locus of P is $\dfrac{3}{4}x^2 - 3xy + 2y^2 = \lambda(y - x)$, where λ is ________.

18. The length of a common tangent to the two hyperbolas $x^2 - 3y^2 = 3$ and $y^2 - 3x^2 = 3$ is ________.

19. The normal to the hyperbola $x^2 - 2y^2 = 2$ meets the axes at M and N, and lines MP and NP are drawn at right angles to the axes. The locus of P is $2x^2 - y^2 = \lambda$, where λ is ________.

20. A triangle is formed by the lines $x + y = 1$ and the two common tangents of the circles $x^2 + y^2 - 2x = 0$ and $x^2 + y^2 - 4x - 2y + 1 = 0$. Its area is ________ sq. units.

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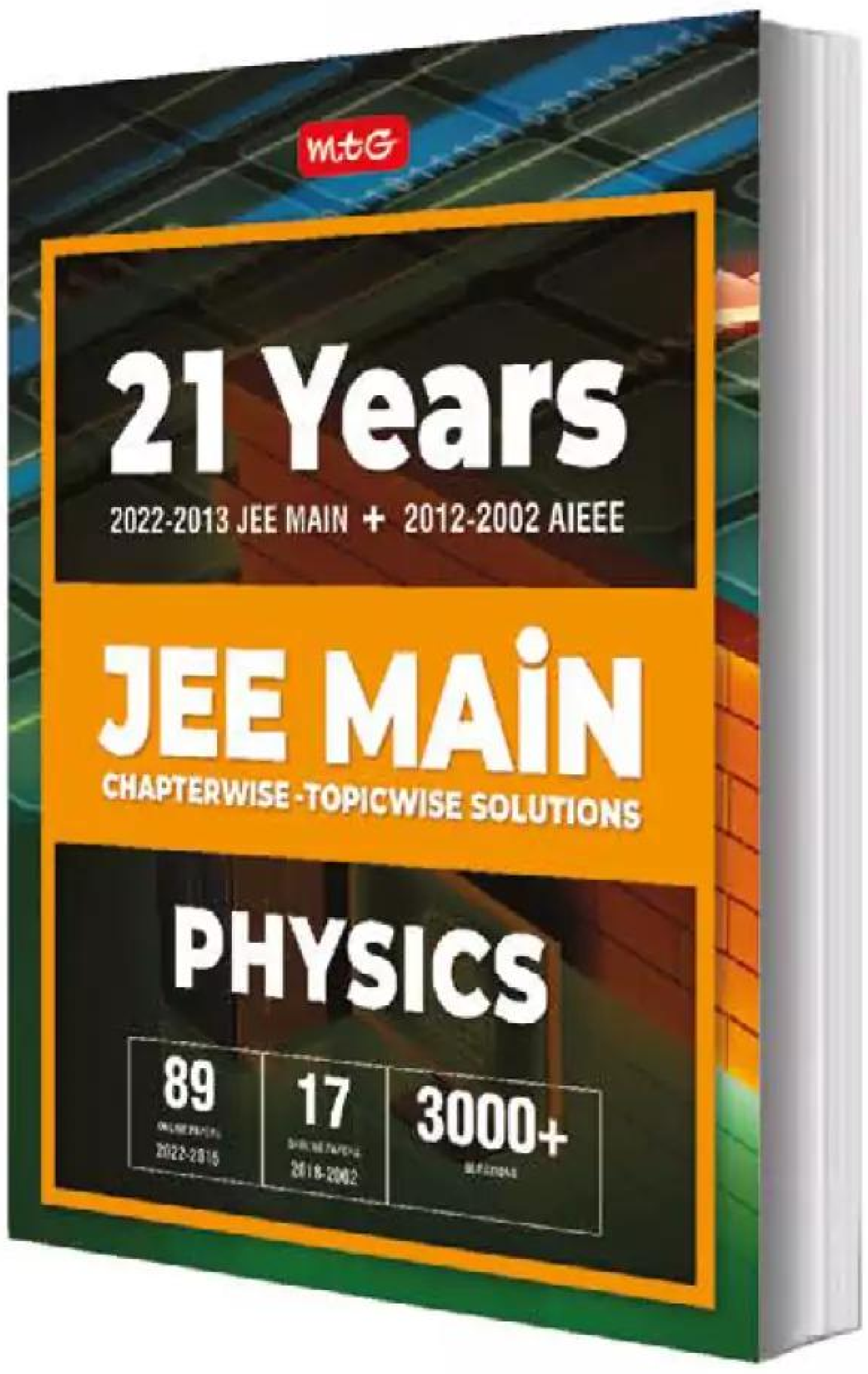
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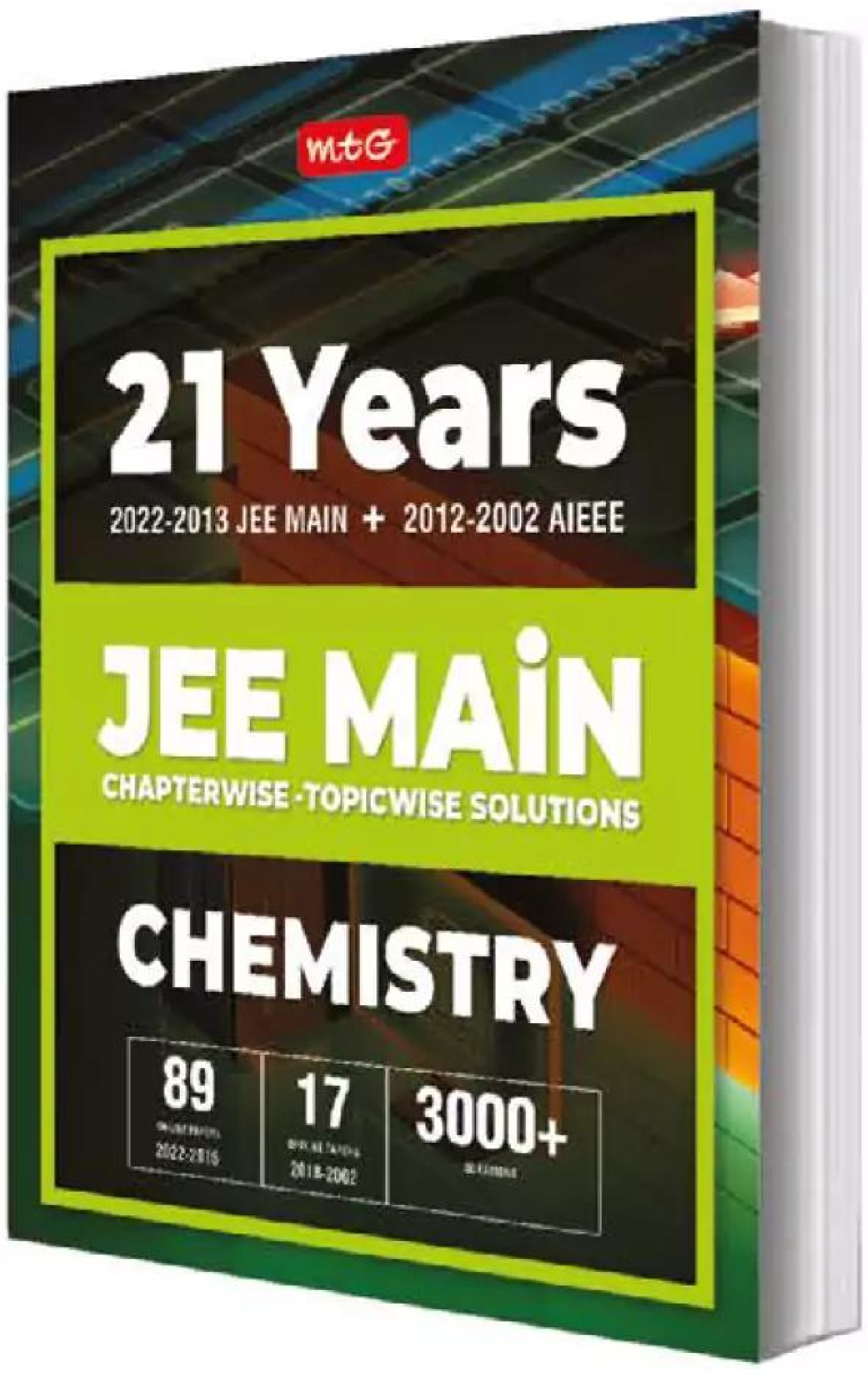
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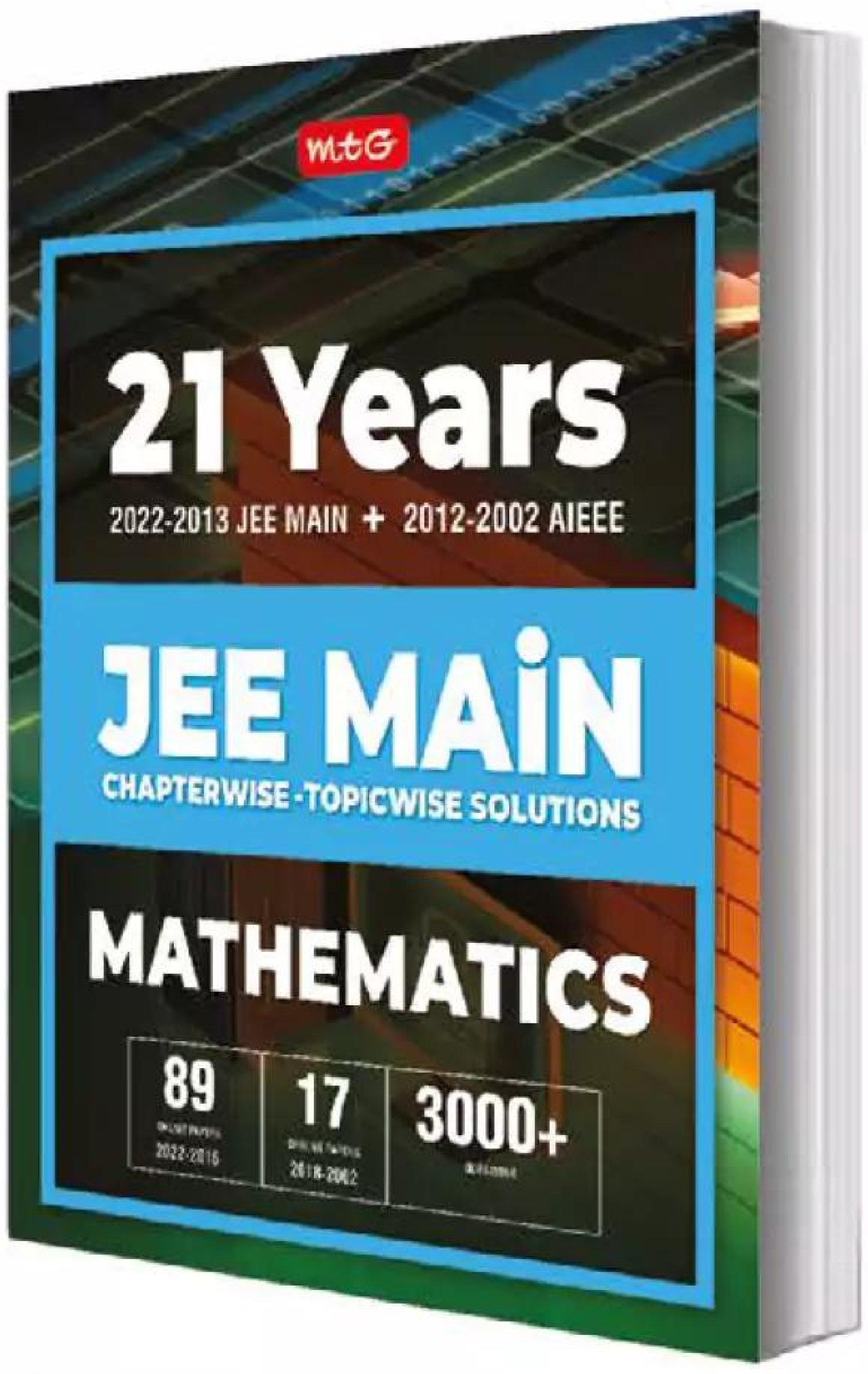
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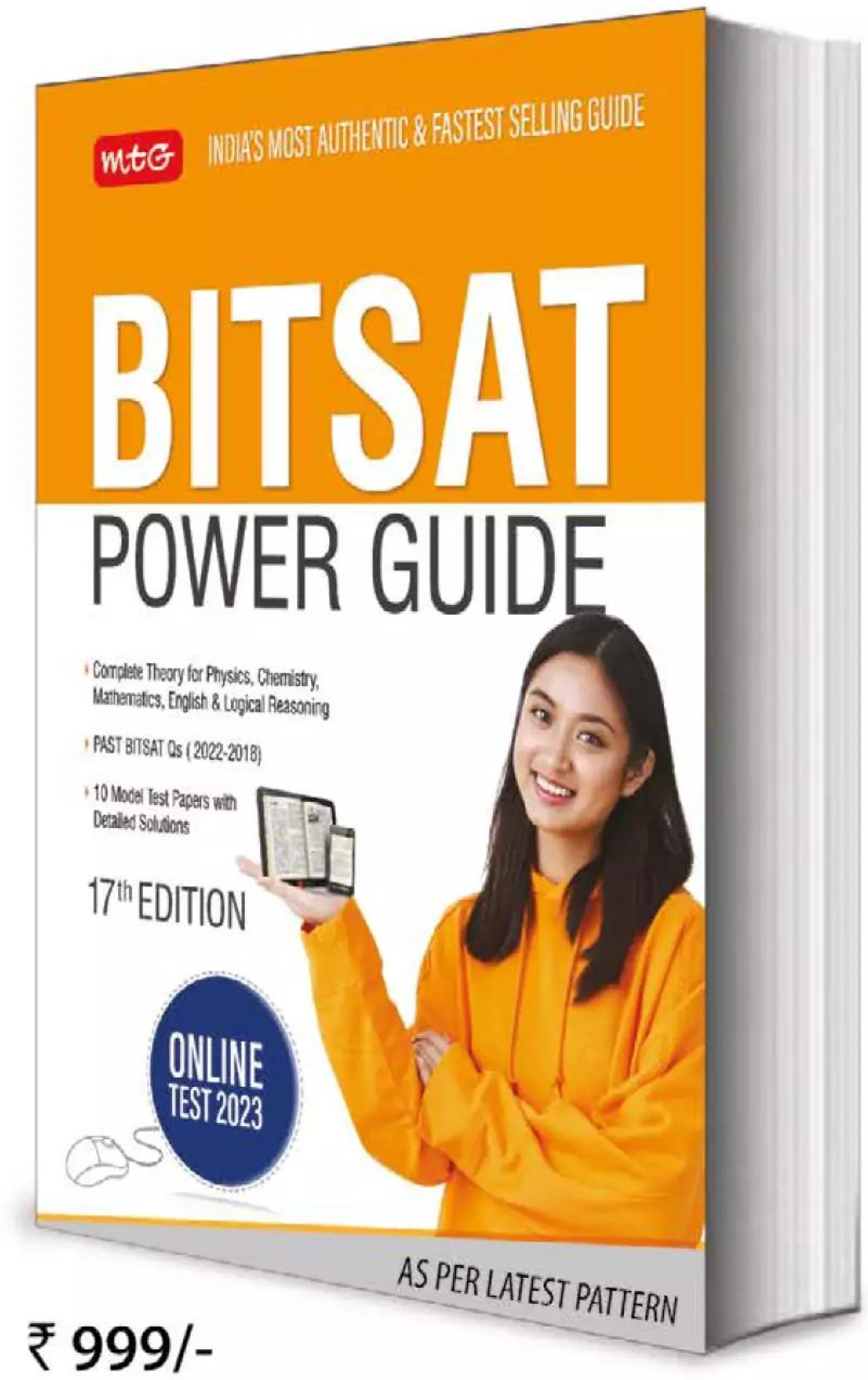
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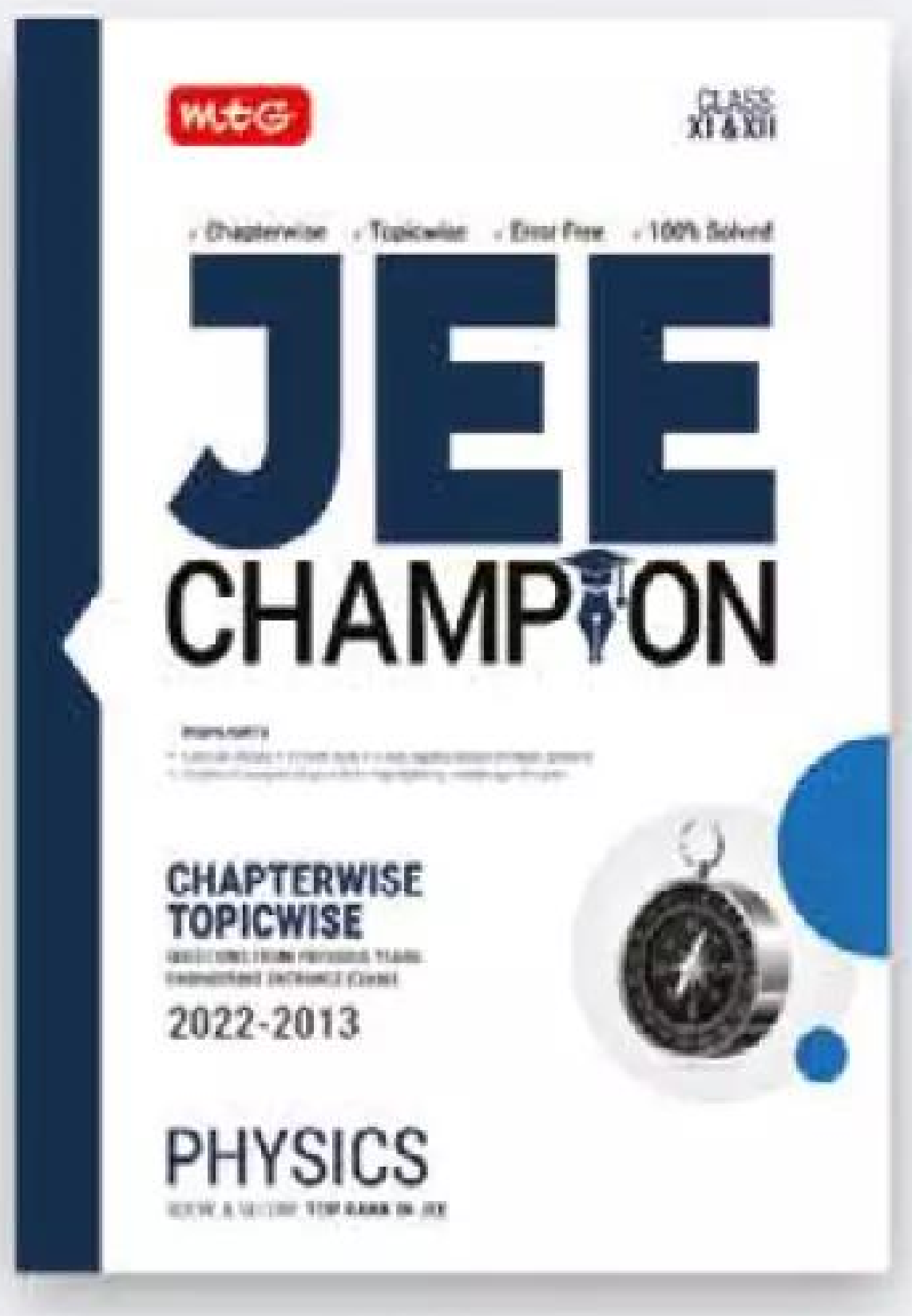

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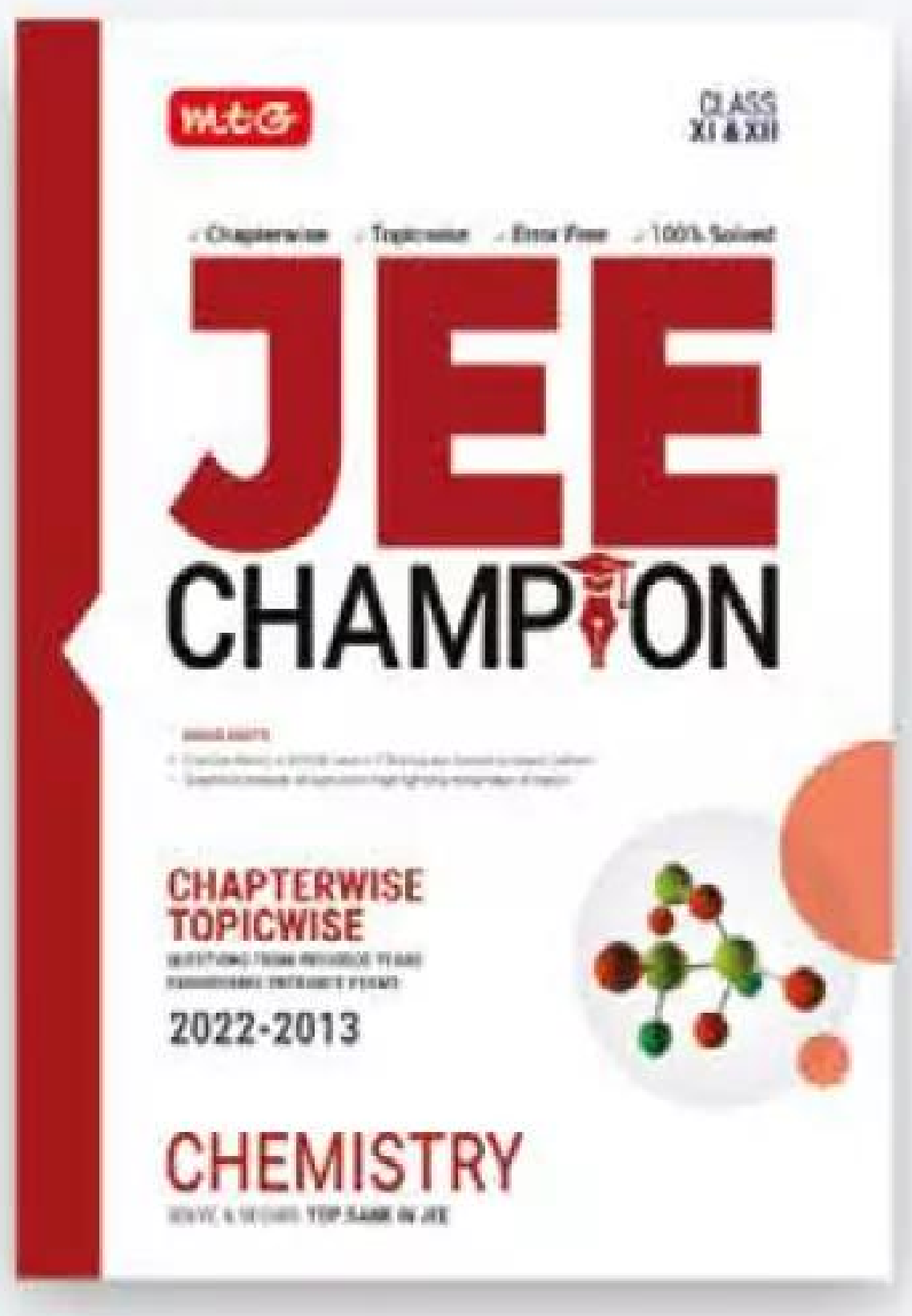

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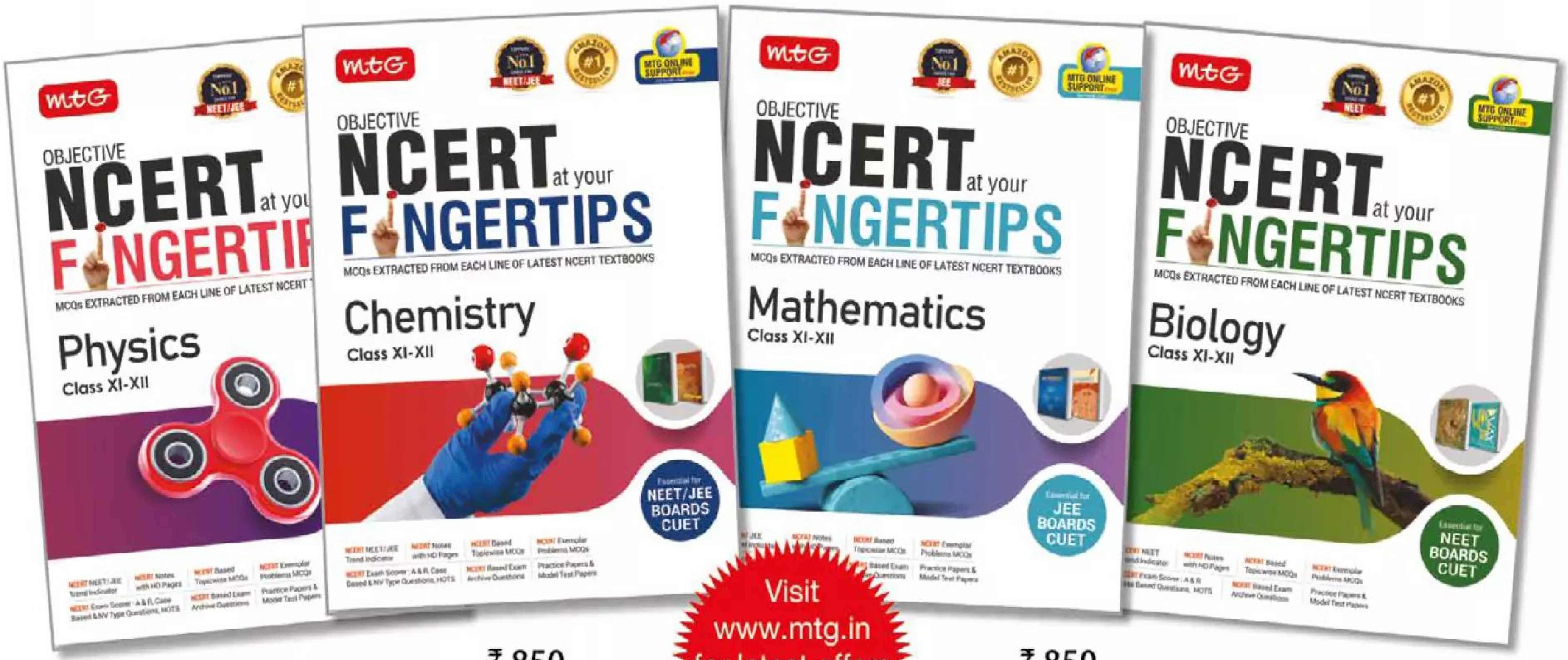

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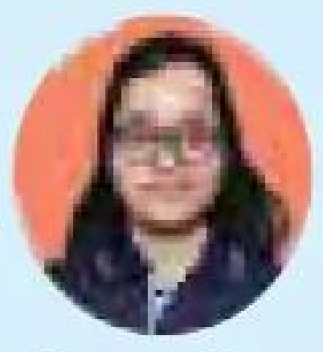
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